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Time : 3 hrs.

Answers & Solutions

M.M. : 300

for

JEE (MAIN)-2021 (Online) Phase-1

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS :

- (1) The test is of **3 hours** duration.
- (2) The Test Booklet consists of 90 questions. The maximum marks are 300.
- (3) There are **three** parts in the question paper A, B, C consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part has two sections.
 - (i) Section-I : This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **-1 mark** for wrong answer.
 - (ii) Section-II : This section contains 10 questions. In Section-II, attempt any **five questions out of 10**. The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and there is no negative marking for wrong answer.

PART-A : PHYSICS

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

- Zener breakdown occurs in a p-n junction having p and n both
 - Lightly doped and have narrow depletion layer
 - Heavily doped and have narrow depletion layer
 - Heavily doped and have wide depletion layer
 - Lightly doped and have wide depletion layer

Answer (2)

Sol. Zener diode has both p and n side highly doped and depletion layer is very thin.

- If the source of light used in a Young's double slit experiment is changed from red to violet
 - The fringes will become brighter
 - The central bright fringe will become a dark fringe
 - Consecutive fringe lines will come closer
 - The intensity of minima will increase

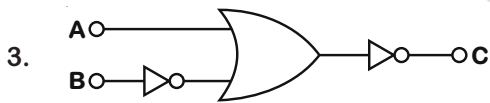
Answer (3)

Sol. $\therefore \beta = \frac{\lambda D}{d}$

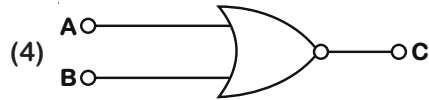
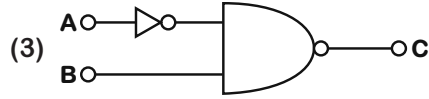
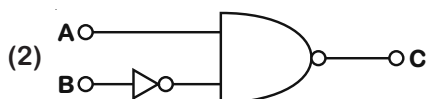
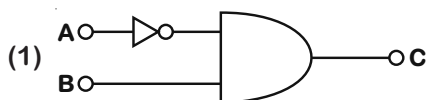
and $\lambda_v < \lambda_{red}$

\Rightarrow Fringe width will decrease.

\Rightarrow Fringe lines will come closer.



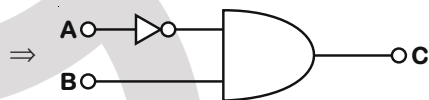
The logic circuit shown above is equivalent to



Answer (1)

Sol. From gates diagram,

$$\overline{(A + B)} = \bar{A} \cdot \bar{B} = \bar{A} \cdot B$$



- Which of the following equations represents a travelling wave?

- $y = A \sin(15x - 2t)$
- $y = A e^x \cos(\omega t - \theta)$
- $y = A \sin x \cos \omega t$
- $y = A e^{-x^2} (vt + \theta)$

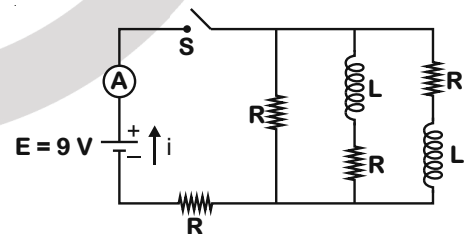
Answer (1)

Sol. Travelling wave equation,

$$y = f(\pm kx \pm \omega t)$$

$\therefore y = A \sin(15x - 2t)$ is a travelling wave.

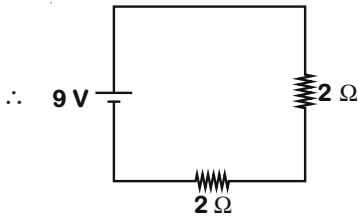
- Figure shows a circuit that contains four identical resistors with resistance $R = 2.0 \Omega$, two identical inductors with inductance $L = 2.0 \text{ mH}$ and an ideal battery with emf $E = 9 \text{ V}$. The current 'i' just after the switch 'S' is closed will be



- 3.37 A
- 3.0 A
- 2.25 A
- 9 A

Answer (3)

Sol. At $t = 0$, inductor behaves as an open switch.



$$\therefore i = \frac{9}{4} = 2.25 \text{ A}$$

6. An X-ray tube is operated at 1.24 million volt. The shortest wavelength of the produced photon will be

- (1) 10^{-3} nm (2) 10^{-2} nm
 (3) 10^{-4} nm (4) 10^{-1} nm

Answer (1)

Sol. $\lambda_{\min} = \frac{hc}{eV}$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19} \times 1.24 \times 10^6}$$

$$= 1.0 \times 10^{-12} \text{ m}$$

$$= 10^{-3} \text{ nm}$$

7. The de Broglie wavelength of a proton and α -particle are equal. The ratio of their velocities is

- (1) 1 : 4 (2) 4 : 3
 (3) 4 : 1 (4) 4 : 2

Answer (3)

Sol. $\lambda_p = \frac{h}{m_p \times v_p}$

$$\lambda_\alpha = \frac{h}{m_\alpha \times v_\alpha}$$

$$\therefore \lambda_p = \lambda_\alpha \Rightarrow m_p v_p = m_\alpha v_\alpha$$

$$\Rightarrow \frac{v_p}{v_\alpha} = \frac{m_\alpha}{m_p} = \frac{4}{1}$$

8. Given below are two statements :

Statement I : PN junction diodes can be used to function as transistor, simply by connecting two diodes, back to back, which acts as the base terminal.

Statement II : In the study of transistor, the amplification factor β indicates ratio of the collector current to the base current.

In the light of the above statements, choose the correct answer from the options given below.

- (1) Both Statement I and Statement II are false
 (2) Statement I is true but Statement II is false
 (3) Statement I is false but Statement II is true
 (4) Both statement I and Statement II are true

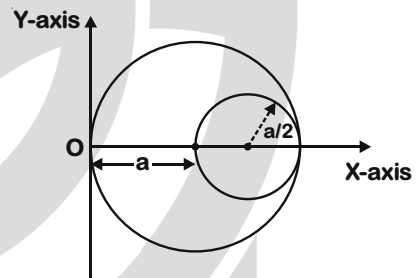
Answer (3)

Sol. Transistor is formed by formation of two p-n junctions by sandwiching a thin base layer. So, statement I is false.

$$\therefore \beta = \frac{\Delta I_C}{\Delta I_B} \text{ and for DC, } \beta = \frac{I_C}{I_B}$$

So, statement II is true.

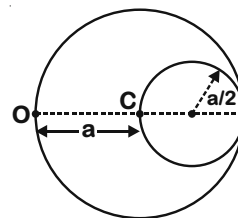
9. A circular hole of radius $\left(\frac{a}{2}\right)$ is cut out of a circular disc of radius 'a' as shown in figure. The centroid of the remaining circular portion with respect to point 'O' will be



- (1) $\frac{1}{6}a$
 (2) $\frac{2}{3}a$
 (3) $\frac{10}{11}a$
 (4) $\frac{5}{6}a$

Answer (4)

Sol. Let mass of hole be $(-m)$, then mass of complete disc will be $4m$.



\therefore Centre of mass w.r.t. point O is

$$X_{CM} = \frac{4m \times a + (-m) \times \left(\frac{3}{2}a\right)}{(4m - m)} = \frac{5}{6}a$$

10. A particle is projected with velocity v_0 along x-axis. A damping force is acting on the particle which is proportional to the square of the distance from the origin i.e. $ma = -\alpha x^2$. The distance at which the particle stops

- (1) $\left(\frac{3v_0^2}{2\alpha}\right)^{\frac{1}{3}}$ (2) $\left(\frac{3v_0^2}{2\alpha}\right)^{\frac{1}{2}}$
 (3) $\left(\frac{2v_0^2}{3\alpha}\right)^{\frac{1}{2}}$ (4) $\left(\frac{2v_0}{3\alpha}\right)^{\frac{1}{3}}$

Answer (1)

Sol. Loss in KE = work against damping

$$\Rightarrow \frac{1}{2}mv_0^2 = \int_{x=0}^x \alpha x^2 dx$$

$$\Rightarrow \frac{1}{2}mv_0^2 = \frac{\alpha x^3}{3}$$

$$\Rightarrow x = \left(\frac{3mv_0^2}{2\alpha}\right)^{\frac{1}{3}}$$

11. When a particle executes SHM, the nature of graphical representation of velocity as a function of displacement is :

- (1) parabolic (2) straight line
 (3) circular (4) elliptical

Answer (4)

Sol. $v = \omega\sqrt{A^2 - x^2}$

$$\frac{v^2}{\omega^2} + x^2 = A^2$$

12. The period of oscillation of a simple pendulum

is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured value of 'L' is 1.0 m

from meter scale having a minimum division of 1 mm and time of one complete oscillation is 1.95 s measured from stopwatch of 0.01 s resolution. The percentage error in the determination of 'g' will be :

- (1) 1.30%
 (2) 1.33%
 (3) 1.13%
 (4) 1.03%

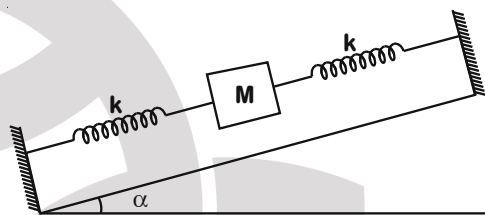
Answer (3)

Sol. $g = 4\pi^2 \frac{\ell}{T^2}$

$$\frac{dg}{g} = \frac{d\ell}{\ell} + \frac{2dT}{T}$$

$$\frac{dg}{g} = \frac{1}{1000} + \frac{2 \times 0.01}{1.95} = \frac{1.13}{100}$$

13. In the given figure, a body of mass M is held between two massless spring, on a smooth inclined plane. The free ends of the springs are attached to firm supports. If each spring has spring constant k, the frequency of oscillation of given body is :



- (1) $\frac{1}{2\pi} \sqrt{\frac{2k}{M}}$ (2) $\frac{1}{2\pi} \sqrt{\frac{k}{2M}}$
 (3) $\frac{1}{2\pi} \sqrt{\frac{2k}{Mg \sin \alpha}}$ (4) $\frac{1}{2\pi} \sqrt{\frac{k}{Mg \sin \alpha}}$

Answer (1)

Sol. $T = 2\pi \sqrt{\frac{M}{k_{eq}}}$

$$f = \frac{1}{2\pi} \sqrt{\frac{k_{eq}}{M}}$$

$$k_{eq} = 2k$$

14. Match List-I with List-II.

- | List-I | List-II |
|-----------------------------------|---------------------------------------|
| (a) Source of microwave frequency | (i) Radioactive decay of nucleus |
| (b) Source of infrared frequency | (ii) Magnetron |
| (c) Source of Gamma Rays | (iii) Inner shell electrons |
| (d) Source of X-rays | (iv) Vibration of atoms and molecules |
| | (v) LASER |
| | (vi) RC circuit |

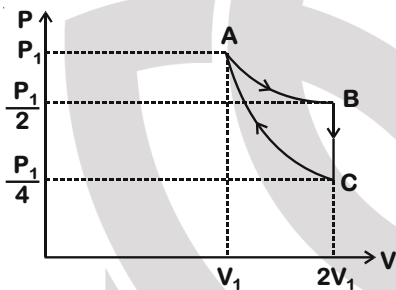
Choose the correct answer from the options given below :

- (1) (a)-(ii), (b)-(iv), (c)-(vi), (d)-(iii)
 (2) (a)-(vi), (b)-(v), (c)-(i), (d)-(iv)
 (3) (a)-(vi), (b)-(iv), (c)-(i), (d)-(v)
 (4) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)

Answer (4)

Sol. Source of Gamma Rays is radioactive decay source X-Ray is transition of inner shell electron and that of microwave is magnetron.

