JEE Main Online Exam 2020

Question with Solutions

3rd September 2020 | Shift-II

MATHEMATICS

Q.1 Let e_1 and e_2 be the eccentricities of the ellipse, $\frac{x^2}{25}+\frac{y^2}{b^2}=1$ (b < 5) and the hyperbola, $\frac{x^2}{16}-\frac{y^2}{b^2}=1$ respectively satisfying $e_1e_2 = 1$. If α and β are the distances between the foci of the ellipse and the foci of the hyperbola respectively, then the ordered pair (α, β) is equal to -

(1) $\left(\frac{24}{5},10\right)$ (2) (8, 12) (3) $\left(\frac{20}{3},12\right)$ (4) (8, 10)

Ans. [4]

Sol. For ellipse $\frac{x^2}{25}+\frac{y^2}{b^2}=1$ (b < 5)

Let e_1 is eccentricity of ellipse

$\therefore\ b^2 = 25(1-e_1^2)$ ....(1)

Again for hyperbola

$$\frac{x^2}{16}-\frac{y^2}{b^2}=1$$

Let e_2 is eccentricity of hyperbola.

$\therefore\ b^2 = 16(e_2^2-1)$ ....(2)

by (1) & (2)

$25(1-e_1^2) = 16(e_2^2-1)$

Now $e_1 . e_2 = 1$ (given)

$$\therefore\ 25(1-e_1^2) = 16\left(\frac{1-e_1^2}{e_1^2}\right)$$

or $e_1 = \frac{4}{5}$ $\therefore$ $e_2 = \frac{5}{4}$

Now distance between foci is 2ae

$\therefore$ distance for ellipse $= 2\times5\times\frac{4}{5} = 8 = \alpha$

distance for hyperbola $= 2\times4\times\frac{5}{4} = 10 = \beta$

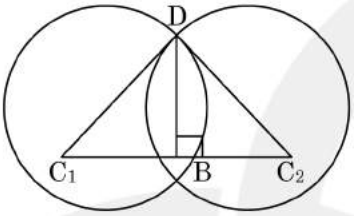
$\therefore\ (\alpha, \beta) \equiv (8, 10)$

Q.2 Let the latus rectum of the parabola $y^2 = 4x$ be the common chord to the circles C_1 and C_2 each of them having radius $2\sqrt{5}$. Then, the distance between the centres of the circles C_1 and C_2 is -

(1) $4\sqrt{5}$ (2) 8 (3) $8\sqrt{5}$ (4) 12

Ans. [2]

Sol. Length of latus rectum = 4



DB = 2

$C_1B = \sqrt{(C_1D)^2 - (DB)^2} = 4$

$C_1C_2 = 8$

Q.3 If the value of the integral $\int_0^{1/2} \frac{x^2}{(1-x^2)^{3/2}}dx$ is $\frac{k}{6}$, then k is equal to -

(1) $2\sqrt{3} + \pi$ (2) $3\sqrt{2} - \pi$ (3) $3\sqrt{2} + \pi$ (4) $2\sqrt{3} - \pi$

Ans. [4]

Sol. $\int_0^{1/2} \frac{((x^2-1)+1)}{(1-x^2)^{3/2}}dx$

$\int_0^{1/2} \frac{dx}{(1-x^2)^{3/2}} - \int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$

$\int_0^{1/2} \frac{x^{-3}}{(x^{-2}-1)^{3/2}} dx - (\sin^{-1} x)_0^{1/2}$

Let $x^{-2} - 1 = t^2 \Rightarrow x^{-3}\,dx = -t\,dt$

$\int_\infty^{\sqrt{3}} \frac{-t\,dt}{t^3} - \frac{\pi}{6} = \int_{\sqrt{3}}^{\infty} \frac{dt}{t^2} - \frac{\pi}{6} = \frac{1}{\sqrt{3}} - \frac{\pi}{6} = \frac{k}{6}$

$k = 2\sqrt{3} - \pi$

Q.4 Let x_i $(1 \le i \le 10)$ be ten observations of a random variable X. If $\sum_{i=1}^{10}(x_i - p) = 3$ and $\sum_{i=1}^{10}(x_i - p)^2 = 9$ where $0 \ne p \in R$, then the standard deviation of these observations is :

