Time : 3 hrs. M.M. : 300

Answers & Solutions

for

JEE (MAIN)-2021 (Online) Phase-1

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS :

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper A, B, C consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part has two sections.

(i) Section-I : This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) Section-II : This section contains 10 questions. In Section-II, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and there is no negative marking for wrong answer.

PART–A : PHYSICS

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. In a Young's double slit experiment two slits are separated by 2 mm and the screen is placed one meter away. When a light of wavelength 500 nm is used, the fringe separation will be :

   (1) 0.25 mm

   (2) 0.75 mm

   (3) 0.50 mm

   (4) 1 mm

Answer (1)

Sol. $\beta = \frac{\lambda D}{d}$

$= \frac{500 \times 10^{-9} \times 1}{2 \times 10^{-3}}$

$= 250 \times 10^{-6}$

$= 0.25$ mm

2. A planet revolving in elliptical orbit has :

   A. a constant velocity of revolution.

   B. has the least velocity when it is nearest to the sun.

   C. its areal velocity is directly proportional to its velocity.

   D. areal velocity is inversely proportional to its velocity.

   E. to follow a trajectory such that the areal velocity is constant.

   Choose the correct answer from the options given below :

   (1) A only

   (2) E only

   (3) D only

   (4) C only

Answer (2)

Sol. $\frac{dA}{dt}$ = constant according to Kepler's law

3. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

   Assertion A : Body 'P' having mass M moving with speed 'u' has head-on collision elastically with another body 'Q' having mass 'm' initially at rest. If m << M, body 'Q' will have a maximum speed equal to '2u' after collision.

   Reason R : During elastic collision, the momentum and kinetic energy are both conserved.

   In the light of the above statements, choose the most appropriate answer from the options given below :

   (1) Both A and R are correct and R is the correct explanation of A

   (2) A is not correct but R is correct

   (3) A is correct but R is not correct

   (4) Both A and R are correct but R is NOT the correct explanation of A

Answer (1)

Sol. $V_2 = \frac{2M}{M+m} \times u$

$\because$ $m << M_1$

$\Rightarrow$ $V_2 \approx \frac{2M}{M} \times u = 2u$

4. LED is constructed from Ga-As-P semiconducting material. The energy gap of this LED is 1.9 eV. Calculate the wavelength of light emitted and its colour.

   [$h = 6.63 \times 10^{-34}$ Js and $c = 3 \times 10^8$ ms^{-1}]

   (1) 654 nm and red colour

   (2) 1046 nm and blue colour

   (3) 1046 nm and red colour

   (4) 654 nm and orange colour

Answer (1)

Sol. $\Delta E_g = 1.9$ eV

$\therefore$ $\lambda = \frac{1242}{1.9}$ nm

= 654 nm of red colour

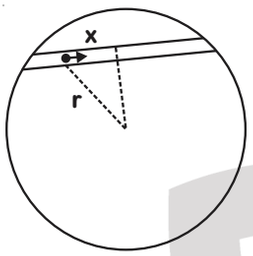
5. Assume that a tunnel is dug along a chord of the earth, at a perpendicular distance (R/2) from the earth's centre, where 'R' is the radius of the Earth. The wall of the tunnel is frictionless. If a particle is released in this tunnel, it will execute a simple harmonic motion with a time period :

(1) $\frac{g}{2\pi R}$ (2) $\frac{2\pi R}{g}$

(3) $2\pi\sqrt{\frac{R}{g}}$ (4) $\frac{1}{2\pi}\sqrt{\frac{g}{R}}$

Answer (3)

Sol.



$$\Delta F = \frac{GMr}{R^3} \times m \times \left(\frac{x}{r}\right)$$

$$\Rightarrow \frac{d^2x}{dt^2} = -\frac{GM}{R^3}x$$

$$\therefore \quad T = 2\pi\sqrt{\frac{R^3}{GM}} = 2\pi\sqrt{\frac{R}{g}}$$

6. A particle is moving with uniform speed along the circumference of a circle of radius R under the action of a central fictitious force F which is inversely proportional to R^3. Its time period of revolution will be given by :

(1) $T \propto R^{\frac{4}{3}}$ (2) $T \propto R^{\frac{3}{2}}$

(3) $T \propto R^2$ (4) $T \propto R^{\frac{5}{2}}$

Answer (3)

Sol. $F = \frac{K}{R^3}$

$$\Rightarrow \frac{mv^2}{R} = \frac{K}{R^3}$$

$$\Rightarrow v \propto \frac{1}{R}$$

$$\therefore \quad T = \frac{2\pi R}{v}$$

$$\Rightarrow T \propto R^2$$

7. The normal density of a material is ρ and its bulk modulus of elasticity is K. The magnitude of increase in density of material, when a pressure P is applied uniformly on all sides, will be :

(1) $\frac{\rho P}{K}$ (2) $\frac{\rho K}{P}$

(3) $\frac{K}{\rho P}$ (4) $\frac{PK}{\rho}$

Answer (1)

Sol. $K = \frac{\Delta P}{\left(-\frac{\Delta V}{V}\right)}$

$$\because \quad -\frac{\Delta V}{V} = \frac{\Delta\rho}{\rho}$$

$$\Rightarrow K = \frac{\Delta P}{\left(\frac{\Delta\rho}{\rho}\right)}$$

$$\Rightarrow \frac{\Delta\rho}{\rho} = \frac{\Delta P}{K}$$

$$\Rightarrow \Delta\rho = \frac{\rho P}{K}$$

8. An alternating current is given by the equation $i = i_1 \sin\omega t + i_2 \cos\omega t$. The rms current will be :

(1) $\frac{1}{\sqrt{2}}(i_1 + i_2)^2$ (2) $\frac{1}{\sqrt{2}}(i_1 + i_2)$

(3) $\frac{1}{2}\left(i_1^2 + i_2^2\right)^{\frac{1}{2}}$ (4) $\frac{1}{\sqrt{2}}\left(i_1^2 + i_2^2\right)^{\frac{1}{2}}$

Answer (4)

Sol. $i = i_1 \sin\omega t + i_2 \cos\omega t$

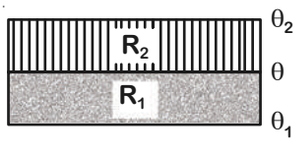
$$\therefore \quad (i_{rms})^2 = \frac{\int_0^T i^2 dt}{T}$$

$$= \frac{\int_0^T \left(i_1^2 \sin^2\omega t + i_2^2 \cos^2\omega t + 2i_1 i_2 \sin\omega t \times \cos\omega t\right) dt}{T}$$

$$= \frac{i_1^2}{2} + \frac{i_2^2}{2} + 0$$

$$\therefore \quad i_{rms} = \frac{1}{\sqrt{2}}\left(i_1^2 + i_2^2\right)^{\frac{1}{2}}$$

9. The temperature θ at the junction of two insulating sheets, having thermal resistances R_1 and R_2 as well as top and bottom temperatures θ_1 and θ_2 (as shown in figure) is given by :

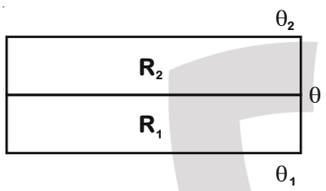


(1) $\frac{\theta_1R_2-\theta_2R_1}{R_2-R_1}$ (2) $\frac{\theta_2R_2-\theta_1R_1}{R_2-R_1}$

