

JEE Main Online Exam 2020

Question with Solutions

3rd September 2020 | Shift-I

MATHEMATICS

- Q.1** The proposition $p \rightarrow \sim(p \wedge \sim q)$ is equivalent to
 (1) $(\sim p) \vee q$ (2) $(\sim p) \vee (\sim q)$ (3) q (4) $(\sim p) \wedge q$

Ans. [1]

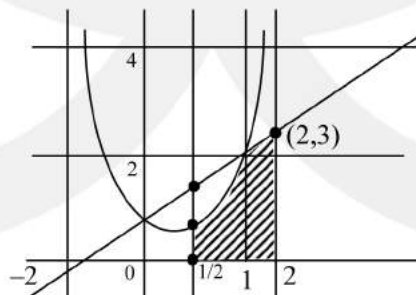
Sol.

$$\begin{aligned}
 p \rightarrow \sim(p \wedge \sim q) &= \sim p \vee \sim(p \wedge \sim q) \\
 &= \sim p \vee \sim p \vee q \\
 &= \sim(p \wedge q) \vee q \\
 &= \sim p \vee q
 \end{aligned}$$

- Q.2** The area (in sq. units) of the region $\{(x, y) : 0 \leq y \leq x^2 + 1, 0 \leq y \leq x + 1, \frac{1}{2} \leq x \leq 2\}$ is
 (1) $\frac{23}{6}$ (2) $\frac{79}{24}$ (3) $\frac{79}{16}$ (4) $\frac{23}{16}$

Ans. [2]

Sol. $0 \leq y \leq x^2 + 1, 0 \leq y \leq x + 1, \frac{1}{2} \leq x \leq 2$



$$\begin{aligned}
 \text{Required area} &= \int_{1/2}^2 (x^2 + 1) dx + \frac{1}{2} (2 + 3) \times 1 \\
 &= \frac{19}{24} + \frac{5}{2} = \frac{79}{24}
 \end{aligned}$$

- Q.3** If the number of integral terms in the expansion of $(3^{1/2} + 5^{1/8})^n$ is exactly 33, then the least value of n is-
 (1) 128 (2) 264 (3) 256 (4) 248

Ans. [3]

Sol. $T_{r+1} = {}^nC_r (3)^{\frac{n-r}{2}} (5)^{\frac{r}{8}} \quad (n \geq r)$

Clearly r should be a multiple of 8.

∴ there are exactly 33 integral terms

possible values of r can be

0, 8, 16,, 32×8

∴ least value of n = 256.

Q.4 $2\pi - \left(\sin^{-1} \frac{4}{5} + \sin^{-1} \frac{5}{13} + \sin^{-1} \frac{16}{65} \right)$ is equal to-

(1) $\frac{3\pi}{2}$

(2) $\frac{7\pi}{4}$

(3) $\frac{\pi}{2}$

(4) $\frac{5\pi}{4}$

Ans. [1]

Sol. $2\pi - \left(\sin^{-1} \left(\frac{4}{5} \right) + \sin^{-1} \left(\frac{5}{13} \right) + \sin^{-1} \left(\frac{16}{65} \right) \right)$
 $= 2\pi - \left(\tan^{-1} \left(\frac{4}{3} \right) + \tan^{-1} \left(\frac{5}{12} \right) + \tan^{-1} \left(\frac{16}{63} \right) \right)$
 $= 2\pi - \left(\tan^{-1} \left(\frac{63}{16} \right) + \tan^{-1} \left(\frac{16}{63} \right) \right)$
 $= 2\pi - \frac{\pi}{2} = \frac{3\pi}{2}$

Q.5 The lines

$\vec{r} = (\hat{i} - \hat{j}) + l(2\hat{i} + \hat{k})$ and

$\vec{r} = (2\hat{i} - \hat{j}) + m(\hat{i} + \hat{j} - \hat{k})$

(1) intersect for all values of l and m

(2) do not intersect for any values of l and m

(3) intersect when $l = 2$ and $m = \frac{1}{2}$

(4) intersect when $l = 1$ and $m = 2$

Ans. [2]

Sol. $\vec{r} = \hat{i}(1 + 2l) + \hat{j}(-1) + \hat{k}(l)$

$\vec{r} = \hat{i}(2 + m) + \hat{j}(m - 1) + \hat{k}(-m)$

For intersection

$1 + 2l = 2 + m \quad \dots(i)$

$-1 = m - 1 \quad \dots(ii)$

$l = -m \quad \dots(iii)$

from (ii) $m = 0$

from (iii) $\ell = 0$

These values of m and ℓ do not satisfy equation (1).