(1) $\sqrt{\frac{3}{5}}$ (2) $\frac{9}{10}$ (3) $\frac{4}{5}$ (4) $\frac{7}{10}$

Ans. [2]

Sol. Variance $= \frac{\Sigma(x_i - p)^2}{n} - \left(\frac{\Sigma(x_i - p)}{n}\right)^2$

$= \frac{9}{10} - \left(\frac{3}{10}\right)^2 = \frac{81}{100}$

S.D. $= \frac{9}{10}$

Q.5 Let R_1 and R_2 be two relations defined as follows :
$R_1 = [(a, b) \in R^2 ; a^2 + b^2 \in Q]$ and $R_2 = [(a, b) \in R^2 : a^2 + b^2 \notin Q]$, where Q is the set of all rational numbers. Then :

(1) Neither R_1 nor R_2 is transitive.
(2) R_1 and R_2 are both transitive
(3) R_1 is transitive but R_2 is not transitive
(4) R_2 is transitive but R_1 is not transitive

Ans. [1]

Sol. Let $a^2 + b^2 \in Q$ & $b^2 + c^2 \in Q$

eg. $a = 2 + \sqrt{3}$ & $b = 2 - \sqrt{3}$

$a^2 + b^2 = 14 \in Q$

Let $c = (1 + 2\sqrt{3})$

$b^2 + c^2 = 20 \in Q$

But $a^2 + c^2 = (2 + \sqrt{3})^2 + (1 + 2\sqrt{3})^2 \notin Q$

for R_2 Let $a^2 = 1, b^2 = \sqrt{3}$ & $c^2 = 2$

$a^2 + b^2 \notin Q$ & $b^2 + c^2 \notin Q$

But $a^2 + c^2 \in Q$

Q.6 The probability that a randomly chosen 5-digit number is made from exactly two digits is :

(1) $\frac{121}{10^4}$ (2) $\frac{134}{10^4}$ (3) $\frac{150}{10^4}$ (4) $\frac{135}{10^4}$

Ans. [4]

Sol. First Case : Choose two non-zero digits ${}^9C_2$.

Now, number of 5-digit numbers containing both digits $= 2^5 - 2$

Second Case : Choose one non-zero & one zero as digit ${}^9C_1$.

Number of 5-digit numbers containing one non zero and one zero both $= (2^4 - 1)$

Required prob.

$= \frac{({}^9C_2 \times (2^5 - 2) + {}^9C_1 \times (2^4 - 1))}{9 \times 10^4}$

$= \frac{36 \times (32 - 2) + 9 \times (16 - 1)}{9 \times 10^4}$

$= \frac{4 \times 30 + 15}{10^4} = \frac{135}{10^4}$

Q.7 Let p, q, r be three statements such that the truth value of $(p \wedge q) \rightarrow (\sim q \vee r)$ is F. Then the truth values of p, q, r are respectively.

(1) T, F, T (2) T, T, T (3) T, T, F (4) F, T, F

Ans. [3]

Sol. $(p \wedge q) \rightarrow (\sim q \vee r)$ = false
when $(p \wedge q)$ = T
and $(\sim q \vee r)$ = F
So $(p \wedge q)$ = T is possible when p = q = true
$\therefore$ ~ q = False (q = true)
So $(\sim q \vee r)$ = False is possible if r is false
$\therefore$ p = T, q = T, r = F

Q.8 Let a, b, c $\in$ R be such that $a^2 + b^2 + c^2 = 1$. If $a\cos\theta = b\cos\left(\theta + \frac{2\pi}{3}\right) = c\cos\left(\theta + \frac{4\pi}{3}\right)$, where $\theta = \frac{\pi}{9}$, then the angle between the vectors $a\hat{i} + b\hat{j} + c\hat{k}$ and $b\hat{i} + c\hat{j} + a\hat{k}$ is -

(1) $\frac{\pi}{2}$ (2) $\frac{2\pi}{3}$ (3) $\frac{\pi}{9}$ (4) 0

Ans. [1]

Sol. $\cos\phi = \frac{\bar{p}.\bar{q}}{|\bar{p}||\bar{q}|} = \frac{ab + bc + ca}{a^2 + b^2 + c^2} = \frac{\Sigma ab}{1}$