(3) $\frac{\theta_1R_1+\theta_2R_2}{R_1+R_2}$ (4) $\frac{\theta_1R_2+\theta_2R_1}{R_1+R_2}$

Answer (4)

Sol.



$$\frac{\theta_2-\theta}{R_2}=\frac{\theta-\theta_1}{R_1}$$

$$\Rightarrow \theta_2R_1-\theta\times R_1=\theta R_2-\theta_1R_2$$

$$\Rightarrow \theta=\frac{\theta_1R_2+\theta_2R_1}{R_1+R_2}$$

10. In a typical combustion engine the work done by a gas molecule is given by $W=\alpha^2\beta e^{-\frac{\beta x^2}{kT}}$, where x is the displacement, k is the Boltzmann constant and T is the temperature, If α and β are constants, dimensions of α will be :

(1) $[MLT^{-1}]$ (2) $[M^0LT^0]$

(3) $[MLT^{-2}]$ (4) $[M^2LT^{-2}]$

Answer (2)

Sol. $W=\alpha^2\beta e^{-\frac{\beta x^2}{kT}}$

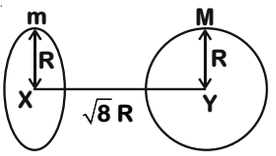
$$[\beta]=\left[\frac{kT}{x^2}\right]=\frac{ML^2T^{-2}}{L^2}=MT^{-2}$$

$$[\alpha^2\beta]=ML^2T^{-2}$$

$$\Rightarrow [\alpha^2]=\frac{ML^2T^{-2}}{MT^{-2}}$$

$$\Rightarrow [\alpha]=M^0LT^0$$

11. Find the gravitational force of attraction between the ring and sphere as shown in the diagram, where the plane of the ring is perpendicular to the line joining the centres. If $\sqrt{8}\,R$ is the distance between the centres of a ring (of mass 'm') and a sphere (mass 'M') where both have equal radius 'R'

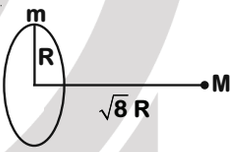


(1) $\frac{1}{3\sqrt{8}}\cdot\frac{GMm}{R^2}$ (2) $\frac{\sqrt{8}}{9}\cdot\frac{GmM}{R}$

(3) $\frac{\sqrt{8}}{27}\cdot\frac{GmM}{R^2}$ (4) $\frac{2\sqrt{2}}{3}\cdot\frac{GMm}{R^2}$

Answer (3)

Sol. Sphere can be supposed to be concentrated at centre.



$$F=\frac{GMm\sqrt{8}\,R}{\left[R^2+8R^2\right]^{3/2}}$$

$$F=\frac{\sqrt{8}\,GMm}{27R^2}$$

12. If λ_1 and λ_2 are the wavelengths of the third member of Lyman and first member of the Paschen series respectively, then the value of $\lambda_1:\lambda_2$ is

(1) 1 : 3 (2) 7 : 108

(3) 7 : 135 (4) 1 : 9

Answer (3)

Sol. $$\frac{1}{\lambda_1}=R\left[\frac{1}{1}-\frac{1}{16}\right]\Rightarrow\lambda_1=\frac{16}{15R}$$

$$\frac{1}{\lambda_2}=R\left[\frac{1}{9}-\frac{1}{16}\right]\Rightarrow\lambda_2=\frac{144}{7R}$$

$$\frac{\lambda_1}{\lambda_2}=\frac{16}{15R}\times\frac{7R}{144}=\frac{7}{135}$$

13. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : An electron microscope can achieve better resolving power than an optical microscope.

Reason R : The de Broglie's wavelength of the electrons emitted from an electron gun is much less than wavelength of visible light.

In the light of the above statements, choose the correct answer from the options given below :

(1) A is false but R is true.

(2) Both A and R are true and R is the correct explanation of A.

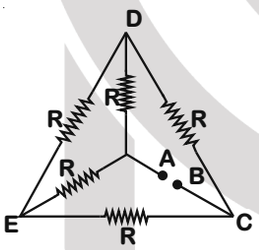
(3) Both A and R are true but R is NOT the correct explanation of A.

(4) A is true but R is false.

Answer (2)

Sol. Resolving power increases on decreasing the wavelength.

14. Five equal resistances are connected in a network as shown in figure. The net resistance between the points A and B is :



(1) $\frac{R}{2}$ (2) $\frac{3R}{2}$

(3) R (4) 2R

Answer (3)

Sol. E and D are at same potential

So, $\frac{1}{R_{eq}} = \frac{1}{2R} + \frac{1}{2R}$

$R_{eq} = R$

15. If two similar springs each of spring constant K_1 are joined in series, the new spring constant and time period would be changed by a factor :

(1) $\frac{1}{4}, 2\sqrt{2}$ (2) $\frac{1}{2}, 2\sqrt{2}$

(3) $\frac{1}{2}, \sqrt{2}$ (4) $\frac{1}{4}, \sqrt{2}$

Answer (3)

Sol. $\frac{1}{K_1} = \frac{1}{K} + \frac{1}{K}$

$K_1 = \frac{K}{2}$

$T \propto \sqrt{\frac{1}{K}}$

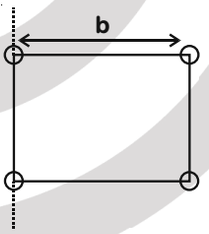
16. Four identical solid spheres each of mass 'm' and radius 'a' are placed with their centres on the four corners of a square of side 'b'. The moment of inertia of the system about one side of square where the axis of rotation is parallel to the plane of the square is :

(1) $\frac{4}{5}ma^2$ (2) $\frac{8}{5}ma^2+mb^2$

(3) $\frac{8}{5}ma^2+2mb^2$ (4) $\frac{4}{5}ma^2+2mb^2$

Answer (3)

Sol.



$I = \frac{2}{5}ma^2 \times 4 + 2 \times mb^2$

$= \frac{8}{5}ma^2 + 2mb^2$

17. Consider the combination of 2 capacitors C_1 and C_2, with $C_2 > C_1$, when connected in parallel, the equivalent capacitance is $\frac{15}{4}$ times the equivalent capacitance of the same connected in series. Calculate the ratio of capacitors, $\frac{C_2}{C_1}$.

(1) $\frac{15}{4}$ (2) $\frac{29}{15}$

(3) $\frac{111}{80}$ (4) $\frac{15}{11}$

Answer (None of the option)

Sol.

$$\frac{C_1C_2}{C_1+C_2}=\frac{4}{15}(C_1+C_2)$$

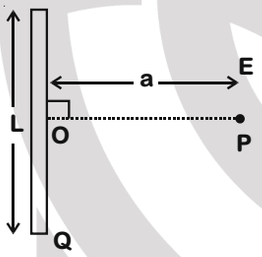
$$4(C_1+C_2)^2=15C_1C_2$$

Put $\frac{C_2}{C_1}=x$

$4(x+1)^2 = 15x$

None of the values of x satisfies the condition.

18. Find the electric field at point P (as shown in figure) on the perpendicular bisector of a uniformly charged thin wire of length L carrying a charge Q. The distance of the point P from the centre of the rod is $a=\frac{\sqrt{3}}{2}L$.