15. If one mole of an ideal gas at (P_1, V_1) is allowed to expand reversibly and isothermally (A to B) its pressure is reduced to one-half of the original pressure (see figure). This is followed by a constant volume cooling till its pressure is reduced to one-fourth of the initial value (B → C). Then it is restored to its initial state by a reversible adiabatic compression (C to A). The net work done by the gas is equal to :



- (1) $-\frac{RT}{2(\gamma-1)}$ (2) $RT \left(\ln 2 - \frac{1}{2(\gamma-1)} \right)$
 (3) 0 (4) $RT \ln 2$

Answer (2)

Sol. $W_{A-B} = P_1 V_1 \ln(2)$

$$W_{B-C} = 0$$

$$W_{C-A} = \frac{-P_1 V_1}{2(\gamma-1)}$$

16. A soft ferromagnetic material is placed in an external magnetic field. The magnetic domains:
 (1) decrease in size and changes orientation.
 (2) may increase or decrease in size and change its orientation.
 (3) increase in size but no change in orientation.
 (4) have no relation with external magnetic field

Answer (2)

Sol. Domains aligned in the direction of field increases in size and those aligned in the direction opposite to the field reduces in size.

17. A body weighs 49 N on a spring balance at the north pole. What will be its weight recorded on the same weighing machine, if it is shifted to the equator?

[Use $g = \frac{GM}{R^2} = 9.8 \text{ ms}^{-2}$ and radius of earth, $R = 6400 \text{ km}$.]

- (1) 49.83 N (2) 49 N
 (3) 48.83 N (4) 49.17 N

Answer (3)

Sol. $g' = g - \omega^2 R$

Weight at equator will decrease.

18. According to Bohr atom model, in which of the following transitions will the frequency be maximum?

- (1) $n = 3$ to $n = 2$ (2) $n = 5$ to $n = 4$
 (3) $n = 4$ to $n = 3$ (4) $n = 2$ to $n = 1$

Answer (4)

Sol. Energy in transition is maximum for $n = 2$ to $n = 1$, hence the frequency.

19. On the basis of kinetic theory of gases, the gas exerts pressure because its molecules :

- (1) suffer change in momentum when impinge on the walls of container.
 (2) continuously stick to the walls of container.
 (3) continuously lose their energy till it reaches wall.
 (4) are attracted by the walls of container.

Answer (1)

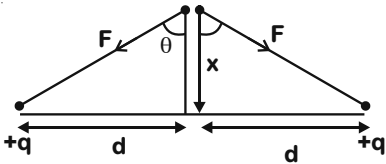
Sol. Pressure is due to force exerted by molecules on wall

20. Two electrons each are fixed at a distance '2d'. A third charge proton placed at the midpoint is displaced slightly by a distance x ($x \ll d$) perpendicular to the line joining the two fixed charges. Proton will execute simple harmonic motion having angular frequency: (m = mass of charged particle)

- (1) $\left(\frac{q^2}{2\pi\epsilon_0 m d^3} \right)^{\frac{1}{2}}$ (2) $\left(\frac{2\pi\epsilon_0 m d^3}{q^2} \right)^{\frac{1}{2}}$
 (3) $\left(\frac{2q^2}{\pi\epsilon_0 m d^3} \right)^{\frac{1}{2}}$ (4) $\left(\frac{\pi\epsilon_0 m d^3}{2q^2} \right)^{\frac{1}{2}}$

Answer (1)

Sol. $F_{\text{net}} = 2F \cos\theta$



$$F = \frac{q^2}{4\pi\epsilon_0(x^2 + d^2)}$$

$$F_{\text{net}} = \frac{2q^2x}{4\pi\epsilon_0(x^2 + d^2)^{3/2}}$$

$$\frac{q^2x}{2\pi\epsilon_0d^3} = -m\omega^2x$$

$$\omega^2 = \frac{q^2}{2\pi\epsilon_0md^3}$$

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. The root mean square speed of molecules of a given mass of a gas at 27°C and 1 atmosphere pressure is 200 ms⁻¹. The root mean square speed of molecules of the gas at 127°C and

2 atmosphere pressure is $\frac{x}{\sqrt{3}}$ ms⁻¹. The value of x will be _____.

Answer (400)

Sol. $u_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

$$\frac{u_2}{u_1} = \sqrt{\frac{T_2}{T_1}}$$

$$u_2 = \sqrt{\frac{273 + 127}{273 + 27}} \cdot u_1$$

$$= \frac{2}{\sqrt{3}} \cdot 200$$

$$= \frac{400}{\sqrt{3}} \text{ m/s}$$

2. An electromagnetic wave of frequency 3 GHz enters a dielectric medium of relative electric permittivity 2.25 from vacuum. The wavelength of this wave in that medium will be _____ × 10⁻² cm.

Answer (667)

Sol. Assuming relative permeability of the medium to be 1.

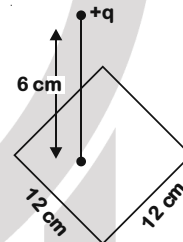
$$n = \sqrt{\mu_r \epsilon_r} = 1.5$$

$$u = \frac{c}{n} = 2 \times 10^8 \text{ m/s}$$

$$\lambda = \frac{u}{f} = \frac{2 \times 10^8}{3 \times 10^9} \text{ m}$$

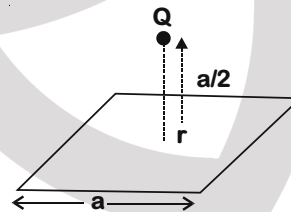
$$= 666.67 \times 10^{-2} \text{ cm}$$

3. A point charge of +12μC is at a distance 6 cm vertically above the centre of a square of side 12 cm as shown in figure. The magnitude of the electric flux through the square will be _____ × 10³ Nm²/C.



Answer (226)

Sol.



$$\phi = \frac{Q}{6\epsilon_0}$$

$$= \frac{12 \times 10^{-6}}{6} \times 4\pi \times 9 \times 10^9$$

$$= 226.28 \times 10^3 \frac{\text{N-m}^2}{\text{C}}$$

4. Two solids A and B of mass 1 kg and 2 kg respectively are moving with equal linear momentum. The ratio of their kinetic energies (K.E.)_A : (K.E.)_B will be $\frac{A}{1}$, so the value of A will be _____.

Answer (2)

Sol. $k = \frac{P^2}{2m}$

$$\frac{k_A}{k_B} = \frac{m_B}{m_A} = \frac{2}{1}$$

5. A uniform metallic wire is elongated by 0.04 m when subjected to a linear force F. The elongation, if its length and diameter is doubled and subjected to the same force will be _____ cm.

Answer (2)

Sol. $\Delta l = \frac{FL}{AY}$

$$\frac{\Delta l_2}{\Delta l_1} = \frac{L_2}{L_1} \frac{A_1}{A_2}$$

$$\Delta l_2 = \left(\frac{L_2}{L_1}\right) \left(\frac{A_1}{A_2}\right) \Delta l_1 = 2 \times \frac{1}{4} \times 4 = 2 \text{ cm}$$

6. A cylindrical wire of radius 0.5 mm and conductivity $5 \times 10^7 \text{ S/m}$ is subjected to an electric field of 10 mV/m. The expected value of current in the wire will be $x^3\pi \text{ mA}$. The value of x is _____.

Answer (5)

Sol. $J = \sigma E$

$$i = \sigma EA$$

$$= 5 \times 10^7 \times 10 \times 10^{-3} \times \pi \times (0.5 \times 10^{-3})^2$$

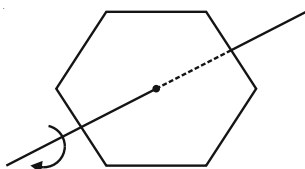
$$= 5^3 \pi \times 10^{-3} \text{ A}$$

$$= 125\pi \text{ mA}$$

7. A uniform thin bar of mass 6 kg and length 2.4 meter is bent to make an equilateral hexagon. The moment of inertia about an axis passing through the centre of mass and perpendicular to the plane of hexagon is _____ $\times 10^{-1} \text{ kg m}^2$.

Answer (8)

Sol. $m = \frac{M}{6}$ and $\ell = \frac{L}{6}$



$$I = 6 \left(\frac{m\ell^2}{12} + \frac{3m}{4}\ell^2 \right)$$

$$= 5 m\ell^2$$

$$= \frac{5}{216} ML^2$$

$$= \frac{5}{216} \times 6 \times (24)^2$$

$$= 0.8$$

8. A series LCR circuit is designed to resonate at an angular frequency $\omega_0 = 10^5 \text{ rad/s}$. The circuit draws 16 W power from 120 V source at resonance. The value of resistance 'R' in the circuit is _____ Ω .