Hence the two lines do not intersect for any values of ℓ and m

Q.6 $\int_{-\pi}^{\pi} |\pi - |x|| dx$ is equal to-

(1) π^2

(2) $\frac{\pi^2}{2}$

(3) $2\pi^2$

(4) $\sqrt{2}\pi^2$

Ans. [1]

Sol.
$$\int_{-\pi}^{\pi} |\pi - |x|| dx = 2 \int_0^{\pi} |\pi - x| dx$$

$$= 2 \int_0^{\pi} (\pi - x) dx$$

$$= 2 \left[\pi x - \frac{x^2}{2} \right]_0^{\pi} = \pi^2$$

Q.7 Consider the two sets :

$A = \{m \in \mathbb{R} : \text{both the roots of } x^2 - (m+1)x + m + 4 = 0 \text{ are real}\}$ and $B = \{-3, 5\}$.

Which of the following is not true ?

(1) $A \cap B = \{-3\}$

(2) $A \cup B = \mathbb{R}$

(3) $A - B = (-\infty, -3) \cup (5, \infty)$

(4) $B - A = (-3, 5)$

Ans. [3]

Sol. $A : B \geq 0$

$$\Rightarrow (m+1)^2 - 4(m+4) \geq 0$$

$$\Rightarrow m^2 + 2m + 1 - 4m - 16 \geq 0$$

$$\Rightarrow m^2 - 2m - 15 \geq 0$$

$$\Rightarrow (m-5)(m+3) \geq 0$$

$$\Rightarrow m \in (-\infty, -3] \cup [5, \infty)$$

$$\therefore A = (-\infty, -3] \cup [5, \infty)$$

$$B = [-3, 5]$$

$$A - B = (-\infty, -3) \cup [5, \infty)$$

$$A \cap B = \{-3\}$$

$$B - A = (-3, 5)$$

$$A \cup B = \mathbb{R}$$

Q.8 If α and β are the roots of the equation $x^2 + px + 2 = 0$ and $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are the roots of the equation

$2x^2 + 2qx + 1 = 0$, then $\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right)\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$ is equal to-

- (1) $\frac{9}{4}(9 - q^2)$ (2) $\frac{9}{4}(9 + q^2)$ (3) $\frac{9}{4}(9 - p^2)$ (4) $\frac{9}{4}(9 + p^2)$

Ans. [3]

Sol.

α, β are roots of $x^2 + px + 2 = 0$

$$\Rightarrow \alpha^2 + p\alpha + 2 = 0 \text{ \& } \beta^2 + p\beta + 2 = 0$$

$$\Rightarrow \frac{1}{\alpha}, \frac{1}{\beta} \text{ are roots of } 2x^2 + px + 1 = 0$$

But $\frac{1}{\alpha}, \frac{1}{\beta}$ are roots of $2x^2 + 2qx + 1 = 0$

$$\Rightarrow p = 2q$$

$$\text{Also } \alpha + \beta = -p \qquad \alpha\beta = 2$$

$$\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right)\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$$

$$= \left(\frac{\alpha^2 - 1}{\alpha}\right)\left(\frac{\beta^2 - 1}{\beta}\right)\left(\frac{\alpha\beta + 1}{\beta}\right)\left(\frac{\alpha\beta + 1}{\alpha}\right)$$

$$= \frac{(-p\alpha - 3)(-p\beta - 3)(\alpha\beta + 1)^2}{(\alpha\beta)^2}$$

$$= \frac{9}{4}(p\alpha\beta + 3p(\alpha + \beta) + 9)$$

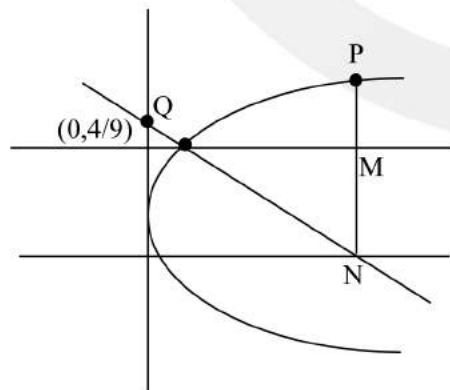
$$= \frac{9}{4}(9 - p^2) = \frac{9}{4}(9 - 4q^2)$$