$= abc\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$

$= \frac{abc}{\lambda}\left(\cos\theta + \cos\left(\theta + \frac{2\pi}{3}\right) + \cos\left(\theta + \frac{4\pi}{3}\right)\right)$

$= \frac{abc}{\lambda}\left(\cos + 2\cos(\theta + \pi)\cos\frac{\pi}{3}\right)$

$= \frac{abc}{\lambda}(\cos\theta - \cos\theta) = 0$

$\phi = \frac{\pi}{2}$

Q.9 If the sum of the series $20 + 19\frac{3}{5} + 19\frac{1}{5} + 18\frac{4}{5} + \ldots$ upto n^{th} term is 488 and the n^{th} term is negative, then-

(1) n^{th} term is $-4\frac{2}{5}$ (2) n = 41 (3) n^{th} term is –4 (4) n = 60

Ans. [3]

Sol. $S = \frac{100}{5} + \frac{98}{5} + \frac{96}{5} + \frac{94}{5} + \ldots. n$

$S_n = \frac{n}{2}\left(2\times\frac{100}{5} + (n-1)\left(\frac{-2}{5}\right)\right) = 188$

$n(100 - n + 1) = 488 \times 5$
$n^2 - 101n + 488 \times 5 = 0$
$n = 61, 40$

$T_n = a + (n-1)d = \frac{100}{5} - \frac{2}{5} \times 60$

$= 20 - 24 = -4$

Q.10 If z_1, z_2 are complex numbers such that $Re(z_1) = |z_1 - 1|$, $Re(z_2) = |z_2 - 1|$ and $\arg(z_1 - z_2) = \frac{\pi}{6}$, then $Im(z_1 + z_2)$ is equal to -

(1) $\frac{2}{\sqrt{3}}$ (2) $2\sqrt{3}$ (3) $\frac{1}{\sqrt{3}}$ (4) $\frac{\sqrt{3}}{2}$

Ans. [2]

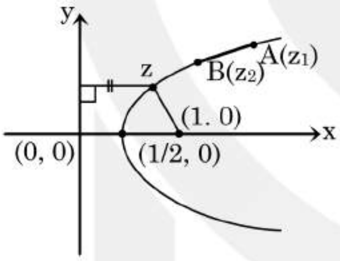
Sol. $Re(z) = |z - 1|$

$\Rightarrow x = \sqrt{(x-1)^2 + (y-0)^2}$ $(x > 0)$

$\Rightarrow y^2 = 2x - 1 = 4\,\frac{1}{2}\left(x - \frac{1}{2}\right)$

$\Rightarrow$ a parabola with focus (1, 0) & directrix as imaginary axis.

$\therefore$ Vertex $= \left(\frac{1}{2}, 0\right)$



A(z_1) & B(z_2) are two points on it such that slope of AB $= \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

$(\arg(z_1 - z_2) = \frac{\pi}{6})$

for $y^2 = 4ax$

Let $A(at_1^2, 2at_1)$ & $B(at_2^2, 2at_2)$

$m_{AB} = \frac{2}{t_1 + t_2} = \frac{4a}{y_1 + y_2} = \frac{1}{\sqrt{3}}$

$\left(\text{Here } a = \frac{1}{2}\right)$

$\Rightarrow y_1 + y_2 = 4a\sqrt{3} = 2\sqrt{3}$

Q.11 If $\int \sin^{-1}\left(\sqrt{\frac{x}{1+x}}\right)dx = A(x)\tan^{-1}(\sqrt{x}) + B(x) + C$, where C is a constant of integration, then the ordered pair (A(x), B(x)) can be -

(1) $(x + 1, \sqrt{x})$ (2) $(x + 1, -\sqrt{x})$ (3) $(x - 1, -\sqrt{x})$ (4) $(x - 1, \sqrt{x})$

Ans. [2]