Answer (900)

Sol. At resonance $P = \frac{V^2}{R}$

$$R = \frac{V^2}{P} = \frac{(120)^2}{16} = 900 \Omega$$

9. A signal of 0.1 kW is transmitted in a cable. The attenuation of cable is -5 dB per km and cable length is 20 km. The power received at receiver is 10^{-x} W . The value of x is _____.

$$\left[\text{Gain in dB} = 10 \log_{10} \left(\frac{P_0}{P_i} \right) \right]$$

Answer (8)

Sol. Total attenuation = -100 dB

$$-100 = 10 \log_{10} \left(\frac{P_{\text{out}}}{P_{\text{in}}} \right)$$

$$P_{\text{out}} = P_{\text{in}} \times 10^{-10}$$

$$= 100 \times 10^{-10} \text{ W}$$

$$= 10^{-8} \text{ W}$$

10. Two cars are approaching each other at an equal speed of 7.2 km/hr. When they see each other, both blow horns having frequency of 676 Hz. The beat frequency heard by each driver will be _____ Hz. [Velocity of sound in air is 340 m/s.]

Answer (8)

$$f' = f f_0 \left(\frac{V_s + V_c}{V_s - V_c} \right)$$

$$= 676 \left(\frac{340 + 2}{340 - 2} \right) = 684 \text{ Hz}$$

$$\text{Beat frequency } \Delta f = f' - f_0 = 8 \text{ Hz}$$

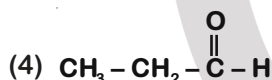
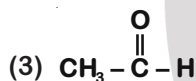
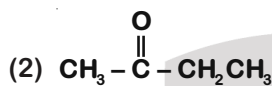
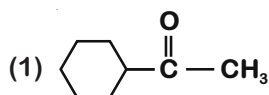
PART-B : CHEMISTRY

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

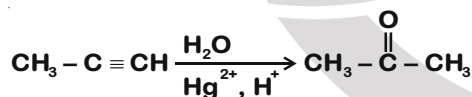
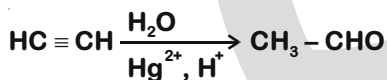
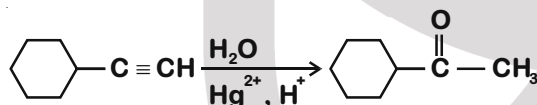
Choose the correct answer :

1. Which one of the following carbonyl compounds cannot be prepared by addition of water on an alkyne in the presence of HgSO_4 and H_2SO_4 ?



Answer (4)

Sol. $\text{CH}_3 - \text{CH}_2 - \text{CHO}$ (Propanaldehyde) cannot be prepared by addition of water on alkyne in the presence of HgSO_4 and H_2SO_4 .



2. Match List-I with List-II.

| List-I (Metal) | List-II (Ores) |
|-------------------|-------------------|
| (a) Aluminium | (i) Siderite |
| (b) Iron | (ii) Calamine |
| (c) Copper | (iii) Kaolinite |
| (d) Zinc | (iv) Malachite |

Choose the correct answer from the options given below:

(1) (a)-(i), (b)-(ii), (c)-(iii), (d)-(iv)

(2) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)

(3) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

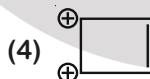
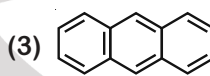
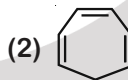
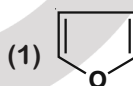
(4) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)

Answer (3)



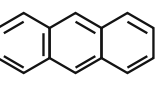

| Sol. | Ores | Formula | Metal present |
|------|-----------|---|---------------|
| | Siderite | FeCO_3 | Fe |
| | Calamine | ZnCO_3 | Zn |
| | Kaolinite | $\text{Al}_2\text{Si}_2\text{O}_5(\text{OH})_4$ | Al |
| | Malachite | $\text{Cu}(\text{OH})_2 \cdot \text{CuCO}_3$ | Cu |

(a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

3. Which one of the following compounds is non-aromatic?



Answer (2)

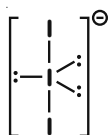
| | | |
|------|--|--------------|
| Sol. |  | Aromatic |
| |  | Non-aromatic |
| | ↑ sp^3 hybridized | |
| |  | Aromatic |
| |  | Aromatic |

4. The correct shape and I – I – I bond angles respectively in I_3^- ion are:

- (1) Distorted trigonal planar; 135° and 90°
- (2) Trigonal planar; 120°
- (3) T-shaped; 180° and 90°
- (4) Linear; 180°

Answer (4)

Sol. I_3^-



Shape = Linear

Angle $\angle I - I - I$ is 180°

5. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Hydrogen is the most abundant element in the Universe, but it is not the most abundant gas in the troposphere.

Reason R : Hydrogen is the lightest element.

In the light of the above statements, choose the correct answer from the options given below.

- (1) Both A and R are true but R is NOT the correct explanation of A
- (2) A is true but R is false
- (3) Both A and R are true and R is the correct explanation of A
- (4) A is false but R is true

Answer (3)

Sol. Both A and R are true and R is the correct explanation of A.

Hydrogen is the lightest element. Because it is lighter than air, and so it can easily escape the earth's gravity.

6. Given below are two statements :

Statement I : The value of the parameter "Biochemical Oxygen Demand (BOD)" is important for survival of aquatic life.

Statement II : The optimum value of BOD is 6.5 ppm.

In the light of the above statements, choose the most appropriate answer from the options given below:

- (1) Both statement I and statement II are true
- (2) Both statement I and statement II are false
- (3) Statement I is false but statement II is true
- (4) Statement I is true but statement II is false

Answer (4)

Sol. The amount of BOD in the water is a measure of the amount of organic material in the water, in terms of how much oxygen will be required to break it down biologically. Clean water would have BOD value of less than 5 ppm, whereas highly polluted water would have a BOD value of 17 ppm or more.

Statement I is true and statement II is false.

7. The calculated magnetic moments (spin only value) for species $[FeCl_4]^{2-}$, $[Co(C_2O_4)_3]^{3-}$ and MnO_4^{2-} respectively are :

- (1) 5.82, 0 and 0 BM
- (2) 4.90, 0 and 1.73 BM
- (3) 5.92, 4.90 and 0 BM
- (4) 4.90, 0 and 2.83 BM

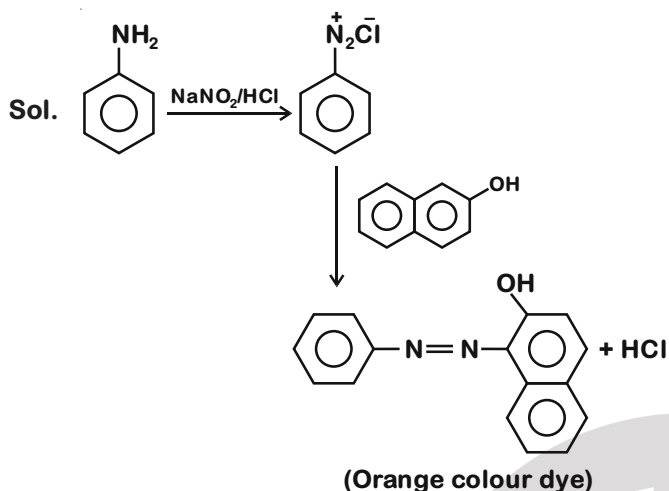
Answer (2)

Sol. $[FeCl_4]^{2-}$ Fe^{2+} + Weak ligand $n = 4$
(High spin complex) $\mu = 4.9$ BM
 $[Co(C_2O_4)_3]^{3-}$ Co^{3+} + Strong field $n = 0$
(Low spin complex) $\mu = 0$ BM
 MnO_4^{2-} Mn^{+6} $n = 1$
(Low spin complex) $\mu = 1.73$ BM
(n is number of unpaired electron and μ is spin only magnetic moment)

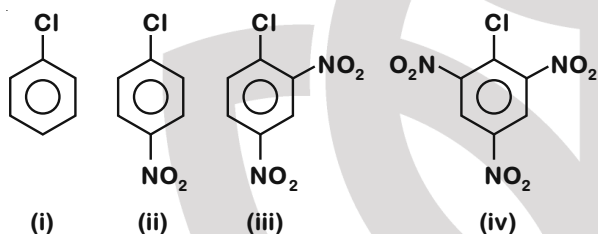
8. Match List-I and List-II.

- | List-I | List-II |
|--|-------------------------|
| (a) Valium | (i) Antifertility drug |
| (b) Morphine | (ii) Pernicious anaemia |
| (c) Norethindrone | (iii) Analgesic |
| (d) Vitamin B_{12} | (iv) Tranquilizer |
| (1) (a)-(iv), (b)-(iii), (c)-(i), (d)-(ii) | |
| (2) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i) | |
| (3) (a)-(ii), (b)-(iv), (c)-(iii), (d)-(i) | |
| (4) (a)-(i), (b)-(iii), (c)-(iv), (d)-(ii) | |

Answer (1)



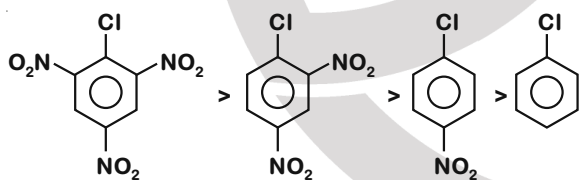
14. The correct order of the following compounds showing increasing tendency towards nucleophilic substitution reaction is :



- (1) (iv) < (i) < (ii) < (iii)
 (2) (iv) < (i) < (iii) < (ii)
 (3) (iv) < (iii) < (ii) < (i)
 (4) (i) < (ii) < (iii) < (iv)

Answer (4)

Sol. More the number of EWG attached to benzene ring, more will be the tendency towards nucleophilic substitution reaction



15. In polymer Buna-S : 'S' stands for:

- (1) Strength
 (2) Sulphonation
 (3) Styrene
 (4) Sulphur

Answer (3)

Sol. In polymer Buna-S : S stands for styrene

Buna-S is a polymer of buta 1,3 diene and styrene.