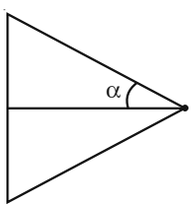


(1) $\frac{Q}{3\pi\varepsilon_0L^2}$ (2) $\frac{Q}{4\pi\varepsilon_0L^2}$

(3) $\frac{\sqrt{3}Q}{4\pi\varepsilon_0L^2}$ (4) $\frac{Q}{2\sqrt{3}\pi\varepsilon_0L^2}$

Answer (4)

Sol.



$$\lambda=\frac{Q}{L}$$

$$E=\frac{\lambda}{4\pi\varepsilon_0a}\times 2\sin\alpha$$

$\alpha = 30°$

$$E=\frac{Q}{4\pi\varepsilon_0L\times\sqrt{3}\frac{L}{2}}=\frac{Q}{2\sqrt{3}\pi\varepsilon_0L^2}$$

19. A large number of water drops, each of radius r, combine to have a drop of radius R. If the surface tension is T and mechanical equivalent of heat is J, the rise in heat energy per unit volume will be

(1) $\frac{2T}{J}\left(\frac{1}{r}-\frac{1}{R}\right)$

(2) $\frac{3T}{rJ}$

(3) $\frac{3T}{J}\left(\frac{1}{r}-\frac{1}{R}\right)$

(4) $\frac{2T}{rJ}$

Answer (3)

Sol.

$nr^3 = R^3$

$H = T[4\pi nr^2 - 4\pi R^2]$

Rise in Heat energy/volume (Q)

$$Q=\frac{4\pi T}{Jn.\frac{4}{3}\pi r^3}\left[nr^2-R^2\right]$$

Solving we get

$$Q=\frac{3T}{J}\left(\frac{1}{r}-\frac{1}{R}\right)$$

20. A short straight object of height 100 cm lies before the central axis of a spherical mirror whose focal length has absolute value |f| = 40 cm. The image of object produced by the mirror is of height 25 cm and has the same orientation of the object. One may conclude from the information :

(1) Image is virtual, opposite side of convex mirror

(2) Image is virtual, opposite side of concave mirror

(3) Image is real, same side of convex mirror

(4) Image is real, same side of concave mirror

Answer (1)

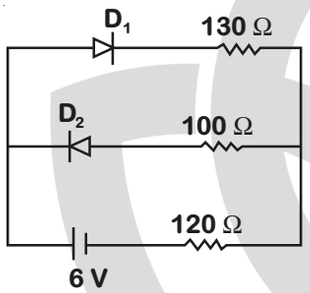
Sol. Image is diminished and magnification is positive.

$\therefore$ It is possible if object is placed in front of convex mirror.

SECTION - II

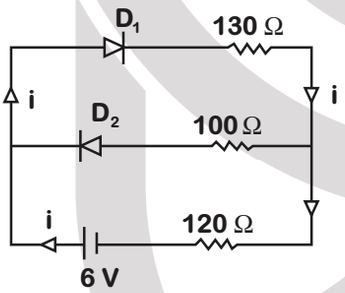
Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. The circuit contains two diodes each with a forward resistance of 50 Ω and with infinite reverse resistance. If the battery voltage is 6 V, the current through the 120 Ω resistance is _____ mA.



Answer (20)

Sol.



$$i = \frac{6}{50 + 130 + 120}$$

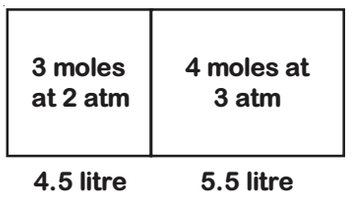
$$= \frac{6000}{300} \times 10^{-3}\ A$$

$$= 20\ mA$$

2. A container is divided into two chambers by a partition. The volume of first chamber is 4.5 litre and second chamber is 5.5 litre. The first chamber contain 3.0 moles of gas at pressure 2.0 atm and second chamber contain 4.0 moles of gas at pressure 3.0 atm. After the partition is removed and the mixture attains equilibrium, then, the common equilibrium pressure existing in the mixture is $x \times 10^{-1}$ atm. Value of x is _____.

Answer (25)

Sol. *Assuming, identical gas in both chamber

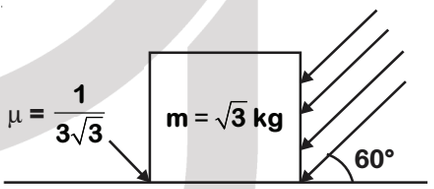


$$T = \frac{n_1T_1 + n_2T_2}{n_1 + n_2}$$

$$P = \frac{P_1V_1 + P_2V_2}{V_1 + V_2} = 2.55\ atm$$

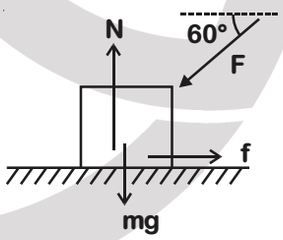
3. As shown in the figure, a block of mass $\sqrt{3}$ kg is kept on a horizontal rough surface of coefficient of friction $\frac{1}{3\sqrt{3}}$. The critical force to be applied on the vertical surface as shown at an angle 60° with horizontal such that it does not move, will be 3x. The value of x will be _____.

[g = 10 m/s^2; $\sin 60° = \frac{\sqrt{3}}{2}$; $\cos 60° = \frac{1}{2}$]



Answer (3.33)

Sol.



$N = mg + F \sin 60°$

For no movement of the block —

$F \cos 60° \le f_l$

$F \cos 60° \le \mu (mg + F \sin 60°)$

$$F \le \frac{\mu mg}{\cos 60° - \mu \sin 60°}$$

$F_{critical} = 10\ N$

4. A person standing on a spring balance inside a stationary lift measures 60 kg. The weight of that person if the lift descends with uniform downward acceleration of 1.8 m/s^2 will be _____ N. [g = 10 m/s^2]

Answer (492)

Sol. M = 60 kg

N = M(g – a)

= 60(10 – 1.8)

= 492 N

5. In an electrical circuit, a battery is connected to pass 20 C of charge through it in a certain given time. The potential difference between two plates of the battery is maintained at 15 V. The work done by the battery is _____ J.

Answer (300)

Sol. $W = q\Delta V$

$= 20 \times 15$

$= 300$ J

6. A boy pushes a box of mass 2 kg with a force $\vec{F} = (20\hat{i} + 10\hat{j})$ N on a frictionless surface. If the box was initially at rest, then _____ m is displacement along the x-axis after 10 s.

Answer (500)

Sol. $\vec{a} = \frac{\vec{F}}{m} = 10\hat{i} + 5\hat{j}$

$x = \frac{1}{2}a_x t^2 = \frac{1}{2} \cdot 10 \cdot 100$

$= 500$ m

7. In a series LCR resonant circuit, the quality factor is measured as 100. If the inductance is increased by two fold and resistance is decreased by two fold, then the quality factor after this change will be _____.

Answer (400)

Sol. $Q = \frac{X_L}{R} = 100$

$Q' = \frac{2X_L}{R/2} = 4Q$

$= 400$

8. The maximum and minimum amplitude of an amplitude modulated wave is 16 V and 8 V respectively. The modulation index for this amplitude modulated wave is $x \times 10^{-2}$. The value of x is _____.