Sol. Put $x = \tan^2\theta \Rightarrow dx = 2\tan\theta\sec^2\theta\, d\theta$

$$\int \theta.(2\tan\theta.\sec^2\theta)d\theta$$

$$\underset{\text{I}}{\downarrow}\ \ \underset{\text{II}}{\downarrow} \quad \text{(By parts)}$$

$$= \theta.\tan^2\theta - \int \tan^2\theta\, d\theta$$

$$= \theta.\tan^2\theta - \int (\sec^2\theta - 1)\, d\theta$$

$$= \theta(1 + \tan^2\theta) - \tan\theta + C$$

$$= \tan^{-1}(\sqrt{x})(1 + x) - \sqrt{x} + C$$

Q.12 $\lim\limits_{x\to a} \dfrac{(a+2x)^{1/3} - (3x)^{1/3}}{(3a+x)^{1/3} - (4x)^{1/3}}$ $(a \neq 0)$ is equal to -

(1) $\left(\frac{2}{3}\right)\left(\frac{2}{9}\right)^{1/3}$ (2) $\left(\frac{2}{9}\right)^{4/3}$ (3) $\left(\frac{2}{9}\right)\left(\frac{2}{3}\right)^{1/3}$ (4) $\left(\frac{2}{3}\right)^{4/3}$

Ans. [1]

Sol. Required limit

$$L = \lim_{h\to 0} \frac{(a+2(a+h))^{1/3} - (3(a+h))^{1/3}}{(3a+a+h)^{1/3} - (4(a+h))^{1/3}}$$

$$= \lim_{h\to 0} \frac{(3a)^{1/3}\left(1+\frac{2h}{3a}\right)^{1/3} - (3a)^{1/3}\left(1+\frac{h}{a}\right)^{1/3}}{(4a)^{1/3}\left(1+\frac{h}{4a}\right)^{1/3} - (4a)^{1/3}\left(1+\frac{h}{a}\right)^{1/3}}$$

$$= \lim_{h\to 0} \left(\frac{3^{1/3}}{4^{1/3}}\right)\left[\frac{\left(1+\frac{2h}{9a}\right) - \left(1+\frac{h}{3a}\right)}{\left(1+\frac{h}{12a}\right) - \left(1+\frac{h}{3a}\right)}\right]$$

$$= \left(\frac{3}{4}\right)^{1/3} \frac{\left(\frac{2}{9} - \frac{1}{3}\right)}{\left(\frac{1}{12} - \frac{1}{3}\right)} = \left(\frac{3}{4}\right)^{1/3}\left(\frac{8-12}{3-12}\right)$$

$$= \left(\frac{3}{4}\right)^{1/3}\left(\frac{-4}{-9}\right) = \frac{4^{1-\frac{1}{3}}}{3^{2-\frac{1}{3}}} = \frac{4^{2/3}}{3^{5/3}}$$

$$= \frac{(8\times 2)^{1/3}}{(27\times 9)^{1/3}} = \frac{2}{3}\left(\frac{2}{9}\right)^{1/3}$$

Q.13 If the term independent of x in the expansion of $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$ is k, then 18 k is equal to -

(1) 5 (2) 11 (3) 7 (4) 9

Ans. [3]

Sol. $T_{r+1} = {}^9C_r \left(\frac{3}{2}x^2\right)^{9-r} \left(-\frac{1}{3x}\right)^r$

$T_{r+1} = {}^9C_r \left(\frac{3}{2}\right)^{9-r} \left(-\frac{1}{3}\right)^r x^{18-3r}$

For independent of x

$18 - 3r = 0, r = 6$

$\therefore \ T_7 = {}^9C_6 \left(\frac{3}{2}\right)^3 \left(-\frac{1}{3}\right)^6 = \frac{21}{54} = k$

$\therefore \ 18k = \frac{21}{54} \times 18 = 7$

Q.14 If $x^3dy + xy\,dx = x^2dy + 2y\,dx$; $y(2) = e$ and $x > 1$, then $y(4)$ is equal to -

(1) $\frac{\sqrt{e}}{2}$ (2) $\frac{3}{2}\sqrt{e}$ (3) $\frac{3}{2} + \sqrt{e}$ (4) $\frac{1}{2} + \sqrt{e}$

Ans. [2]

Sol. $x^3dy + xy\,dx = x^2\,dy + 2y\,dx$

$\Rightarrow dy(x^3 - x^2) = dx\,(2y - xy)$

$\Rightarrow -\int \frac{1}{y}dx = \int \frac{x-2}{x^2(x-1)}dx$

$\Rightarrow -\ell ny = \int \left(\frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x-1)}\right)dx$