16. According to Bohr's atomic theory:

- (A) Kinetic energy of electron is $\propto \frac{Z^2}{n^2}$
 (B) The product of velocity (v) of electron and principal quantum number (n), 'vn' $\propto Z^2$
 (C) Frequency of revolution of electron in an orbit is $\propto \frac{Z^3}{n^3}$
 (D) Coulombic force of attraction on the electron is $\propto \frac{Z^3}{n^4}$

Choose the most appropriate answer from the options given below:

- (1) (A) only (2) (A) and (D) only
 (3) (C) only (4) (A), (C) and (D) only

Answer (2)

Sol. In Bohr's atomic theory:

$$\text{K.E.} \propto \frac{Z^2}{n^2}$$

$$\text{Velocity (v)} \propto \frac{Z}{n}$$

$$\therefore v \cdot n \propto Z$$

$$\text{Frequency of revolution} = \frac{v}{2\pi r}$$

$$\begin{aligned} \text{Frequency of revolution} &\propto \frac{v}{r} \\ &\propto \frac{Z \times Z}{n \times n^2} \\ &\propto \frac{Z^2}{n^3} \end{aligned}$$

$$\text{Force of attraction} = \frac{mV^2}{r}$$

$$\begin{aligned} \text{Force of attraction} &\propto \frac{v^2}{r} \\ &\propto \frac{Z^2 \times Z}{n^2 \times n^2} \\ &\propto \frac{Z^3}{n^4} \end{aligned}$$

Correct statements : (A) and (D).

17. Most suitable salt which can be used for efficient clotting of blood will be

- (1) FeCl_3
- (2) FeSO_4
- (3) NaHCO_3
- (4) $\text{Mg}(\text{HCO}_3)_2$

Answer (1)

Sol. More the positive charge, more effective is the coagulation.

Hardy-Schulzke rule - "Higher the valency or charge greater its coagulating power".

18. Match List - I with List - II.

| List - I (Salt) | List - II (Flame colour wavelength) |
|--------------------|--|
| (a) LiCl | (i) 455.5 nm |
| (b) NaCl | (ii) 670.8 nm |
| (c) RbCl | (iii) 780.0 nm |
| (d) CsCl | (iv) 589.2 nm |

Choose the correct answer from the options given below:

- (1) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)
- (2) (a)-(ii), (b)-(iv), (c)-(iii), (d)-(i)
- (3) (a)-(iv), (b)-(ii), (c)-(iii), (d)-(i)
- (4) (a)-(i), (b)-(iv), (c)-(ii), (d)-(iii)

Answer (2)

| Sol. | Salt | Flame colour wavelength |
|------|------|-------------------------|
| (a) | LiCl | 670.8 nm (Crimson red) |
| (b) | NaCl | 589.2 nm (Yellow) |
| (c) | RbCl | 780.0 nm (Red violet) |
| (d) | CsCl | 455.5 (Blue) |

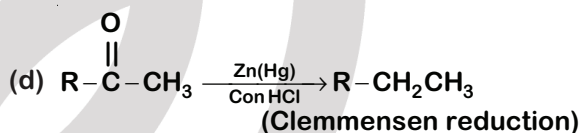
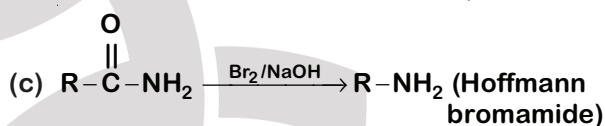
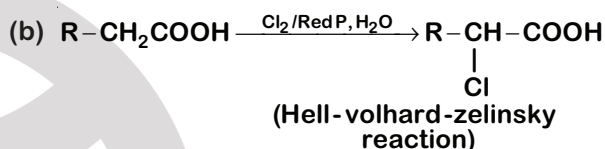
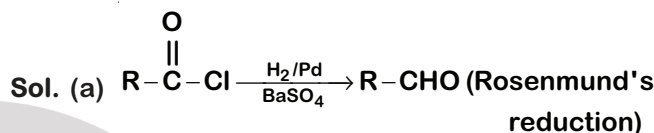
19. Match List - I and List - II.

| List - I | List - II |
|---|---|
| (a) $\text{R}-\overset{\text{O}}{\parallel}{\text{C}}-\text{Cl} \rightarrow \text{R}-\text{CHO}$ | (i) Br_2/NaOH |
| (b) $\text{R}-\text{CH}_2-\text{COOH} \rightarrow$ $\text{R}-\underset{\text{Cl}}{\text{CH}}-\text{COOH}$ | (ii) $\text{H}_2/\text{Pd} - \text{BaSO}_4$ |
| (c) $\text{R}-\overset{\text{O}}{\parallel}{\text{C}}-\text{NH}_2 \rightarrow \text{R}-\text{NH}_2$ | (iii) $\text{Zn}(\text{Hg})/\text{Conc. HCl}$ |
| (d) $\text{R}-\overset{\text{O}}{\parallel}{\text{C}}-\text{CH}_3 \rightarrow \text{R}-\text{CH}_2-\text{CH}_3$ | (iv) $\text{Cl}_2/\text{Red P, H}_2\text{O}$ |

Choose the correct answer from the options given below:

- (1) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)
- (2) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)
- (3) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)
- (4) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)

Answer (2)



20. What is the correct order of the following elements with respect to their density?

- (1) $\text{Cr} < \text{Zn} < \text{Co} < \text{Cu} < \text{Fe}$
- (2) $\text{Cr} < \text{Fe} < \text{Co} < \text{Cu} < \text{Zn}$
- (3) $\text{Zn} < \text{Cu} < \text{Co} < \text{Fe} < \text{Cr}$
- (4) $\text{Zn} < \text{Cr} < \text{Fe} < \text{Co} < \text{Cu}$

Answer (4)

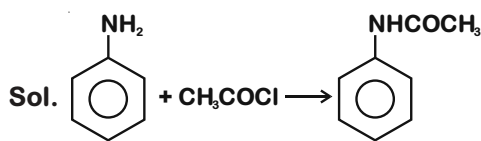
| | | | | | | |
|------|-----|-----|-----|------|-----|-----------------------------------|
| Sol. | Cu | Co | Fe | Cr | Zn | density in g/cm^3 |
| | 8.9 | 8.7 | 7.8 | 7.19 | 7.1 | |

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. 1.86 g of aniline completely reacts to form acetanilide. 10% of the product is lost during purification. Amount of acetanilide obtained after purification (in g) is _____ $\times 10^{-2}$.

Answer (243)



$$\text{No. of moles of aniline} = \frac{1.86}{93} = 0.02$$

$$\text{Moles of acetanilide formed} = 0.9 \times 0.02 = 0.018$$

$$\text{Mass of acetanilide formed (ing)} = 0.018 \times 135 = 243 \times 10^{-2}$$

2. C_6H_6 freezes at $5.5^\circ C$. The temperature at which a solution of 10 g of C_4H_{10} in 200 g of C_6H_6 freeze is $\quad^\circ C$. (The molal freezing point depression constant of C_6H_6 is $5.12^\circ C/m$.)

Answer (1)

Sol. $\Delta T_f = ik_f m$

$$i = 1 \text{ for } C_4H_{10}; T_f^\circ = 5.5^\circ C$$

$$m = \frac{10 \times 1000}{58 \times 200} = \frac{50}{58}$$

$$K_f = 5.12^\circ C/m$$

$$\Delta T_f = 5.12 \times \frac{50}{58} = 4.41^\circ C$$

$$T_f^\circ - T_f = 4.41$$

$$T_f = 5.50 - 4.41 = 1.09 = 1^\circ C$$

3. The volume occupied by 4.75 g of acetylene gas at $50^\circ C$ and 740 mmHg pressure is \quad L. (Rounded off to the nearest integer)
[Given $R = 0.0826 \text{ L atm K}^{-1} \text{ mol}^{-1}$]

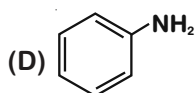
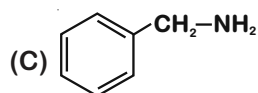
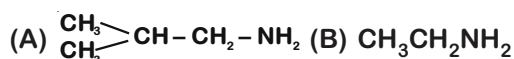
Answer (5)

Sol. $PV = nRT$

$$n = (4.75/26); P = 740/760 \text{ atm}; T = 323 \text{ K}$$

$$V = \frac{4.75 \times 0.0826 \times 323 \times 760}{26 \times 740} = 5 \text{ L}$$

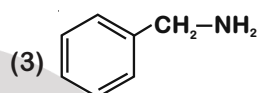
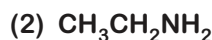
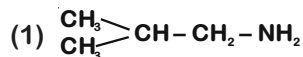
4. The total number of amines among the following which can be synthesized by Gabriel synthesis is \quad



Answer (3)

Sol. Only aliphatic primary amines can be synthesised by Gabriel phthalimide synthesis.

Out of the given amines the following amines can be synthesised by Gabriel synthesis.



5. Among the following allotropic forms of sulphur, the number of allotropic forms, which will show paramagnetism is \quad .

(A) α -sulphur

(B) β -sulphur

(C) S_2 -form

Answer (1)

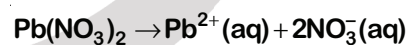
Sol. α -sulphur (Rhombic sulphur) and β -sulphur (Monoclinic sulphur) are the two allotropes of sulphur which are diamagnetic. But the S_2 -form which exists at high temperature and has structure similar to O_2 is paramagnetic.