Answer (33)

Sol. $\mu = \frac{A_m}{A_C} = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$

$= \frac{16-8}{16+8} = \frac{1}{3}$

$= 0.33$

$= 33 \times 10^{-2}$

9. The mass per unit length of a uniform wire is 0.135 g/cm. A transverse wave of the form y = –0.21 sin (x + 30 t) is produced in it, where x is in meter and t is in second. Then, the expected value of tension in the wire is $x \times 10^{-2}$ N. Value of x is _____.

(Round-off to the nearest integer)

Answer (1215)

Sol. $v = \frac{\omega}{k} = 30$ m/s

$T = \mu v^2$

$= 0.0135 \times 900$

$= 12.15$ N

10. A radiation is emitted by 1000 W bulb and it generates an electric field and magnetic field at P, placed at a distance of 2 m. The efficiency of the bulb is 1.25%. The value of peak electric field at P is $x \times 10^{-1}$ V/m. Value of x is _____. (Rounded-off to the nearest integer)

[Take $\varepsilon_0 = 8.85 \times 10^{-12} C^2 N^{-1} m^{-2}$, $c = 3 \times 10^8 ms^{-1}$]

Answer (137)

Sol. $I = \frac{\eta P}{4\pi r^2} = \frac{1}{2}\varepsilon_0 E_0^2 \cdot c$

$= \frac{0.0125 \times 1000}{4 \times 3.14 \times 2^2} = \frac{1}{2} \times 8.85 \times 10^{-12} \times 3 \times 10^8 E_0^2$

$\Rightarrow E_0 = 13.69$ V/m

PART–B : CHEMISTRY

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Match List -I with List - II.

List - I (Ore)	List - II (Element Present)
(a) Kernite	(i) Tin
(b) Cassiterite	(ii) Boron
(c) Calamine	(iii) Fluorine
(d) Cryolite	(iv) Zinc

Choose the most appropriate answer from the options given below:

(1) (a) → (ii), (b) → (i), (c) → (iv), d → (iii)

(2) (a) → (iii), (b) → (i), (c) → (ii), d → (iv)

(3) (a) → (ii), (b) → (iv), (c) → (i), d → (iii)

(4) (a) → (i), (b) → (iii), (c) → (iv), d → (ii)

Answer (1)

Sol. Kernite : $Na_2B_4O_7{\cdot}4H_2O$ (Boron)

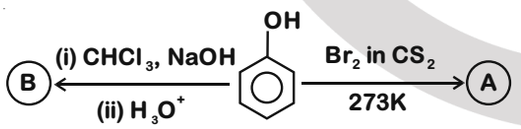
Cassiterite : SnO_2 (Tin)

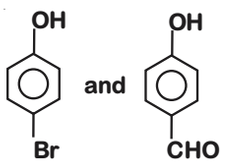
Calamine : $ZnCO_3$ (Zinc)

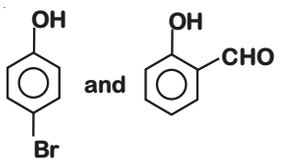
Cryolite : Na_3AlF_6 (Fluorine)

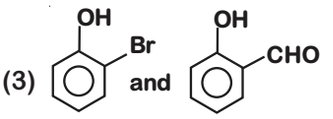
(a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)

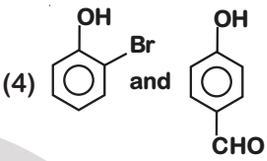
2. Identify the major products A and B respectively in the following reactions of phenol:



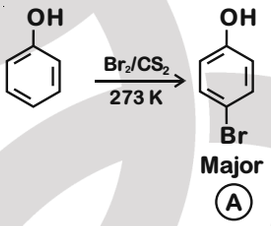
(1) 

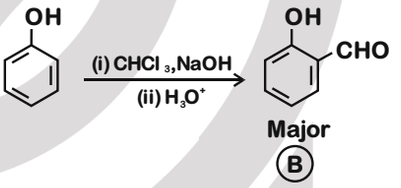
(2) 

(3) 

(4) 

Answer (2)

Sol. 



So option (2) is the correct answer

3. $\underset{(C_4H_8Cl_2)}{A} \xrightarrow[373K]{\text{Hydrolysis}} \underset{(C_4H_8O)}{B}$

B reacts with Hydroxyl amine but does not give Tollen's test. Identify A and B.

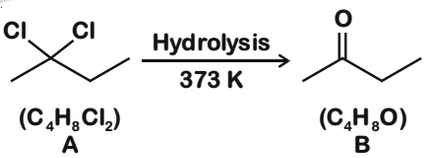
(1) 2,2-Dichlorobutane and Butanal

(2) 1,1-Dichlorobutane and Butanal

(3) 1,1-Dichlorobutane and 2-Butanone

(4) 2,2-Dichlorobutane and Butan-2-one

Answer (4)

Sol. 

B is a ketone, cannot give Tollen's test.

A → 2, 2-Dichlorobutane

B → Butan-2-one

So correct option should be (4)

4. Given below are two statements : One is labelled as Assertion A and the other is labelled as Reason R

Assertion A : Dipole-dipole interactions are only non-covalent interactions, resulting in hydrogen bond formation.

Reason R : Fluorine is the most electronegative element and hydrogen bonds in HF are symmetrical.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) A is false but R is true

(2) Both A and R are true and R is the correct explanation of A

(3) A is true but R is false

(4) Both A and R are true but R is NOT the correct explanation of A

Answer (1)

Sol.

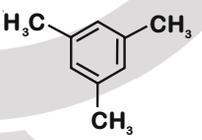
- Dipole - Dipole are not only the interaction responsible for hydrogen bond formation. Ion-dipole can also be responsible for hydrogen bond formation.
- F is most electronegative element and anhydrous HF in solid phase has symmetrical hydrogen bonding

So the correct option is (1).

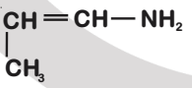
5. For the given reaction :

What is 'A'?

(1) 

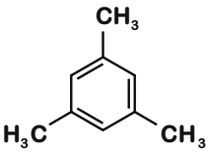
(2) 

(3) $CH_3CH_2CH_2NH_2$

(4) 

Answer (2)

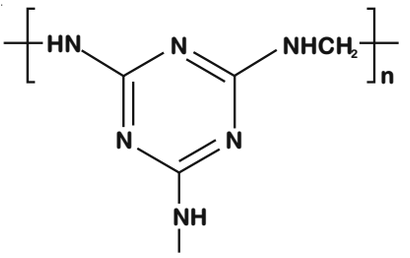
Sol. $CH_3 - CH = CH - Br \xrightarrow[\text{(ii) Red hot iron tube, 873 K}]{\text{(i) } NaNH_2}$

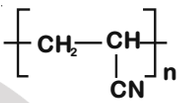


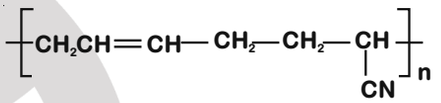
Mesitylene

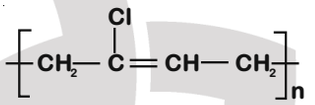
So the correct option should be (2).

6. The structure of Neoprene is :

(1) 

(2) 

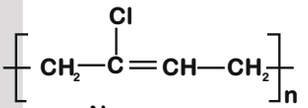
(3) 

(4) 

Answer (4)

Sol. Neoprene is a polymer of monomer chloroprene

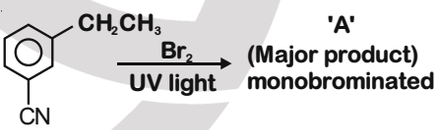
Chloroprene



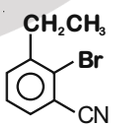
Neoprene

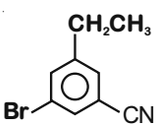
So the correct option should be (4)

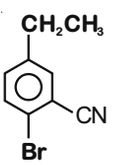
7. For the given reaction :

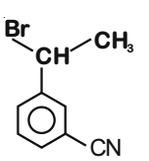


What is 'A'?