Q.9 Let P be a point on the parabola, $y^2 = 12x$ and N be the foot of the perpendicular drawn from P on the axis of the parabola. A line is now drawn through the mid-point M of PN, parallel to its axis which meets the parabola at Q. If the y-intercept of the line NQ is $\frac{4}{3}$, then-

- (1) PN = 4 (2) MQ = $\frac{1}{4}$ (3) PN = 3 (4) MQ = $\frac{1}{3}$

Ans. [2]

Sol.



$$\Rightarrow \frac{40}{2} [2 \times 3 + (39d)] = 2 \times \frac{25}{2} [2 \times 2 + 24d]$$

$$\Rightarrow d = \frac{1}{6}$$

Q.12 A dice is thrown two times and the sum of the scores appearing on the dice is observed to be a multiple of 4. Then the conditional probability that the score 4 has appeared atleast once is-

- (1) $\frac{1}{8}$ (2) $\frac{1}{9}$ (3) $\frac{1}{3}$ (4) $\frac{1}{4}$

Ans. [2]

Sol.

A : Sum obtained is a multiple of 4.

$A = \{(1, 3), (2, 2), (3, 1), (2, 6), (3, 5), (4, 4), (5, 3), (6, 2), (6, 6)\}$

B : Score of 4 has appeared at least once.

$B = \{(1, 4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4), (4, 1), (4, 2), (4, 3), (4, 5), (4, 6)\}$

$$\text{Required probability} = P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}$$

$$= \frac{1/36}{9/36} = \frac{1}{9}$$

Q.13 The value of $(2 \cdot {}^1P_0 - 3 \cdot {}^2P_1 + 4 \cdot {}^3P_2 - \dots$ up to 51th term) + $(1! - 2! + 3! - \dots$ up to 51th term) is equal to-

- (1) $1 + (52)!$ (2) $1 - 51(51)!$ (3) 1 (4) $1 + (51)!$

Ans. [1]

Sol.

$S = (2 \cdot {}^1p_0 - 3 \cdot {}^2P_1 + 4 \cdot {}^3p_2 \dots \dots \dots$ upto 51 terms)

+ $(1! + 2! + 3! \dots \dots \dots$ upto 51 terms)

$$[\because {}^n p_{n-1} = n!]$$

$$\therefore S = (2 \times 1! - 3 \times 2! + 4 \times 3! \dots \dots \dots 52 \cdot 51!)$$

$$+ (1! - 2! + 3! \dots \dots \dots (51) !)$$

$$= (2! - 3! + 4! \dots \dots \dots + 52!)$$

$$+ (1! - 2! + 3! - 4! + \dots \dots \dots + (51) !)$$

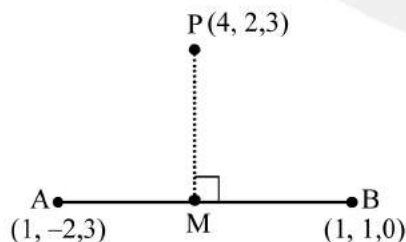
$$= 1! + 52!.$$

Q.14 The foot of the perpendicular drawn from the point (4, 2, 3) to the line joining the points (1, -2, 3) and (1, 1, 0) lies on the plane.