6. The solubility product of PbI_2 is 8.0×10^{-9} . The solubility of lead iodide in 0.1 molar solution of lead nitrate is $x \times 10^{-6} \text{ mol/L}$. The value of x is \quad (Rounded off to the nearest integer)

$$[\text{Given } \sqrt{2} = 1.41]$$

Answer (141)

Sol. $PbI_2(s) \rightleftharpoons Pb^{2+}(aq) + 2I^-(aq) \quad K_{sp} = 8.0 \times 10^{-9}$



$$[Pb^{2+}] = 0.1 \text{ M}$$

$$[Pb^{2+}][I^-]^2 = 8.0 \times 10^{-9}$$

$$[I^-]^2 = \frac{8.0 \times 10^{-9}}{0.1} = 8.0 \times 10^{-8}$$

$$[I^-] = 2\sqrt{2} \times 10^{-4} \text{ M}$$

Solubility of PbI_2 in 0.1 M $Pb(NO_3)_2$ solution

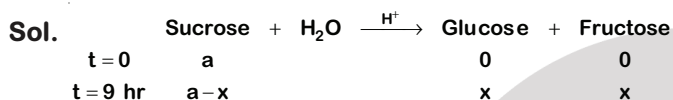
$$= \frac{[I^-]}{2} = \sqrt{2} \times 10^{-4} = 141 \times 10^{-6} \text{ M}$$

7. Sucrose hydrolyses in acid solution into glucose and fructose following first order rate law with a half-life of 3.33 h at 25°C. After 9 h, the fraction of sucrose remaining is f. The value

of $\log_{10}\left(\frac{1}{f}\right)$ is _____ $\times 10^{-2}$. (Rounded off to the nearest integer)

[Assume : $\ln 10 = 2.303$, $\ln 2 = 0.693$]

Answer (81)



$$k = \frac{0.693}{3.33} \text{hr}^{-1}$$

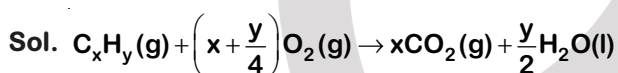
$$k = \frac{2.303}{t} \log \frac{a}{a-x}$$

$$\frac{0.693 \times 9}{3.33 \times 2.303} = \log_{10} \left(\frac{1}{f} \right)$$

$$\log_{10} \left(\frac{1}{f} \right) = 81$$

8. The formula of a gaseous hydrocarbon which requires 6 times of its own volume of O_2 for complete oxidation and produces 4 times its own volume of CO_2 is C_xH_y . The value of y is _____.

Answer (8)

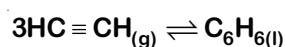


$$x + \frac{y}{4} = 6$$

$$x = 4$$

$$y = 8$$

9. Assuming ideal behaviour, the magnitude of $\log K$ for the following reaction at 25°C is $x \times 10^{-1}$. The value of x is _____ (Integer answer)



[Given : $\Delta_f G^\circ(\text{HC} \equiv \text{CH}) = -2.04 \times 10^5 \text{ J mol}^{-1}$;

$\Delta_f G^\circ(\text{C}_6\text{H}_6) = -1.24 \times 10^5 \text{ J mol}^{-1}$; $R = 8.314 \text{ J K}^{-1} \text{ mol}^{-1}$]

Answer (855)



$$\Delta G^\circ = \Delta G_f^\circ(\text{C}_6\text{H}_6) - 3\Delta G_f^\circ(\text{HC} \equiv \text{CH})$$

$$= -1.24 \times 10^5 - 3(-2.04 \times 10^5)$$

$$= 4.88 \times 10^5 \text{ J mol}^{-1}$$

$$\Delta G^\circ = -RT \ln K$$

$$= -2.303 RT \log K$$

$$|\log K| = \frac{4.88 \times 10^5}{2.303 \times 8.314 \times 298}$$

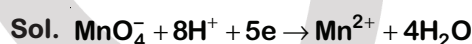
$$x \times 10^{-1} = 85.5$$

$$x = 855$$

10. The magnitude of the change in oxidising power of the $\text{MnO}_4^-/\text{Mn}^{2+}$ couple is $x \times 10^{-4} \text{ V}$, if the H^+ concentration is decreased from 1 M to 10^{-4} M at 25°C. (Assume concentration of MnO_4^- and Mn^{2+} to be same on change in H^+ concentration). The value of x is _____ (Rounded off to the nearest integer)

$$\left[\text{Given; } \frac{2.303RT}{F} = 0.059 \right]$$

Answer (3776)



$$E_{\text{MnO}_4^-/\text{Mn}^{2+}} = E^\circ_{\text{MnO}_4^-/\text{Mn}^{2+}} - \frac{0.059}{5} \log \frac{[\text{Mn}^{2+}]}{[\text{MnO}_4^-][\text{H}^+]^8}$$

$$\text{If } [\text{H}^+] = 1 \text{ M } E_{\text{MnO}_4^-/\text{Mn}^{2+}} = E^\circ_{\text{MnO}_4^-/\text{Mn}^{2+}}$$

$$\text{If } [\text{H}^+] = 10^{-4} \text{ M}$$

$$E_{\text{MnO}_4^-} = E^\circ_{\text{MnO}_4^-/\text{Mn}^{2+}} - \frac{0.059}{5} \log 10^{32}$$

$$= E^\circ_{\text{MnO}_4^-/\text{Mn}^{2+}} - 0.3776$$

Magnitude of change in oxidising power

$$= 3776 \times 10^{-4}$$

PART-C : MATHEMATICS

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

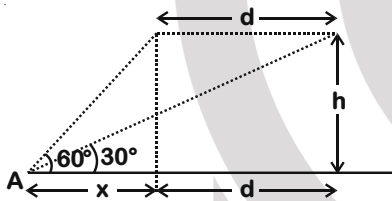
Choose the correct answer :

1. The angle of elevation of a jet plane from a point A on the ground is 60° . After a flight of 20 seconds at the speed of 432 km/hour, the angle of elevation changes to 30° . If the jet plane is flying at a constant height, then its height is :

- (1) $3600\sqrt{3}$ m (2) $2400\sqrt{3}$ m
 (3) $1800\sqrt{3}$ m (4) $1200\sqrt{3}$ m

Answer (4)

Sol.



$$\text{Given } \tan 30^\circ = \frac{h}{x+d} \text{ and } \tan 60^\circ = \frac{h}{x}$$

$$\Rightarrow x+d = \sqrt{3}h \text{ and } x = \frac{h}{\sqrt{3}}$$

$$\Rightarrow d = \left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)h = \frac{2h}{\sqrt{3}}$$

$$\text{Given } \frac{d}{20} = \frac{432 \times 1000}{3600}$$

$$\Rightarrow d = 2400 \text{ m}$$

$$\Rightarrow 2400 = \frac{2h}{\sqrt{3}} \Rightarrow h = 1200\sqrt{3} \text{ m}$$

2. The probability that two randomly selected subsets of the set $\{1, 2, 3, 4, 5\}$ have exactly two elements in their intersection, is :

- (1) $\frac{65}{2^7}$ (2) $\frac{135}{2^9}$
 (3) $\frac{65}{2^8}$ (4) $\frac{35}{2^7}$

Answer (2)

Sol. Number of ways of selecting elements common to both A and B = 5C_2

$$\therefore \text{ Required probability} = \frac{{}^5C_2 \cdot 3^3}{4^5} = \frac{135}{2^9}$$

3. If P is a point on the parabola $y = x^2 + 4$ which is closest to the straight line $y = 4x - 1$, then the co-ordinates of P are :

- (1) $(-2, 8)$ (2) $(1, 5)$
 (3) $(3, 13)$ (4) $(2, 8)$

Answer (4)

Sol. Closest point will be point of tangency of tangent of same slope i.e. 4

Let equation of tangent $y = 4x + c$

$$\Rightarrow 4x + c = x^2 + 4 \text{ have } D = 0$$

$$\text{i.e. } x^2 - 4x + (4 - c) = 0$$

$$D = 0 \Rightarrow 16 - 4(4 - c) = 0 \Rightarrow c = 0$$

Tangent is $y = 4x$ gives $x = 2$ and $y = 8$ as point of tangency

\therefore Nearest point $(2, 8)$

4. If $n \geq 2$ is a positive integer, then the sum of the series

${}^{n+1}C_2 + 2({}^2C_2 + {}^3C_2 + {}^4C_2 + \dots + {}^nC_2)$ is :

- (1) $\frac{n(2n+1)(3n+1)}{6}$ (2) $\frac{n(n-1)(2n+1)}{6}$
 (3) $\frac{n(n+1)^2(n+2)}{12}$ (4) $\frac{n(n+1)(2n+1)}{6}$

Answer (4)

Sol. Sum of ${}^2C_2 + {}^3C_2 + \dots + {}^nC_2$ is coefficient of x^2 in $(1+x)^2 + (1+x)^3 + \dots + (1+x)^n$

i.e. coefficient of x^2 in

$$(1+x)^2 \frac{((1+x)^{n-1} - 1)}{(1+x) - 1} = {}^{n+1}C_3$$

Hence required sum = ${}^{n+1}C_2 + 2 \cdot {}^{n+1}C_3$

$$= \frac{(n+1)(n)}{2} + \frac{2(n+1)n(n-1)}{6}$$

$$= \frac{n(n+1)(3+2n-2)}{2 \cdot 3} = \frac{n(n+1)(2n+1)}{6}$$

5. Let $f(x)$ be a differentiable function defined on $[0, 2]$ such that $f(x) = f(2 - x)$ for all $x \in (0, 2)$,

$f(0) = 1$ and $f(2) = e^2$. Then the value of $\int_0^2 f(x) dx$ is :

- (1) $2(1 + e^2)$ (2) $1 + e^2$
 (3) $2(1 - e^2)$ (4) $1 - e^2$

Answer (2)

Sol. Given $f(x) = f(2 - x)$

$$\Rightarrow f(x) - f(2 - x) = 0$$

Integrating both sides, we get

$$f(x) + f(2 - x) = c \quad \dots(i)$$

Put $x = 0$, we get

$$c = f(0) + f(2) = 1 + e^2$$

Integrating 0 to 2 equation (i) both sides, we get

$$\int_0^2 f(x) dx + \int_0^2 f(2 - x) dx = (1 + e^2) \times 8 \Big|_0^2$$

$$\text{Also } \int_0^2 f(x) dx = \int_0^2 f(2 - x) dx$$

$$\text{Hence } 2 \int_0^2 f(x) dx = 2(1 + e^2)$$

$$\Rightarrow \int_0^2 f(x) dx = 1 + e^2$$

6. For the statements p and q , consider the following compound statements:

(a) $(\sim q \wedge (p \rightarrow q)) \rightarrow \sim p$

(b) $((p \vee q) \wedge \sim p) \rightarrow q$

Then which of the following statements is correct?

- (1) (a) and (b) both are tautologies.
 (2) (a) is a tautology but not (b).
 (3) (b) is a tautology but not (a).
 (4) (a) and (b) both are not tautologies.