(1) 

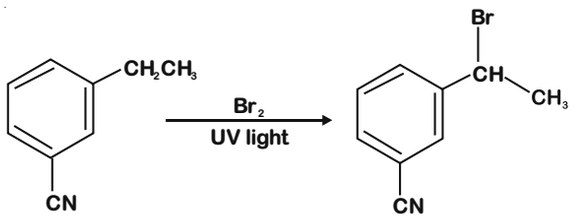
(2) 

(3) 

(4) 

Answer (4)

Sol.



(Major Product)
Monobrominated

So the correct option should be (4)

8. The orbital having two radial as well as two angular nodes is :

(1) 3p (2) 4d

(3) 5d (4) 4f

Answer (3)

Sol. Number of radial nodes = (n – l – 1)

Number of angular nodes = l

for 5d; n = 5, l = 2

5d orbital has two radial nodes and two angular nodes

So, the correct option should be (3)

9. Given below are two statements:

Statement I : o-Nitrophenol is steam volatile due to intramolecular hydrogen bonding.

Statement II : o-Nitrophenol has high melting due to hydrogen bonding.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both statement I and statement II are true

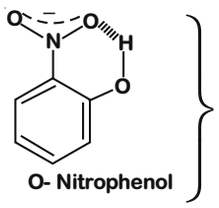
(2) Statement I is false but statement II is true

(3) Statement I is true but statement II is false

(4) Both statement I and statement II are false

Answer (3)

Sol.



O- Nitrophenol

- Has intramolecular hydrogen bonding and steam volatile
- Melting point is not affected by intramolecular hydrogen bonding

So the correct option should be (3)

10. Compound A is used as a strong oxidizing agent is amphoteric in nature. It is the part of lead storage batteries. Compound A is

(1) Pb_3O_4 (2) $PbSO_4$

(3) PbO (4) PbO_2

Answer (4)

Sol. PbO_2 is strong oxidizing agent because Pb^{+4} is not stable and can be easily reduced to Pb^{+2}.

PbO_2 is used in lead storage batteries. It is also amphoteric in nature

So, the answer should be (4)

11. On treating a compound with warm dil. H_2SO_4, gas X is evolved which turns $K_2Cr_2O_7$ paper acidified with dil. H_2SO_4 to a green compound Y. X and Y respectively are :

(1) X = SO_3, Y = Cr_2O_3

(2) X = SO_3, Y = $Cr_2(SO_4)_3$

(3) X = SO_2, Y = $Cr_2(SO_4)_3$

(4) X = SO_2, Y = Cr_2O_3

Answer (3)

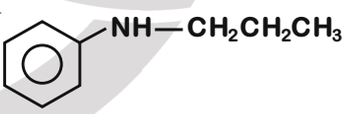
Sol. $SO_2 + K_2Cr_2O_7 + \text{dil. } H_2SO_4 \rightarrow SO_3 + \underset{\text{(green)}}{Cr_2(SO_4)_3}$

12. An amine on reaction with benzenesulphonyl chloride produces a compound insoluble in alkaline solution. This amine can be prepared by ammonolysis of ethyl chloride. The correct structure of amine is :

(1) $CH_3CH_2CH_2\overset{H}{\underset{|}{N}}-CH_2CH_3$

(2) $CH_3CH_2NH_2$

(3) $CH_3CH_2CH_2NHCH_3$

(4) 

Answer (1)

Sol. Given amine on reaction with $PhSO_2Cl$ produces a compound insoluble in alkaline solution it means it is a 2° amine.

$\because$ It is prepared by ammonolysis of C_2H_5Cl, it must contain an ethyl group

$CH_3CH_2CH_2\overset{H}{\underset{|}{N}}-CH_2CH_3$ is a 2° amine as well as it contain ethyl group.

13. Which of the following is 'a' FALSE statement?

(1) Carius method is used for the estimation of nitrogen in an organic compound

(2) Phosphoric acid produced on oxidation of phosphorus present in an organic compound is precipitated as $Mg_2P_2O_7$ by adding magnesia mixture

(3) Kjeldahl's method is used for the estimation of nitrogen in an organic compound

(4) Carius tube is used in the estimation of sulphur in an organic compound

Answer (1)

Sol. Carius method is used for the estimation for halogens and sulphur in organic compound.

14. Find A, B and C in the following reactions :

$NH_3 + A + CO_2 \rightarrow (NH_4)_2CO_3$

$(NH_4)_2CO_3 + H_2O + B \rightarrow NH_4HCO_3$

$NH_4HCO_3 + NaCl \rightarrow NH_4Cl + C$

(1) $A - H_2O$; $B - CO_2$; $C - NaHCO_3$

(2) $A - H_2O$; $B - O_2$; $C - Na_2CO_3$

(3) $A - H_2O$; $B - O_2$; $C - NaHCO_3$

(4) $A - O_2$; $B - CO_2$; $C - Na_2CO_3$

Answer (1)

Sol. These reactions are from Solvay process (formation of washing soda)

$A = H_2O$

$B = CO_2$

$C = NaHCO_3$

15. The presence of ozone in troposphere :

(1) Protects us from greenhouse effect

(2) Protects us from the X-ray radiation

(3) Generates photochemical smog

(4) Protects us from the UV radiation

Answer (3)

Sol. Ozone in stratosphere (not in troposphere) prevent us from U.V. radiation.

Ozone in troposphere generates photochemical smog.

16. Given below are two statements:

Statement I : A mixture of chloroform and aniline can be separated by simple distillation.

Statement II : When separating aniline from a mixture of aniline and water by steam distillation aniline boils below its boiling point.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Both statement I and statement II are false

(2) Both statement I and statement II are true

(3) Statement I is true but statement II is false

(4) Statement I is false but statement II is true

Answer (2)

Sol.
- Mixture of chloroform and aniline can be separated by simple distillation as these two liquids have sufficient difference in boiling point.

  Chloroform (b.p. 334 K), aniline (b.p. 457 K)
- In steam distillation, if one of the substances is water and the other, a water insoluble substance (like aniline) then the mixture will boil close to but below 373 K.

17. Statements about heavy water are given below.

A. Heavy water is used in exchange reactions for the study of reaction mechanisms.

B. Heavy water is prepared by exhaustive electrolysis of water.

C. Heavy water has higher boiling point than ordinary water.

D. Viscosity of H_2O is greater than D_2O.

Choose the most appropriate answer from the options given below:

(1) A and B only (2) A and C only

(3) A and D only (4) A, B and C only

Answer (4)

Sol.
- Viscosity of D_2O is greater than H_2O.
- B.P. of D_2O is greater than H_2O.

18. Which of the following vitamin is helpful in delaying the blood clotting?

(1) Vitamin B (2) Vitamin E

(3) Vitamin K (4) Vitamin C

Answer (3)

Sol. Vitamin K is helpful in delaying the blood clotting.