- (1) $x + 2y - z = 1$ (2) $x - y - 2z = 1$ (3) $x - 2y + z = 1$ (4) $2x + y - z = 1$

Ans. [4]

Sol.



$$\text{Equation of } AB = \vec{r} = (\hat{i} + \hat{j}) + \lambda(3\hat{j} - 3\hat{k})$$

$$\text{Let coordinates of } M = (1, (1 + 3\lambda), -3\lambda),$$

$$\vec{PM} = -3\hat{i} + (3\lambda - 1)\hat{j} - 3(\lambda + 1)\hat{k}$$

$$\vec{AB} = 3\hat{j} - 3\hat{k}$$

$$\therefore \vec{PM} \perp \vec{AB} \Rightarrow \vec{PM} \cdot \vec{AB} = 0$$

$$\Rightarrow 3(3\lambda - 1) + 9(\lambda + 1) = 0$$

$$\Rightarrow \lambda = -\frac{1}{3}$$

$$\therefore M = (1, 0, 1)$$

Clearly M lies on $2x + y - z = 1$.

Q.15 Let $[t]$ denote the greatest integer $\leq t$. If for some $\lambda \in \mathbb{R} - \{0, 1\}$, $\lim_{x \rightarrow 0} \frac{|1-x+|x||}{|\lambda-x+[x]|} = L$, then L is equal to-

(1) $\frac{1}{2}$

(2) 0

(3) 2

(4) 1

Ans. [3]

Sol. LHL : $\lim_{x \rightarrow 0^-} \frac{|1-x-x|}{|\lambda-x-1|} = \frac{1}{|\lambda-1|}$

RHL : $\lim_{x \rightarrow 0^+} \frac{|1-x+x|}{|\lambda-x+1|} = \frac{1}{|\lambda|}$

For existence of limit

$$\text{LHL} = \text{RHL}$$

$$\Rightarrow \frac{1}{|\lambda-1|} = \frac{1}{|\lambda|} \Rightarrow \lambda = \frac{1}{2}$$

$$\therefore L = \frac{1}{|\lambda|} = 2$$

Q.16 The function, $f(x) = (3x - 7)x^{2/3}$, $x \in \mathbb{R}$, is increasing for all x lying in

(1) $(-\infty, 0) \cup \left(\frac{3}{7}, \infty\right)$

(2) $(-\infty, 0) \cup \left(\frac{14}{15}, \infty\right)$

(3) $\left(-\infty, \frac{14}{15}\right)$

(4) $\left(-\infty, \frac{14}{15}\right) \cup (0, \infty)$

Ans. [2]

Sol. $f(x) = (3x - 7)x^{2/3}$
 $\Rightarrow f(x) = 3x^{5/3} - 7x^{2/3}$
 $\Rightarrow f'(x) = 5x^{2/3} - \frac{14}{3x^{1/3}}$
 $= \frac{15x - 14}{3x^{1/3}} > 0$

$$\begin{array}{c} + \qquad \qquad - \qquad \qquad + \\ | \qquad \qquad | \qquad \qquad | \\ 0 \qquad \qquad \qquad \qquad \qquad 14/15 \end{array}$$

$$\therefore f'(x) > 0 \forall x \in (-\infty, 0) \cup \left(\frac{14}{15}, \infty\right)$$

Q.17 The solution curve of differential equation, $(1 + e^{-x})(1 + y^2) \frac{dy}{dx} = y^2$, which passes through the point $(0, 1)$ is-

(1) $y^2 + 1 = y \left(\log_e \left(\frac{1 + e^x}{2} \right) + 2 \right)$

(2) $y^2 = 1 + y \log_e \left(\frac{1 + e^x}{2} \right)$

(3) $y^2 = 1 + y \log_e \left(\frac{1 + e^{-x}}{2} \right)$

(4) $y^2 + 1 = y \left(\log_e \left(\frac{1 + e^{-x}}{2} \right) + 2 \right)$

Ans. [2]

Sol. $(1 + e^{-x})(1 + y^2) \frac{dy}{dx} = y^2$

$$\Rightarrow (1 + y^{-2}) dy = \left(\frac{e^x}{1 + e^x} \right) dx$$

$$\Rightarrow \left(y - \frac{1}{y} \right) = \ln(1 + e^x) + c$$

\therefore It passes through $(0, 1) \Rightarrow c = -\ln 2$

$$\Rightarrow y^2 = 1 + y \ln \left(\frac{1 + e^x}{2} \right)$$

Q.18 A hyperbola having the transverse axis of length $\sqrt{2}$ has the same foci as that of the ellipse $3x^2 + 4y^2 = 12$, then this hyperbola does not pass through which of the following points ?