Answer (1)

Sol. Truth table for required statements

| p | q | $\sim p$ | $\sim q$ | $p \rightarrow q$ | $(\sim q) \wedge (p \rightarrow q)$ | $(\sim q \wedge (p \rightarrow q)) \rightarrow \sim p$ | $p \vee q$ | $(p \vee q) \wedge \sim p$ | $((p \vee q) \wedge \sim p) \rightarrow q$ |
|-----|-----|----------|----------|-------------------|-------------------------------------|--|------------|----------------------------|--|
| T | T | F | F | T | F | T | T | F | T |
| T | F | F | T | F | T | T | T | F | T |
| F | T | T | F | T | F | T | T | T | T |
| F | F | T | T | T | T | T (a) is tautology | F | F | T (b) is tautology |

7. For the system of linear equations :

$$x - 2y = 1, \quad x - y + kz = -2, \quad ky + 4z = 6, \quad k \in \mathbb{R},$$

consider the following statements :

- (A) The system has unique solution if $k \neq 2, k \neq -2$.
 (B) The system has unique solution if $k = -2$.
 (C) The system has unique solution if $k = 2$
 (D) The system has no solution if $k = 2$.
 (E) The system has infinite number of solutions if $k \neq -2$.

Which of the following statements are correct?

- (1) (A) and (E) only (2) (A) and (D) only
 (3) (B) and (E) only (4) (C) and (D) only

Answer (2)

Sol. Using Cramer's Rule, we have

$$\Delta = \begin{vmatrix} 1 & -2 & 0 \\ 1 & -1 & k \\ 0 & k & 4 \end{vmatrix} = 1(-4 - k^2) + 2(4) = 4 - k^2$$

$$\Delta_x = \begin{vmatrix} 1 & -2 & 0 \\ -2 & -1 & k \\ 6 & k & 4 \end{vmatrix} = 1(-4 - k^2) + 2(-2k + 6) = 8 - 4k - k^2$$

Now, $\Delta = 0$ if $k = \pm 2$

if $k = -2$, $\Delta = 0$ and $\Delta_x \neq 0$

Hence no solution

Also if $k = 2$, $\Delta = 0$ and $\Delta_x = 0$

Now

$$\Delta_y = \begin{vmatrix} 1 & 1 & 0 \\ 1 & -2 & k \\ 0 & 6 & 4 \end{vmatrix} = 1(-8 - 6k) - 1(4) = -6k - 12 \neq 0$$

Hence, the system has no solution if $k = \pm 2$

and unique solution if $k \neq \pm 2$

8. Let a, b, c be in arithmetic progression. Let the centroid of the triangle with vertices

(a, c), (2, b) and (a, b) be $\left(\frac{10}{3}, \frac{7}{3}\right)$. If α, β are

the roots of the equation $ax^2 + bx + 1 = 0$, then the value of $\alpha^2 + \beta^2 - \alpha\beta$ is :

- (1) $\frac{69}{256}$ (2) $-\frac{71}{256}$
 (3) $-\frac{69}{256}$ (4) $\frac{71}{256}$

Answer (2)

Sol. Here, $2b = a + c$... (i)

and centroid of Δ is $(\frac{10}{3}, \frac{7}{3})$

$$\Rightarrow \frac{a+2+a}{3} = \frac{10}{3} \Rightarrow a = 4$$

$$\text{and } \frac{c+b+b}{3} = \frac{7}{3} \Rightarrow \frac{c+(a+c)}{3} = \frac{7}{3} \Rightarrow 2c + a = 7$$

$$\Rightarrow 2c + 4 = 7$$

$$\Rightarrow c = \frac{3}{2}$$

$$\text{So from (i) } 2b = \frac{11}{2} \Rightarrow \boxed{b = \frac{11}{4}}$$

So the Q.E. if $4x^2 + \frac{11}{4}x + 1 = 0$

$$\Rightarrow 16x^2 + 11x + 4 = 0 \Rightarrow \alpha + \beta = \frac{-11}{16}, \alpha\beta = \frac{1}{4}$$

$$\text{Now, } \alpha^2 + \beta^2 - \alpha\beta = \alpha^2 + \beta^2 + 2\alpha\beta - 3\alpha\beta$$

$$= (\alpha + \beta)^2 - 3\alpha\beta$$

$$= \left(\frac{-11}{16}\right)^2 - \frac{3}{4}$$

$$= \frac{121}{256} - \frac{3}{4} = \frac{71}{256}$$

9. A possible value of $\tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$ is :

(1) $2\sqrt{2} - 1$

(2) $\sqrt{7} - 1$

(3) $\frac{1}{\sqrt{7}}$

(4) $\frac{1}{2\sqrt{2}}$

Answer (3)

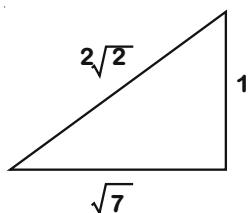
$$\text{Sol. } \tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right) \Rightarrow \tan\left(\frac{\theta}{4}\right) = ?$$

Let

$$\sin^{-1}\frac{\sqrt{63}}{8} = \theta \text{ and } \cos\theta = \frac{1}{8} \Rightarrow \sin\theta = \frac{\sqrt{63}}{8} \Rightarrow \tan\theta = \sqrt{63} \text{ and } 6$$

$$\Rightarrow 2\cos^2\frac{\theta}{2} - 1 = \frac{1}{8} \Rightarrow 2\cos^2\frac{\theta}{2} = \frac{9}{8} \Rightarrow \cos^2\frac{\theta}{2} = \frac{9}{16}$$

$$\Rightarrow \cos\frac{\theta}{2} = \frac{3}{4}$$



$$\Rightarrow 2\cos^2\frac{\theta}{4} - 1 = \frac{3}{4}$$

$$\Rightarrow 2\cos^2\frac{\theta}{4} = \frac{7}{4}$$

$$\Rightarrow \cos^2\frac{\theta}{4} = \frac{7}{8} \Rightarrow \cos\frac{\theta}{4} = \frac{\sqrt{7}}{2\sqrt{2}}$$

$$\Rightarrow \tan\frac{\theta}{4} = \frac{1}{\sqrt{7}}$$

10. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined as

$$f(x) = \begin{cases} -55x, & \text{if } x < -5 \\ 2x^3 - 3x^2 - 120x, & \text{if } -5 \leq x \leq 4 \\ 2x^3 - 3x^2 - 36x - 336, & \text{if } x > 4, \end{cases}$$

Let $A = \{x \in \mathbb{R} : f \text{ is increasing}\}$. Then A is equal to:

(1) $(-5, \infty)$

(2) $(-\infty, -5) \cup (4, \infty)$

(3) $(-5, -4) \cup (4, \infty)$

(4) $(-\infty, -5) \cup (-4, \infty)$

Answer (3)

Sol.

$$f(x) = \begin{cases} -55x, & x < -5 \\ 2x^3 - 3x^2 - 120x, & -5 \leq x \leq 4 \\ 2x^3 - 3x^2 - 36x - 336, & x > 4 \end{cases}$$

$$\text{Now, } f'(x) = \begin{cases} -55, & x < -5 \\ 6x^2 - 6x - 120, & -5 \leq x < 4 \\ 6x^2 - 6x - 36, & x > 4 \end{cases}$$

$$\Rightarrow f'(x) = \begin{cases} -55, & x < -5 \\ 6(x-5)(x+4), & -5 < x < 4 \\ 6(x-3)(x+2), & x > 4 \end{cases}$$

For increasing $f'(x) > 0$

So clearly $f(x)$ is increasing for $x \in (-5, -4) \cup (4, \infty)$

11. The value of the integral, $\int_1^3 [x^2 - 2x - 2] dx$, where $[x]$ denotes the greatest integer less than or equal to x , is:

(1) -5

(2) $-\sqrt{2} - \sqrt{3} + 1$

(3) $-\sqrt{2} - \sqrt{3} - 1$

(4) -4

Answer (3)

$$\text{Sol. } I = \int_1^3 [x^2 - 2x - 2] dx$$

$$= \int_1^3 [(x^2 - 2x + 1) - 3] dx = \int_1^3 [(x-1)^2] dx - \int_1^3 3 dx$$

Now when $x \in [1, 3]$

we see that $(x - 1)^2 \in [0, 4]$

$$\begin{aligned} \text{So, } I &= \int_0^1 0 dx + \int_1^{\sqrt{2}} [(x-1)^2] dx + \int_{\sqrt{2}}^{\sqrt{3}} [(x-1)^2] dx + \\ &\quad \int_{\sqrt{3}}^2 [(x-1)^2] dx - \int_1^3 3 dx \\ &= 0 + \int_1^{\sqrt{2}} 1 dx + \int_{\sqrt{2}}^{\sqrt{3}} 2 dx + \int_{\sqrt{3}}^2 3 dx - 6 \\ &= 0 + [x]_1^{\sqrt{2}} + 2[x]_{\sqrt{2}}^{\sqrt{3}} + 3[x]_{\sqrt{3}}^2 - 6 \\ &= \sqrt{2} - 1 + 2\sqrt{3} - 2\sqrt{2} + 6 - 3\sqrt{3} - 6 \\ &= -\sqrt{3} - \sqrt{2} - 1 \\ &= -\sqrt{2} - \sqrt{3} - 1 \end{aligned}$$

12. The negation of the statement

$\sim p \wedge (p \vee q)$ is :

- (1) $p \wedge \sim q$ (2) $\sim p \vee q$
 (3) $\sim p \wedge q$ (4) $p \vee \sim q$

Answer (4)

$$\begin{aligned} \text{Sol. } \sim(\sim p \wedge (p \vee q)) &= p \vee \sim(p \vee q) \\ &= p \vee (\sim p \wedge \sim q) \\ &= (p \vee \sim p) \wedge (p \vee \sim q) \\ &= p \vee \sim q \end{aligned}$$

13. If the curve $y = ax^2 + bx + c$, $x \in \mathbb{R}$, passes through the point (1, 2) and the tangent line to this curve at origin is $y = x$, then the possible values of a, b, c are :

- (1) $a = \frac{1}{2}, b = \frac{1}{2}, c = 1$
 (2) $a = 1, b = 0, c = 1$
 (3) $a = 1, b = 1, c = 0$
 (4) $a = -1, b = 1, c = 1$

Answer (3)

$$\begin{aligned} \text{Sol. } y &= ax^2 + bx + c \text{ passes through } (1, 2) \\ \text{So } a + b + c &= 2 \quad \dots(1) \\ \text{also } (0, 0) &\text{ satisfies } \Rightarrow c = 0 \quad \dots(2) \\ \text{also slope of tangent at origin is } 1 &\text{ i.e.} \\ y' = 2ax + b &\Rightarrow b = 1 \text{ and } a = 1 \\ \therefore a = 1 = b, c = 0 \end{aligned}$$