19. Which one of the following lanthanoids does not form MO_2?

[M is lanthanoid metal]

(1) Nd

(2) Pr

(3) Dy

(4) Yb

Answer (4)

Sol. Nd (60) = $4f^4\ 6s^2$

Pr (59) = $4f^3\ 6s^2$

Dy (66) = $4f^{10}\ 6s^2$

Yb (70) = $4f^{14}\ 6s^2$

Yb^{+2} has fully-filled 4f orbital, it will require very large amount of energy to reach +4 oxidation state.

20. Match List-I with List-II.

List-I	List-II
Electronic configuration of elements	$\Delta_i H$ in kJ mol^{-1}
(a) $1s^2\ 2s^2$	(i) 801
(b) $1s^2\ 2s^2\ 2p^4$	(ii) 899
(c) $1s^2\ 2s^2\ 2p^3$	(iii) 1314
(d) $1s^2\ 2s^2\ 2p^1$	(iv) 1402

Choose the most appropriate answer from the options given below :

(1) (a) → (i), (b) → (iii), (c) → (iv), (d) → (ii)

(2) (a) → (i), (b) → (iv), (c) → (iii), (d) → (ii)

(3) (a) → (iv), (b) → (i), (c) → (ii), (d) → (iii)

(4) (a) → (ii), (b) → (iii), (c) → (iv), (d) → (i)

Answer (4)

Sol. On moving left to right in periodic table, ionisation energy increases (generally) but group-13 elements have lesser I.E than group-2 due to stable ns^2 electronic configuration of group-2 elements and group-15 elements have greater I.E than group-16 elements due to half-filled stable np^3 configuration of group-15 elements.

∴ Overall order of I.E should be

c > b > a > d

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a NUMERICAL VALUE. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. For a chemical reaction A + B ⇌ C + D

($\Delta_r H^\ominus$ = 80 kJ mol^{-1}) the entropy change $\Delta_r S^\ominus$ depends on the temperature T (in K) as $\Delta_r S^\ominus = 2T$ (J K^{-1} mol^{-1}).

Minimum temperature at which it will become spontaneous is _____ K. (Integer)

Answer (200)

Sol. A + B ⇌ C + D

For a reaction to be spontaneous

$\Delta_r G < 0$

or $\Delta_r G° < 0$ (For the given case)

$\Rightarrow \Delta_r H° - T\Delta_r S° < 0$

$\Rightarrow 80 \times 1000 - T \times 2T < 0$

$\Rightarrow T^2 > 40000$

$T > 200$

$\therefore \boxed{T_{min} = 200\,K}$

2. 224 mL of $SO_{2(g)}$ at 298 K and 1 atm is passed through 100 mL of 0.1 M NaOH solution. The non-volatile solute produced is dissolved in 36 g of water. The lowering of vapour pressure of solution (assuming the solution is dilute) $\left(P^\circ_{(H_2O)} = 24\,mm\,of\,Hg\right)$ is x × 10^{-2} mm of Hg, the value of x is _____. (Integer answer)

Answer (24)

Sol. $n_{SO_2} = \frac{1\times0.224}{0.082\times298} \approx 0.0092 \approx 0.01\,moles$

$NaOH + SO_2 \rightarrow NaHSO_3$

$n_{NaHSO_3} = 0.01$

$NaHSO_3 \rightarrow Na^+ + HSO_3^-$

Ignoring the dissociation of HSO_3^- into H^+ and SO_3^{2-}

van't Hoff factor (i) = 2

$$\frac{P^o_{H_2O} - P_S}{P^o_{H_2O}} = \frac{in_{NaHSO_3}}{n_{H_2O} + in_{NaHSO_3}}$$

$\left(\text{as } n_{HSO_3^-} << n_{H_2O}\right)$

Lowering in vapour presssure

$= \frac{2\times0.01}{2+2\times0.01}\times24$

$= 23.76\times10^{-2}\text{ mmHg} \approx 24\times10^{-2}\text{ mmHg}$

3. 3.12 g of oxygen is adsorbed on 1.2 g of platinum metal. The volume of oxygen adsorbed per gram of the adsorbent at 1 atm and 300 K in L is _____.
[R = 0.0821 L atm K^{-1} mol^{-1}]

Answer (2)

Sol. Using ideal gas equation : PV = nRT

$V = \frac{3.12\times0.0821\times300}{32\times1} = 2.40\,L$

$\therefore$ Volume of $O_2(g)$ adsorbed per gram of the adsorbent $= \frac{2.4}{1.2} = 2$

4. Dichromate ion is treated with base, the oxidation number of Cr in the product formed is _____.

Answer (6)

Sol. $Cr_2O_7^{2-} \xrightarrow{OH^-} CrO_4^{2-}$

$\therefore$ Oxidation state of Cr in CrO_4^{2-} is +6.

5. A homogeneous ideal gaseous reaction $AB_{2(g)} \rightleftharpoons A_{(g)} + 2B_{(g)}$ is carried out in a 25 litre flask at 27°C. The initial amount of AB_2 was 1 mole and the equilibrium pressure was 1.9 atm. The value of K_p is $x \times 10^{-2}$. The value of x is _____. (Integer answer)
[R = 0.08206 dm^3atm K^{-1} mol^{-1}]

Answer (72)

Sol.

	$AB_2(g)$	$\rightleftharpoons$	$A(g)$	+	$2B(g)$
t = 0	1		–		–
t = eq^m	1-α		α		2α

$\therefore$ No. of moles at equilibrium

$= 1 + 2\alpha = \frac{PV}{RT} = \frac{1.9\times25}{0.08206\times300} \approx 1.93$

$\therefore$ $\alpha = 0.465$

$\therefore P_{AB_2} = \frac{1-\alpha}{1+2\alpha}\times P_T \approx 0.53\text{ atm}$

$P_A = \frac{\alpha}{1+2\alpha}\times P_T \approx 0.46\text{ atm}$

$P_B = \frac{2\alpha}{1+2\alpha}\times P_T \approx 0.91\text{ atm}$

$K_P = \frac{P_A\cdot(P_B)^2}{P_{AB_2}}$

$= \frac{0.46\times(0.91)^2}{0.53} \approx 0.72 \approx 72\times10^{-2}$

6. A certain gas obeys $P(V_m - b) = RT$. The value of $\left(\frac{\partial Z}{\partial P}\right)_T$ is $\frac{xb}{RT}$. The value of x is _____.
(Integer answer) (Z : compressibility factor)

Answer (1)

Sol. $P(V_m - b) = RT$

$\Rightarrow PV_m - Pb = RT$

$\Rightarrow \frac{PV_m}{RT} = 1 + \frac{Pb}{RT}$

$\Rightarrow Z = 1 + \frac{Pb}{RT}$

$\Rightarrow \left(\frac{\partial Z}{\partial P}\right)_T = \frac{b}{RT}$

$\therefore$ $\boxed{x = 1}$

7. The number of significant figures in 50000.020×10^{-3} is _____.

Answer (8)

Sol. No. of significant figures in $50000.020 \times 10^{-3} = 8$

8. An exothermic reaction $X \rightarrow Y$ has an activation energy 30 kJ mol^{-1}. If energy change ΔE during the reaction is –20 kJ, then the activation energy for the reverse reaction in kJ is _____. (Integer answer)

Answer (50)

Sol. $X \rightarrow Y$

$\Delta E = (Ea)_f - (Ea)_b$

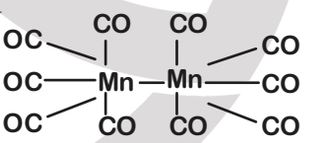
$-20 = 30 - (Ea)_b$

$\Rightarrow (Ea)_b = 50$ kJ

9. Number of bridging CO ligands in $[Mn_2(CO)_{10}]$ is _____.

Answer (0)

Sol. $[Mn_2(CO)_{10}]$



$\therefore$ No. of bridging CO ligands = 0

10. Consider the following reaction
$MnO_4^- + 8H^+ + 5e^- \rightarrow Mn^{+2} + 4H_2O$, $E^\circ = 1.51$ V.
The quantity of electricity required in Faraday to reduce five moles of MnO_4^- is _____.
(Integer answer)

Answer (25)

Sol. $MnO_4^- + 8H^+ + 5e^- \rightarrow Mn^{+2} + 4H_2O$, $E^\circ = 1.51$ V

$\because$ 1 mole of MnO_4^- required 5 moles of electrons or 5 F electricity.