(1) $\left(\frac{\sqrt{3}}{2}, \frac{1}{\sqrt{2}} \right)$

(2) $\left(-\sqrt{\frac{3}{2}}, 1 \right)$

(3) $\left(\frac{1}{\sqrt{2}}, 0 \right)$

(4) $\left(1, -\frac{1}{\sqrt{2}} \right)$

Ans. [1]

Sol. Ellipse : $\frac{x^2}{4} + \frac{y^2}{3} = 1$

$$\text{eccentricity} = \sqrt{1 - \frac{3}{4}} = \frac{1}{2}$$

\therefore foci = $(\pm 1, 0)$

for hyperbola, given $2a = \sqrt{2} \Rightarrow a = \frac{1}{\sqrt{2}}$

\therefore hyperbola will be

$$\frac{x^2}{1/2} - \frac{y^2}{b^2} = 1$$

$$\text{eccentricity} = \sqrt{1 + 2b^2}$$

$$\therefore \text{foci} = \left(\pm \sqrt{\frac{1 + 2b^2}{2}}, 0 \right)$$

\therefore Ellipse and hyperbola have same foci

$$\Rightarrow \sqrt{\frac{1+2b^2}{2}} = 1$$

$$\Rightarrow b^2 = \frac{1}{2}$$

$$\therefore \text{Equation of hyperbola : } \frac{x^2}{1/2} - \frac{y^2}{1/2} = 1$$

$$\Rightarrow x^2 - y^2 = \frac{1}{2}$$

Clearly $\left(\sqrt{\frac{3}{2}}, \frac{1}{\sqrt{2}}\right)$ does not lie on it.

Q.19 If $y^2 + \log_e(\cos^2 x) = y$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, then-

(1) $|y''(0)| = 2$

(2) $|y'(0)| + |y''(0)| = 3$

(3) $|y'(0)| + |y''(0)| = 1$

(4) $y''(0) = 0$

Ans. [1]

Sol. $y^2 + \ln(\cos^2 x) = y$ $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

for $x = 0$ $y = 0$ or 1

Differentiating wrt x

$$\Rightarrow 2yy' - 2 \tan x = y'$$

At $(0, 0)$ $y' = 0$

At $(0, 1)$ $y' = 0$

Differentiating wrt x

$$2yy'' + 2(y')^2 - 2 \sec^2 x = y''$$

At $(0, 0)$ $y'' = -2$

At $(0, 1)$ $y'' = 2$

$$\therefore |y''(0)| = 2$$

Q.20 For the frequency distribution :

Variate (x) : $x_1 \quad x_2 \quad x_3 \dots x_{15}$

Frequency (f) : $f_1 \quad f_2 \quad f_3 \dots f_{15}$

where $0 < x_1 < x_2 < x_3 < \dots < x_{15} = 10$ and $\sum_{t=1}^{15} f_t > 0$, the standard deviation cannot be

(1) 1

(2) 4

(3) 2

(4) 6

Ans. [4]

Sol. $\therefore \sigma^2 \leq \frac{1}{4} (M - m)^2$

Where M and m are upper and lower bounds of values of any random variable.

$$\therefore \sigma^2 \leq \frac{1}{4} (10 - 0)^2$$

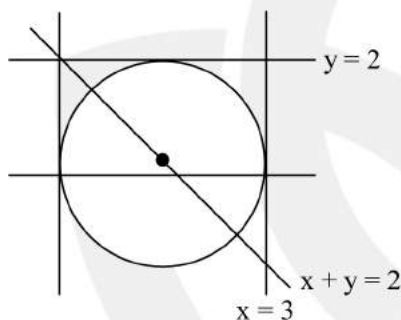
$$\Rightarrow 0 < \sigma < 5$$

$$\therefore \sigma \neq 6.$$

Q.21 The diameter of the circle, whose centre lies on the line $x + y = 2$ in the first quadrant and which touches both the lines $x = 3$ and $y = 2$, is _____.