14. For which of the following curves, the line $x + \sqrt{3}y = 2\sqrt{3}$ is the tangent at the point

$$\left(\frac{3\sqrt{3}}{2}, \frac{1}{2}\right)?$$

- (1) $x^2 + 9y^2 = 9$ (2) $y^2 = \frac{1}{6\sqrt{3}}x$
 (3) $2x^2 - 18y^2 = 9$ (4) $x^2 + y^2 = 7$

Answer (1)

$$\text{Sol. } x + \sqrt{3}y = 2\sqrt{3}$$

$$m_t = \frac{-1}{\sqrt{3}} \text{ and point of tangency } \left(\frac{3\sqrt{3}}{2}, \frac{1}{2}\right)$$

$$\text{Option (1) } x^2 + 9y^2 = 9 \Rightarrow 2x + 18yy' = 0$$

$$\Rightarrow m_t \left(\frac{3\sqrt{3}}{2}, \frac{1}{2}\right)$$

$$\text{i.e. } y' = \frac{-x}{9y} = \frac{-3\sqrt{3}}{9\left(\frac{1}{2}\right)} = \frac{-1}{\sqrt{3}}$$

$$\text{Option (2) } y' = \frac{x}{6\sqrt{3}} \Rightarrow y' = \frac{1}{12\sqrt{3}y}$$

$$\text{i.e. } m_t = \frac{1}{6\sqrt{3}}$$

$$\text{Option (3) } 2x^2 - 18y^2 = 9 \Rightarrow 4x - 36yy' = 0$$

$$\text{i.e. } m_t = \frac{x}{9y} = \frac{3\sqrt{3}}{2.9\frac{1}{2}} = \frac{1}{\sqrt{3}}$$

$$\text{Option (4) } x^2y^2 = 7 \Rightarrow y' = -\frac{x}{y}$$

$$\text{i.e. } m_t = -3\sqrt{3}$$

Hence only option (1) is correct

15. If a curve $y = f(x)$ passes through the point

(1, 2) and satisfies $x \frac{dy}{dx} + y = bx^4$, then for what

$$\text{value of } b, \int_1^2 f(x) dx = \frac{62}{5}?$$

- (1) $\frac{31}{5}$ (2) $\frac{62}{5}$
 (3) 10 (4) 5

Answer (3)

Sol. $x \frac{dy}{dx} + y = bx^4$

$$\Rightarrow \frac{dy}{dx} + \frac{y}{x} = bx^3$$

$$\text{I.F} = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$$

$$\Rightarrow yx = \int bx^4 dx$$

$$\Rightarrow xy = \frac{bx^5}{5} + c$$

$$\downarrow (1, 2)$$

$$\Rightarrow 2 = \frac{b}{5} + c \Rightarrow c = 2 - \frac{b}{5}$$

$$\therefore y = \frac{bx^4}{5} + \frac{1}{x} \left(2 - \frac{b}{5} \right)$$

$$\int_1^2 f(x) dx = \left. \frac{bx^5}{25} \right|_1^2 + \left(2 - \frac{b}{5} \right) \ln x \Big|_1^2$$

$$\frac{31b}{25} + \left(2 - \frac{b}{5} \right) \ln 2 = \frac{62}{5}$$

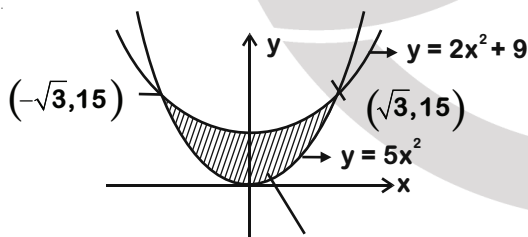
$$\Rightarrow \left(2 - \frac{b}{5} \right) \ln 2 = \left(2 - \frac{b}{5} \right) \frac{31}{5}$$

$$\Rightarrow 2 - \frac{b}{5} = 0 \Rightarrow b = 10$$

16. The area of the region : $R = \{(x, y) : 5x^2 \leq y \leq 2x^2 + 9\}$ is :

- (1) $6\sqrt{3}$ square units
- (2) $11\sqrt{3}$ square units
- (3) $12\sqrt{3}$ square units
- (4) $9\sqrt{3}$ square units

Answer (3)



Sol.

Required Area

$$\text{Required Area} = 2 \int_0^{\sqrt{3}} \left((2x^2 + 9) - (5x^2) \right) dx$$

$$= 2 \left(9x - \frac{3x^3}{3} \right) \Big|_0^{\sqrt{3}}$$

$$= 2(9\sqrt{3} - 3\sqrt{3}) = 12\sqrt{3} \text{ sq.units}$$

17. The vector equation of the plane passing through the intersection of the planes

$$\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1 \text{ and } \vec{r} \cdot (\hat{i} - 2\hat{j}) = -2, \text{ and the point } (1, 0, 2) \text{ is :}$$

$$(1) \vec{r} \cdot (\hat{i} + 7\hat{j} + 3\hat{k}) = \frac{7}{3} \quad (2) \vec{r} \cdot (3\hat{i} + 7\hat{j} + 3\hat{k}) = 7$$

$$(3) \vec{r} \cdot (\hat{i} - 7\hat{j} + 3\hat{k}) = \frac{7}{3} \quad (4) \vec{r} \cdot (\hat{i} + 7\hat{j} + 3\hat{k}) = 7$$

Answer (4)

Sol. $P_1 + \lambda P_2 = 0$

$$\Rightarrow (\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) - 1) + \lambda (\vec{r} \cdot (\hat{i} - 2\hat{j}) + 2) = 0 \quad \dots(1)$$

$$\downarrow (\hat{i} + 2\hat{k})$$

$$\Rightarrow (1 + 2 - 1) + \lambda (1 + 2) = 0 \Rightarrow \lambda = -\frac{2}{3} \quad \dots(2)$$

by (1) and (2)

$$\vec{r} \cdot \left(\hat{i} + \hat{j} + \hat{k} - \frac{2}{3}(\hat{i} - 2\hat{j}) \right) - 1 - \frac{4}{3} = 0$$

$$\Rightarrow \vec{r} \cdot \left(\frac{\hat{i}}{3} + \frac{7\hat{j}}{3} + \hat{k} \right) = \frac{7}{3}$$

$$\Rightarrow \vec{r} \cdot (\hat{i} + 7\hat{j} + 3\hat{k}) = 7$$

18. Let A and B be 3×3 real matrices such that A is symmetric matrix and B is skew-symmetric matrix. Then the system of linear equations $(A^2B^2 - B^2A^2)X = O$, where X is a 3×1 column matrix of unknown variables and O is a 3×1 null matrix, has :

- (1) exactly two solutions
- (2) infinitely many solutions
- (3) no solution
- (4) a unique solution

Answer (2)

Sol. Let $C = A^2B^2 - B^2A^2$

$$\text{Then } C^T = (A^2B^2 - B^2A^2)^T$$

$$= (B^T)^2 \cdot (A^T)^2 - (A^T)^2 \cdot (B^T)^2$$

$$= (-B)^2 A^2 - A^2 \cdot (-B)^2 \quad \{\because A^T = A \text{ and } B^T = -B\}$$

$$= (B)^2 A^2 - A^2 B^2$$

$$\therefore C + C^T = 0$$

$$\therefore C \text{ is a skew symmetric odd order matrix}$$

$$\therefore |C| = |A^2B^2 - B^2A^2| = 0$$

$$\therefore \text{Equation } (A^2B^2 - B^2A^2)X = O \text{ has}$$

Infinite many solution

19. Let $a, b \in \mathbb{R}$. If the mirror image of the point $P(a, 6, 9)$ with respect to the line

$$\frac{x-3}{7} = \frac{y-2}{5} = \frac{z-1}{-9} \text{ is } (20, b, -a-9), \text{ then } |a+b|$$

is equal to :

- (1) 86 (2) 88
(3) 90 (4) 84

Answer (2)

Sol. \therefore Mirror image of $P(a, 6, 9)$ is $Q(20, b, -a-9)$. so

Mid-point of PQ (i.e., $(\frac{a}{2}+10, \frac{b}{2}+3, -\frac{a}{2})$) lies on the given line

$$\frac{\frac{a}{2}+7}{7} = \frac{\frac{b}{2}+1}{5} = \frac{-\frac{a}{2}-1}{-9} \Rightarrow a = -56 \text{ and } b = -32$$

$$a+b = -88$$

$$|a+b| = 88$$

20. Let f be a twice differentiable function defined on \mathbb{R} such that $f(0) = 1$, $f'(0) = 2$ and $f'(x) \neq 0$ for

all $x \in \mathbb{R}$. If $\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix} = 0$, for all $x \in \mathbb{R}$, then

the value of $f(1)$ lies in the interval :

- (1) (0, 3) (2) (9, 12)
(3) (3, 6) (4) (6, 9)

Answer (4)

Sol. $\therefore \begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix} = 0$,

$$\Rightarrow \frac{f(x) \cdot f''(x) - (f'(x))^2}{(f'(x))^2} = 0 \quad \therefore f(x) \neq 0.$$

$$\therefore \frac{d}{dx} \left(\frac{f'(x)}{f(x)} \right) = 0.$$

$$\Rightarrow \frac{f'(x)}{f(x)} = c, \quad \therefore f(0) = 1 \text{ and } f'(0) = 2$$

$$\therefore c = 2$$

$$\therefore f'(x) = 2f(x)$$

$$\Rightarrow \frac{f'(x)}{f(x)} = 2$$

$$\Rightarrow \ln|f(x)| = 2x + d.$$

$$\Rightarrow |f(x)| = e^{2x}. \quad \therefore f(0) = 1.$$

$$\therefore f(1) = e^2$$

$$\therefore f(1) \in (6, 9)$$

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. The sum of first four terms of a geometric progression (G.P) is $\frac{65}{12}$ and the sum of their respective reciprocals is $\frac{65}{18}$. If the product of first three terms of the G.P. is 1, and the third term is α , then 2α is _____.