Where A = 1, B = + 2, C = –1

$\Rightarrow -\ell ny = \ell nx - \frac{2}{x} - \ell n\,(x-1) + \lambda$

$\Rightarrow y(2) = e$

$\Rightarrow -1 = \ell n\,2 - 1 - 0 + \lambda$

$\therefore \ \lambda = -\ell n\,2$

$\Rightarrow \ell ny = -\ell nx + \frac{2}{x} + \ell n\,(x-1) + \ell n\,2$

Now put x = 4 in equation

$\Rightarrow \ell ny = -\,\ell n4 + \frac{1}{2} + \ell n\,3 + \ell n\,2$

$\Rightarrow \ell ny = \ell n\left(\frac{3}{2}\right) + \frac{1}{2}\ \ell ne$

$\Rightarrow y = \frac{3}{2}\sqrt{e}$

Q.15 Let A be a 3 × 3 matrix such that adj A = $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & -2 & -1 \end{bmatrix}$ and B = adj (adj A). If $|A| = \lambda$ and $|(B^{-1})^T| = \mu$, then the ordered pair, $(|\lambda|, \mu)$ is equal to :

(1) $\left(9, \frac{1}{81}\right)$ (2) (3, 81) (3) $\left(9, \frac{1}{9}\right)$ (4) $\left(3, \frac{1}{81}\right)$

Ans. [4]

Sol. $C = \text{adj. } A = \begin{vmatrix} +2 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & -2 & -1 \end{vmatrix}$

$|C| = |\text{adj } A| = +2(0+4) + 1 . (1-2) + 1. (2, 4)$

$= +8 - 1 + 2$

$|\text{adj } A| = |A|^2 = 9 = 9$

$\lambda = |A| = \pm 3$

$|\lambda| = 3$

$B = \text{adj } C$

$|B| = |\text{adj } C| = |C|^2 = 81$

$|(B^{-1})^T| = |B|^{-1} = \frac{1}{81}$

$(|\lambda|, \mu) = \left(3, \frac{1}{81}\right)$

Q.16 If the surface area of a cube is increasing at a rate of 3.6 cm^2/sec, retaining its shape, then the rate of change of its volume (in cm^3/sec.), when the length of a side of the cube is 10 cm, is :

(1) 18 (2) 20 (3) 10 (4) 9

Ans. [4]

Sol. $\frac{d}{dt}(6a^2) = 3.6 \Rightarrow 12a\frac{da}{dt} = 3.6$

$a\frac{da}{dt} = 0.3$

$\frac{dv}{dt} = \frac{d}{dt}(a^3) = 3a\left(a\frac{da}{dt}\right)$

$= 3 \times 10 \times 0.3 = 9$

Q.17 Suppose f(x) is a polynomial of degree four, having critical points at –1, 0, 1. If $T = \{x \in R | f(x) = f(0)\}$, then the sum of squares of all the elements of T is :

(1) 8 (2) 2 (3) 6 (4) 4

Ans. [4]

Sol. $f'(x) = x(x + 1)(x - 1) = x^3 - x$

$\int df(x) = \int x^3 - x\,dx$

$f(x) = \frac{x^4}{4} - \frac{x^2}{2} + C$

$f(x) = f(0)$

$\frac{x^4}{4} - \frac{x^2}{2} = 0$

$x^2(x^2 - 2) = 0$

$x = 0, 0, \sqrt{2}, -\sqrt{2}$

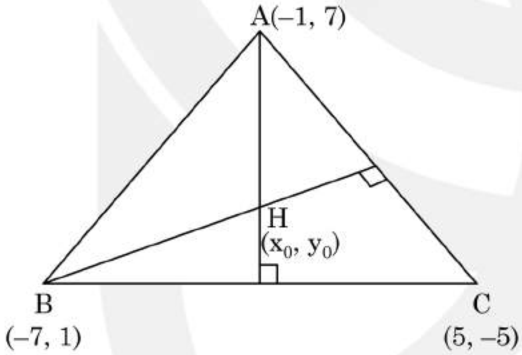
$x_1^2 + x_2^2 + x_3^2 = 0 + 2 + 2 = 4$

Q.18 If a ΔABC has vertices $A(-1, 7)$, $B(-7,1)$ and $C(5, -5)$, then its orthocentre has coordinates -