Ans. [3]

Sol.



\therefore center lies on $x + y = 2$ and in 1st quadrant

$$\text{center} = (\alpha, 2 - \alpha)$$

$$\text{where } \alpha > 0 \text{ and } 2 - \alpha > 0 \Rightarrow 0 < \alpha < 2$$

\therefore circle touches $x = 3$ and $y = 2$

$$\Rightarrow |3 - \alpha| = |2 - (2 - \alpha)| = \text{radius}$$

$$\Rightarrow |3 - \alpha| = |\alpha| \Rightarrow \alpha = \frac{3}{2}$$

$$\therefore \text{radius} = \alpha$$

$$\Rightarrow \text{Diameter} = 2\alpha = 3$$

Q.22 If $\lim_{x \rightarrow 0} \left\{ \frac{1}{x^8} \left(1 - \cos \frac{x^2}{2} - \cos \frac{x^2}{4} + \cos \frac{x^2}{2} \cos \frac{x^2}{4} \right) \right\} = 2^{-k}$, then the value of k is _____.

Ans. [8]

Sol.
$$\lim_{x \rightarrow 0} \left\{ \frac{1}{x^8} \left(1 - \cos \frac{x^2}{2} - \cos \frac{x^2}{4} + \cos \frac{x^2}{2} \cos \frac{x^2}{4} \right) \right\} = 2^{-k}$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{\left(1 - \cos \frac{x^2}{2} \right) \left(1 - \cos \frac{x^2}{4} \right)}{4 \left(\frac{x^2}{2} \right)^2 \cdot 16 \left(\frac{x^2}{4} \right)^2} = \frac{1}{8} \times \frac{1}{32} = 2^{-k}$$

$$\Rightarrow 2^{-8} = 2^{-k} \Rightarrow k = 8.$$

Q.23 Let $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$, $x \in \mathbb{R}$ and $A^4 = [a_{ij}]$. If $a_{11} = 109$, then a_{22} is equal to _____.

Ans. [10]

Sol. $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$

$$A^2 = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix} \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (x^2+1)^2+x^2 & x(x^2+1)+x \\ x(x^2+1)+x & x^2+1 \end{bmatrix}$$

$$a_{11} = (x^2+1)^2+x^2 = 109$$

$$\Rightarrow x = \pm 3$$

$$a_{22} = x^2+1 = 10$$

Q.24 The value of $(0.16)^{\log_{2.5}\left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots \text{to } \infty\right)}$ is equal to _____.

(1)

(2)

(3)

(4)

Ans. [4]

Sol. $(0.16)^{\log_{2.5}\left(\frac{1}{3} + \frac{1}{3^2} + \dots \text{to } \infty\right)}$

$$= \left(\frac{4}{25}\right)^{\log_{\left(\frac{5}{2}\right)}\left(\frac{1}{2}\right)}$$

$$= \left(\frac{1}{2}\right)^{\log_{\left(\frac{5}{2}\right)}\left(\frac{4}{25}\right)} = \left(\frac{1}{2}\right)^{-2} = 4$$

Q.25 If $\left(\frac{1+i}{1-i}\right)^{m/2} = \left(\frac{1+i}{1-i}\right)^{n/3} = 1$, ($m, n \in \mathbb{N}$), then the greatest common divisor of the least values of m and n is

(1)

(2)

(3)

(4)

Ans. [4]

Sol. $\left(\frac{1+i}{1-i}\right)^{m/2} = \left(\frac{1+i}{1-i}\right)^{n/3} = 1$

$$\Rightarrow \left(\frac{(1+i)^2}{2}\right)^{m/2} = \left(\frac{(1+i)^2}{-2}\right)^{n/3} = 1$$

$$\Rightarrow (i)^{m/2} = (-i)^{n/3} = 1$$

$$\Rightarrow \frac{m}{2} = 4k_1 \text{ and } \frac{n}{3} = 4k_2$$

$$\Rightarrow m = 8k_1 \text{ and } n = 12k_2$$

Least value of $m = 8$ and $n = 12$.

$\therefore \text{GCD} = 4$