Answer (3)

Sol. Let the G.P be $ar^3, ar, \frac{a}{r}, \frac{a}{r^3}, \dots$

$$\therefore a \left(r^3 + r + \frac{1}{r} + \frac{1}{r^3} \right) = \frac{65}{12} \quad \dots (i)$$

$$\frac{1}{a} \left(r^3 + r + \frac{1}{r} + \frac{1}{r^3} \right) = \frac{65}{18} \quad \dots (ii)$$

$$\Rightarrow a^2 = \frac{3}{2}$$

$$\text{Also } a^3 r^3 = 1 \Rightarrow r = \frac{1}{a}$$

$$\text{and } \alpha = \frac{a}{r} = a^2 = \frac{3}{2}$$

$$\Rightarrow 2\alpha = 3$$

2. Let λ be an integer. If the shortest distance between the lines $x - \lambda = 2y - 1 = -2z$ and $x = y + 2\lambda = z - \lambda$ is $\frac{\sqrt{7}}{2\sqrt{2}}$, the the value of $|\lambda|$ is _____.

Answer (01)

$$\text{Sol. } L_1: \frac{x-\lambda}{1} = \frac{y-\frac{1}{2}}{\frac{1}{2}} = \frac{z}{-\frac{1}{2}}$$

$$L_2: \frac{x}{1} = \frac{y-2\lambda}{1} = \frac{z-\lambda}{1}$$

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & \frac{1}{2} & -\frac{1}{2} \\ 1 & 1 & 1 \end{vmatrix} = \hat{i} - \frac{3}{2}\hat{j} + \frac{1}{2}\hat{k}$$

$$\overline{a_1 - a_2} = \lambda \hat{i} + \left(\frac{1}{2} + 2\lambda\right) \hat{j} - \lambda \hat{k}$$

$$d = \left| \frac{(\overline{a_1 - a_2}) \cdot (\overline{b_1 \times b_2})}{(\overline{b_1 \times b_2})} \right| = \left| \frac{\frac{5\lambda}{2} + \frac{3}{4}}{\frac{\sqrt{7}}{2}} \right| = \frac{\sqrt{7}}{2\sqrt{2}}$$

$$\Rightarrow \left| \frac{5\lambda}{2} + \frac{3}{4} \right| = \frac{7}{4} \Rightarrow \lambda = \frac{2}{5} \text{ or } -1$$

$$\Rightarrow |\lambda| = 1$$

3. Let $i = \sqrt{-1}$. If $\frac{(-1+i\sqrt{3})^{21}}{(1-i)^{24}} + \frac{(1+i\sqrt{3})^{21}}{(1+i)^{24}} = k$, and

$n = [k]$ be the greatest integral part of $|k|$.

Then $\sum_{j=0}^{n+5} (j+5)^2 - \sum_{j=0}^{n+5} (j+5)$ is equal to _____.

Answer (310)

Sol. $k = \frac{(-1+i\sqrt{3})^{21}}{(1-i)^{24}} + \frac{(1+i\sqrt{3})^{21}}{(1+i)^{24}}$

$$= \frac{\left(2e^{\frac{2\pi i}{3}}\right)^{21}}{\left(\sqrt{2}e^{-\frac{\pi i}{4}}\right)^{24}} + \frac{\left(2e^{\frac{\pi i}{3}}\right)^{21}}{\left(\sqrt{2}e^{\frac{\pi i}{4}}\right)^{24}}$$

$$= 2^9 \left[e^{(14\pi + 6\pi)i} + e^{(7\pi - 6\pi)i} \right]$$

$$= 2^9 [0] = 0$$

Now $\sum_{j=0}^5 \left\{ (j+5)^2 - (j+5) \right\} = \sum_{j=0}^5 (j+4)(j+5)$

$$= \sum_{j=0}^5 \frac{1}{3} \left\{ (j+4)(j+5)(j+6) - (j+3)(j+4)(j+5) \right\}$$

$$= \frac{1}{3} [9 \times 10 \times 11 - 3 \times 4 \times 5] = 310$$

4. The number of the real roots of the equation

$$(x+1)^2 + |x-5| = \frac{27}{4} \text{ is } \underline{\hspace{2cm}}.$$

Answer (2)

Sol. If $x \geq 5$

$$(x^2 + 2x + 1) + (x - 5) = \frac{27}{4}$$

$$\Rightarrow x^2 + 3x - \frac{43}{4} = 0$$

$$\Rightarrow x = \frac{-3 \pm \sqrt{52}}{2} \text{ (No value is greater than or equal to 5)}$$

If $x < 5$

$$x^2 + 2x + 1 - x + 5 = \frac{27}{4}$$

$$\Rightarrow x^2 + x - \frac{3}{4} = 0$$

$$\Rightarrow x = \frac{1}{2}, -\frac{3}{2}$$

So there will be two real roots.

5. The students S_1, S_2, \dots, S_{10} are to be divided into 3 groups A, B and C such that each group has at least one student and the group C has at most 3 students. Then the total number of possibilities of forming such groups is _____.

Answer (31650)

Sol. Number of possible ways when

(i) There is one student in group C = ${}^{10}C_1 \cdot (2^9 - 2) = 5100$

(ii) There are two students in group C = ${}^{10}C_2 \cdot (2^8 - 2) = 11430$

(iii) There are three students in group C = ${}^{10}C_3 \cdot (2^7 - 2) = 15120$

Total number of ways = 31650

6. Let a point P be such that its distance from the point (5, 0) is thrice the distance of P from the point (-5, 0). If the locus of the point P is a circle of radius r, then $4r^2$ is equal to _____.

Answer (56.25*)

Sol. Let P(x, y)

$$\sqrt{(x-5)^2 + y^2} = 3\sqrt{(x+5)^2 + y^2}$$

$$\Rightarrow x^2 + 25 - 10x + y^2 = 9(x^2 + y^2 + 10x + 25)$$

$$\Rightarrow 8x^2 + 8y^2 + 100x + 200 = 0$$

$$\Rightarrow x^2 + y^2 + \frac{25}{2}x + 25 = 0$$

$$r = \sqrt{\left(\frac{25}{4}\right)^2 - 25} = 5\left(\frac{3}{4}\right) = \frac{15}{4}$$

$$\Rightarrow 4r^2 = 56.25$$

7. For integers n and r, let

$$\binom{n}{r} = \begin{cases} {}^n C_r, & \text{if } n \geq r \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

The maximum value of k for which the sum

$$\sum_{i=0}^k \binom{10}{i} \binom{15}{k-i} + \sum_{i=0}^{k+1} \binom{12}{i} \binom{13}{k+i-i}$$
 exists, is equal to _____.

Answer (12)

Sol. $\sum_{i=0}^k {}^{10}C_i \cdot {}^{15}C_{k-i} + \sum_{i=0}^{k+1} {}^{12}C_i \cdot {}^{13}C_{k+1-i}$

$$= {}^{25}C_k + {}^{25}C_{k+1}$$

$$= {}^{26}C_{k+1}$$

$$0 \leq k+1 \leq 25$$

$$-1 \leq k \leq 24$$

But ${}^{13}C_{k+1-i}$ exists for $0 \leq i \leq k+1$

then $0 \leq i \leq k+1$

$$\Rightarrow k \leq 12$$

Hence $k_{\max} = 12$

8. If $a + \alpha = 1$, $b + \beta = 2$ and

$$af(x) + \alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x}, \quad x \neq 0, \text{ then the value of}$$

$$\text{the expression } \frac{f(x) + f\left(\frac{1}{x}\right)}{x + \frac{1}{x}} \text{ is } \underline{\hspace{2cm}}.$$

Answer (2)

Sol. $af(x) + \alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x}$... (i)

Replace x by $\frac{1}{x}$

$$af\left(\frac{1}{x}\right) + \alpha f(x) = \frac{b}{x} + \beta x$$
 ... (ii)

(i) + (ii)

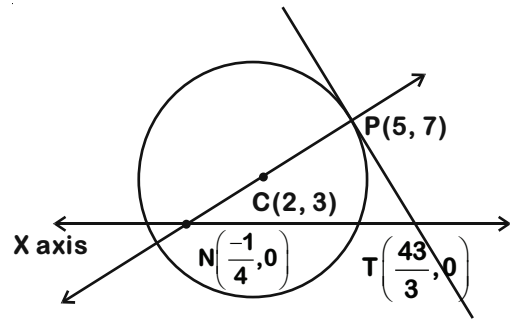
$$\Rightarrow (a + \alpha) \left(f(x) + f\left(\frac{1}{x}\right) \right) = (b + \beta) \left(x + \frac{1}{x} \right)$$

$$\Rightarrow \frac{f(x) + f\left(\frac{1}{x}\right)}{x + \frac{1}{x}} = \frac{b + \beta}{a + \alpha} = \frac{2}{1} = 2$$

9. If the area of the triangle formed by the positive x-axis, the normal and the tangent to the circle $(x - 2)^2 + (y - 3)^2 = 25$ at the point (5, 7) is A, then 24A is equal to _____.

Answer (1225)

Sol. Equation of normal PN



$$Y - 7 = \frac{7-3}{5-2}(x-5)$$

$$4x - 3y + 1 = 0 \quad \dots(i)$$

$$N\left(-\frac{1}{4}, 0\right)$$

Equation of Tangent PT

$$3x + 4y = 43 \quad \dots(ii)$$

$$T\left(\frac{43}{3}, 0\right)$$

$$PT = \frac{43}{3} + \frac{1}{4} = \frac{175}{12}$$

$$\text{Area of triangle PNT} = \frac{1}{2} \times \frac{175}{12} \times 7 = A$$

$$24A = 1225$$

10. If the variance of 10 natural numbers 1, 1, 1, ..., 1, k is less than 10, then the maximum possible value of k is _____.

Answer (11)

Sol. $\sigma^2 = \frac{9+k^2}{10} - \left(\frac{9+k}{10}\right)^2 < 10$

$$10(k^2 + 9) - (k^2 + 18k + 81) < 1000$$

$$9k^2 - 18k + 9 < 1000$$

$$9(k-1)^2 < 1000$$

$$|k-1| < \frac{10\sqrt{10}}{3} = \frac{10 \times 3.162}{3} = 10.54$$

$$-10.54 < k-1 < 10.54$$

$$-9.54 < k < 11.54$$

But $k \in \mathbb{N}$, $\therefore k_{\max} = 11$