(1) $\left(\frac{3}{5}, -\frac{3}{5}\right)$ (2) $(3, -3)$ (3) $\left(-\frac{3}{5}, \frac{3}{5}\right)$ (4) $(-3, 3)$

Ans. [4]

Sol. Let orthocentre is $H(x_0, y_0)$



$m_{AH} \cdot m_{BC} = -1$

$\Rightarrow \left(\frac{y_0 - 7}{x_0 + 1}\right)\left(\frac{1+5}{-7-5}\right) = -1$

$\Rightarrow 2x_0 - y_0 + 9 = 0$ ....(1)

and $m_{BH} \cdot m_{AC} = -1$

$\Rightarrow \left(\frac{y_0 - 1}{x_0 + 7}\right)\left(\frac{7-(-5)}{-1-5}\right) = -1$

$\Rightarrow x_0 - 2y_0 + 9 = 0$ ....(2)

Solving equation (1) and (2) we get

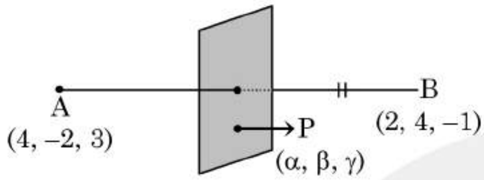
$(x_0, y_0) \equiv (-3, 3)$

Q.19 The plane which bisects the line joining the points $(4, -2, 3)$ and $(2, 4, -1)$ at right angles also passes through the point -

(1) $(4, 0, -1)$ (2) $(4, 0, 1)$ (3) $(0, 1, -1)$ (4) $(0, -1, 1)$

Ans. [1]

Sol.



PA = PB

$\Rightarrow$ $PA^2 = PB^2$

$\Rightarrow$ $(\alpha - 4)^2 + (\beta + 2)^2 + (\gamma + 3)^2$

$= (\alpha - 2)^2 + (\beta - 4)^2 + (\gamma + 1)^2$

$\Rightarrow$ $-4\alpha + 12\beta - 8\gamma = -8$

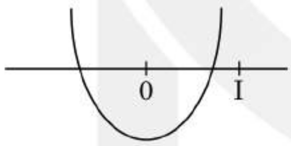
$\Rightarrow$ $2x - 6y + 4z = 4$

Q.20 The set of all real values of λ for which the quadratic equations, $(\lambda^2 + 1)x^2 - 4\lambda x + 2 = 0$ always have exactly one root in the interval (0, 1) is :

(1) (–3, –1) (2) (2, 4] (3) (1, 3] (4) (0, 2)

Ans. **[3]**

Sol. If exactly one root in (0, 1) then



$\Rightarrow$ $f(0) . f(1) < 0$

$\Rightarrow$ $2(\lambda^2 - 4\lambda + 3) < 0$

$\Rightarrow$ $1 < \lambda < 3$

Now for $\lambda = 1, 2x^2 - 4x + 2 = 0$

$(x - 1)^2 = 0, x = 1, 1$

So both roots doesn't lie between (0, 1)

$\therefore$ $\lambda \neq 1$

Again for $\lambda = 3$

$10x^2 - 12x + 2 = 0$

$\Rightarrow$ $x = 1, \frac{1}{5}$

So if one root is 1 then second root lie between (0, 1) so $\lambda = 3$ is correct.

$\therefore$ $\lambda \in (1, 3]$.

Q.21 If the tangent to the curve, $y = e^x$ at a point (c, e^c) and the normal to the parabola, $y^2 = 4x$ at the point (1, 2) intersect at the same point on the x-axis, then the value of c is ....