$\therefore$ 5 moles of MnO_4^- required 25 F electricity.

PART–C : MATHEMATICS

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. If $\vec{a}$ and $\vec{b}$ are perpendicular, then $\vec{a}\times\left(\vec{a}\times\left(\vec{a}\times\left(\vec{a}\times\vec{b}\right)\right)\right)$ is equal to :

(1) $\vec{a}\times\vec{b}$ (2) $\vec{0}$

(3) $\frac{1}{2}|\vec{a}|^4\vec{b}$ (4) $|\vec{a}|^4\vec{b}$

Answer (4)

Sol. Let $\hat{c}$ be a unit vector in the direction of $\vec{a}\times\vec{b}$.

$\Rightarrow \hat{a}\times\hat{b}=\hat{c},\ \hat{b}\times\hat{c}=\hat{a}\ \&\ \hat{c}\times\hat{a}=\hat{b}$

$\vec{a}\times\vec{b}=|\vec{a}|\times|\vec{b}|\times\hat{c}$

$\vec{a}\times\left(\vec{a}\times\vec{b}\right)=-|\vec{a}|^2|\vec{b}|\hat{b}$

$\vec{a}\times\left(\vec{a}\times\left(\vec{a}\times\vec{b}\right)\right)=-|\vec{a}|^3|\vec{b}|\hat{c}$

$\vec{a}\times\left(\vec{a}\times\left(\vec{a}\times\left(\vec{a}\times\vec{b}\right)\right)\right)=|\vec{a}|^4|\vec{b}|\hat{b}$

$=|\vec{a}|^4\vec{b}$

2. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{\cos^2 x}{1+3^x}dx$ is :

(1) $\frac{\pi}{4}$ (2) 2π

(3) $\frac{\pi}{2}$ (4) 4π

Answer (1)

Sol. $\int_{-a}^{a}f(x)dx=\int_{0}^{a}(f(x)+f(a-x))dx$

$\int_{-\pi/2}^{\pi/2}\frac{\cos^2 x}{1+3^x}dx=\int_{0}^{\pi/2}\frac{\cos^2 x}{1+3^x}+\frac{\cos^2(-x)}{1+3^{-x}}dx$

$=\int_{0}^{\pi/2}\cos^2 x\left(\frac{1}{1+3^x}+\frac{3^x}{1+3^x}\right)dx=\int_{0}^{\pi/2}\cos^2 x\,dx$

$=\frac{1}{2}\int_{0}^{\pi/2}(1+\cos 2x)dx=\frac{\pi}{4}$

3. The value of $\begin{vmatrix}(a+1) & (a+2) & a+2 & 1\\ (a+2) & (a+3) & a+3 & 1\\ (a+3) & (a+4) & a+4 & 1\end{vmatrix}$ is :

(1) 0 (2) $(a+2)(a+3)(a+4)$

(3) –2 (4) $(a+1)(a+2)(a+3)$

Answer (3)

Sol. Given determinant is

$$D=\begin{vmatrix}a^2+3a+2 & a+2 & 1\\ a+5a+6 & a+3 & 1\\ a^2+7a+12 & a+4 & 1\end{vmatrix}$$

$R_3\to R_3-R_2;\ R_2\to R_2-R_1$

$$=\begin{vmatrix}a^2+3a+2 & a+2 & 1\\ 2a+4 & 1 & 0\\ 2a+6 & 1 & 0\end{vmatrix}$$

Expanding by C_3

$D = (2a + 4) – (2a + 6) = –2$

4. The maximum slope of the curve $y=\frac{1}{2}x^4-5x^3+18x^2-19x$ occurs at the point :

(1) (2, 2) (2) (0, 0)

(3) $\left(3,\frac{21}{2}\right)$ (4) (2, 9)

Answer (1*)

Sol. $y=\frac{1}{2}x^4-5x^3+18x^2-19x$

Slope = $y' = 2x^3 – 15x^2 + 36x – 19 = g(x)$ say

$g'(x) = 6x^2 – 30x + 36 = 6(x – 2)(x – 3)$

$g'(x) = 0 \Rightarrow x = 2, 3$

Slope g(x) has local maximum at x = 2

$x = 2 \Rightarrow y = 2$

Local maximum at (2, 2)

[Note : Overall maximum (Absolute maximum) value of slope is far greater than that at (2, 2)].

5. In an increasing geometric series, the sum of the second and the sixth term is $\frac{25}{2}$ and the product of the third and fifth term is 25. Then, the sum of $4^{th}, 6^{th}$ and 8^{th} term is equal to :

(1) 26 (2) 35

(3) 30 (4) 32

Answer (2)

Sol. $a_2 + a_6 = \frac{25}{2}$

$a_3 \times a_5 = 25 = a_2 \times a_6 = a_4^2$

$a_4^2 = 25 \Rightarrow a_4 = 5$

a_2 & a_6 are roots of $x^2 - \frac{25}{2}x + 25 = 0$

$x = \frac{5}{2}, 10$

$a_2 = \frac{5}{2}, a_6 = 10$ ($\because$ GP is increasing)

$a_4 = 5$

$a_4 = a_2 r^2 \Rightarrow 5 = \frac{5}{2} r^2 \Rightarrow r^2 = 2$

$a_8 = a_6 r^2 = 10 \times 2 = 20$

$a_4 + a_6 + a_8 = 5 + 10 + 20 = 35$

6. The number of seven digit integers with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only is :

(1) 77 (2) 42

(3) 82 (4) 35

Answer (1)

Sol. Combination of digits

3, 2, 1, 1, 1, 1, 1 $\rightarrow \frac{7!}{5!} = 42$

2, 2, 2, 1, 1, 1, 1 $\rightarrow \frac{7!}{4!\,3!} = 35$

Total = 42 + 35 = 77

7. The sum of infinite series

$1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \frac{17}{3^4} + \frac{22}{3^5} + ...$ is equal to :

(1) $\frac{13}{4}$ (2) $\frac{9}{4}$

(3) $\frac{11}{4}$ (4) $\frac{15}{4}$

Answer (1)

Sol. $S = 1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \frac{17}{3^4} + \frac{22}{3^5} + ...$

$\frac{1}{3}S = \frac{1}{3} + \frac{2}{3^2} + \frac{7}{3^3} + \frac{12}{3^4} + \frac{17}{3^5} + ...$

$- \quad - \quad - \quad - \quad - \quad -$

$\frac{2}{3}S = 1 + \frac{1}{3} + \left(\frac{5}{3^2} + \frac{5}{3^3} + \frac{5}{3^4} + \frac{5}{3^5} + ...\right)$

$= \frac{4}{3} + \frac{\frac{5}{9}}{1 - \frac{1}{3}} = \frac{5}{3} + \frac{\frac{5}{9}}{\frac{2}{3}}$

$\Rightarrow \frac{2}{3}S = \frac{4}{3} + \frac{5}{6} = \frac{13}{6}$

$S = \frac{13}{4}$

8. Consider the three planes

$P_1 : 3x + 15y + 21z = 9$,

$P_2 : x - 3y - z = 5$, and

$P_3 : 2x + 10y + 14z = 5$

Then, which one of the following is true ?