Ans. **[4]**

Sol. $y = e^x \Rightarrow \frac{dy}{dx} = e^x$

$m = \left(\frac{dy}{dx}\right)_{(c, e^c)} = e^c$

$\Rightarrow$ Tangent at (c, e^c)

$y - e^c = e^c (x - c)$

it intersect x-axis

Put $y = 0 \Rightarrow x = c - 1$ ....(1)

Now $y^2 = 4x \Rightarrow \frac{dy}{dx} = \frac{2}{y} \Rightarrow \left(\frac{dy}{dx}\right)_{(1, 2)} = 1$

$\Rightarrow$ Slope of normal $= -1$

Equation of normal $y - 2 = -1(x - 1)$

$x + y = 3$ it intersect x-axis

Put $y = 0 \Rightarrow x = 3$ ....(2)

Points are same

$\Rightarrow x = c - 1 = 3$

$\Rightarrow c = 4$

Q.22 If m arithmetic means (A. Ms) and three geometric means (G. Ms) are inserted between 3 and 243 such that 4^{th} A.M. is equal to 2^{nd} G.M., then m is equal to ....

Ans. **[39]**

Sol. $3, A_1, A_2 \ldots\ldots A_m, 243$

$d = \frac{243 - 3}{m + 1} = \frac{240}{m + 1}$

Now $3, G_1, G_2, G_3, 243$

$r = \left(\frac{243}{3}\right)^{\frac{1}{3+1}} = 3$

$\therefore A_4 = G_2$

$\Rightarrow a + 4d = at^2$

$3 + 4\left(\frac{240}{m + 1}\right) = 3\,(3)^2$

$m = 39$

Q.23 The total number of 3-digit numbers, whose sum of digits is 10, is ....

Ans. **[54]**

Sol. Let three digit number is xyz

$x + y + z = 10; x \geq 1, y \geq 0\ z \geq 0$ ....(1)

Let $T = x - 1 \Rightarrow x = T + 1$ where $T \geq 0$

Put in (1)

$T + y + z = 9;\ 0 \leq T \leq 8,\ 0 \leq y,\ z \leq 9$

No. of non negative integral solution

$= {}^{9+3-1}C_{3-1} - 1$ (when $T = 9$)

$= 55 - 1 = 54$

Q.24 Let S be the set of all integer solutions, (x, y, z), of the system of equations

$x - 2y + 5z = 0$

$-2x + 4y + z = 0$

$-7x + 14y + 9z = 0$

such that $15 \le x^2 + y^2 + z^2 \le 150$. Then, the number of elements in the set S is equal to ....

Ans. **[8]**

Sol. $\Delta = \begin{vmatrix} 1 & -2 & 5 \\ -2 & 4 & 1 \\ -7 & 14 & 9 \end{vmatrix} = 0$

Let x = k

$\Rightarrow$ Put in (1) & (2)

$k - 2y + 5z = 0$

$-2k + 4y + z = 0$

$z = 0, y = \frac{k}{2}$

$\therefore$ x, y, z are integer

$\Rightarrow$ k is even integer

Now $x = k, y = \frac{k}{2}, z = 0$ put in condition

$15 \le k^2 + \left(\frac{k}{2}\right)^2 + 0 \le 150$

$12 \le k^2 \le 120$

$\Rightarrow k = \pm 4, \pm 6, \pm 8, \pm 10$

$\Rightarrow$ Number of element in S = 8.

Q.25 Let a plane P contain two lines $\vec{r} = \hat{i} + \lambda(\hat{i} + \hat{j}), \lambda \in R$ and $\vec{r} = -\hat{j} + \mu(\hat{j} - \hat{k}), \mu \in R$. If Q (α, β, γ) is the foot of the perpendicular drawn from the point M(1, 0, 1) to P, then $3(\alpha + \beta + \gamma)$ equals ....

Ans. **[5]**

Sol. Dr's normal to plane

$= \begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix} = -\hat{i} + \hat{j} + \hat{k}$

Equation of plane

$-1(x - 1) + 1(y - 0) + 1(z - 0) = 0$

$x - y - z - 1 = 0$ ....(1)

Now $\frac{\alpha - 1}{1} = \frac{\beta - 0}{-1} = \frac{\gamma - 1}{-1} = \frac{(1 - 0 - 1 - 1)}{3}$

$\frac{\alpha - 1}{1} = \frac{\beta}{-1} = \frac{\gamma - 1}{-1} = \frac{1}{3}$

$\alpha = \frac{4}{3}, \beta = -\frac{1}{3}, \gamma = \frac{2}{3}$

$3(\alpha + \beta + \gamma) = 3\left(\frac{4}{3} - \frac{1}{3} + \frac{2}{3}\right) = 5$