(1) P_2 and P_3 are parallel

(2) P_1 and P_3 are parallel

(3) P_1 and P_2 are parallel

(4) P_1, P_2 and P_3 all are parallel

Answer (2)

Sol. Ratios of DRs of normals of P_1 & P_3 are

$\frac{3}{2} = \frac{15}{10} = \frac{21}{14}$

$\Rightarrow$ Normals are parallel

$\Rightarrow P_1 \parallel P_3$

9. Let A be a symmetric matrix of order 2 with integer entries. If the sum of the diagonal elements of A^2 is 1, then the possible number of such matrices is :

(1) 6 (2) 1

(3) 4 (4) 12

Answer (3)

Sol. Let $A = \begin{bmatrix} a & c \\ c & b \end{bmatrix}$

$$A^2 = \begin{bmatrix} a & c \\ c & b \end{bmatrix}\begin{bmatrix} a & c \\ c & b \end{bmatrix} = \begin{bmatrix} a^2+c^2 & ac+bc \\ ac+bc & c^2+b^2 \end{bmatrix}$$

$a^2 + b^2 + 2c^2 = 1$ as $a, b, c \in z$

$c = 0$ and $a, b = \pm 1$

Total 4 matrices are possible

10. The maximum value of the term independent of 't' in the expansion of $\left(tx^{\frac{1}{5}} + \frac{(1-x)^{\frac{1}{10}}}{t}\right)^{10}$ where $x \in (0,1)$ is :

(1) $\frac{2.10!}{3(5!)^2}$ (2) $\frac{2.10!}{3\sqrt{3}(5!)^2}$

(3) $\frac{10!}{\sqrt{3}(5!)^2}$ (4) $\frac{10!}{2(5!)^2}$

Answer (2)

Sol. $T_{r+1} = {}^{10}C_r (tx^{\frac{1}{5}})^{10-r}\left(\frac{(1-x)^{\frac{1}{10}}}{t}\right)^r$

For term independent of f

$10 - r - r = 0 \Rightarrow r = 5$

$T_6 = {}^{10}C_5\ x(1-x)^{\frac{1}{2}} = f(x)$ (Let)

$\therefore\ f'(x) = {}^{10}C_5\left((1-x)^{\frac{1}{2}} - \frac{x}{2(1-x)^{\frac{1}{2}}}\right) = 0$

$2 - 2x = x \Rightarrow x = \frac{2}{3}$

$f''(x) < 0$ at $x = \frac{2}{3}$

$T_{6(max)} = {}^{10}C_5 \cdot \frac{2}{3}\left(\frac{1}{3}\right)^{\frac{1}{2}} = \frac{2.10!}{(5!)^2 3\sqrt{3}}$

11. A fair coin is tossed a fixed number of times. If the probability of getting 7 heads is equal to probability of getting 9 heads, then the probability of getting 2 heads is :

(1) $\frac{15}{2^{13}}$ (2) $\frac{15}{2^{12}}$

(3) $\frac{15}{2^8}$ (4) $\frac{15}{2^{14}}$

Answer (1)

Sol. Let n number of tosses

Given,

$${}^nC_7\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^{n-7} = {}^nC_9 = \left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right)^{n-9}$$

$\Rightarrow$ n = 16

$\therefore$ Probability of getting 2 heads = $16C_2\left(\frac{1}{2}\right)^{16}$

$= \frac{15}{2^{13}}$

12. The value of $\sum_{n=1}^{100}\int_{n-1}^{n} e^{x-[x]}dx$, where [x] is the greatest integer $\le$ x, is :

(1) 100(e – 1) (2) 100(1 – e)

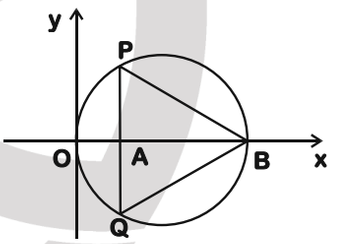
(3) 100e (4) 100(1 + e)

Answer (1)

Sol. $\int_{n-1}^{n} e^{x-[x]}\,dx = \int_0^1 e^x\,dx = (e-1)$

$\therefore\ \sum_{n=1}^{100}(e-1) = 100(e-1)$

13. In the circle given below, let OA = 1 unit, OB = 13 unit and $PQ \perp OB$. Then, the area of the triangle PQB (in square units) is :



(1) $24\sqrt{2}$ (2) $24\sqrt{3}$

(3) $26\sqrt{3}$ (4) $26\sqrt{2}$

Answer (2)

Sol. Assume that OB is diameter of the given circle

Using Ptolemy's Theorem,

$OP \cdot QB + OQ \cdot PB = PQ \times OB$

$\Rightarrow 2OP \cdot PB = 13PQ$

Also $PA^2 = OP^2 - 1 = PB^2 - 12^2$

$\Rightarrow PB^2 - OP^2 = 143$

and $OP^2 + PB^2 = 13^2$

then $PB^2 = 156$ and $OP^2 = 13$

So, $PQ = \frac{2\sqrt{13}\cdot\sqrt{156}}{13} = 4\sqrt{3}$

Area of $\Delta PQB = \frac{1}{2}\cdot 4\sqrt{3}\cdot 12 = 24\sqrt{3}$

14. The value of

$$\lim_{h\to 0} 2\left\{\frac{\sqrt{3}\sin\left(\frac{\pi}{6}+h\right)-\cos\left(\frac{\pi}{6}+h\right)}{\sqrt{3}h\left(\sqrt{3}\cos h-\sin h\right)}\right\}$$ is

(1) $\frac{4}{3}$

(2) $\frac{3}{4}$

(3) $\frac{2}{3}$

(4) $\frac{2}{\sqrt{3}}$

Answer (1)

Sol. $$\lim_{h\to 0} 2\left\{\frac{\frac{\sqrt{3}}{2}\sin\left(\frac{\pi}{6}+h\right)-\frac{1}{2}\cos\left(\frac{\pi}{6}+h\right)}{\sqrt{3}h\left(\frac{\sqrt{3}}{2}\cosh-\frac{1}{2}\sinh\right)}\right\}$$

$$\lim_{h\to 0} 2\left\{\frac{\sin(h)}{\sqrt{3}h\left(\sin\frac{\pi}{3}-h\right)}\right\} = \frac{2}{\sqrt{3}}\cdot\frac{2}{\sqrt{3}} = \frac{4}{3}$$

15. Let R = {(P, Q) | P and Q are at the same distance from the origin} be a relation, then the equivalence class of (1, –1) is the set :

(1) $S = \{(x, y) \mid x^2 + y^2 = 2\}$

(2) $S = \{(x, y) \mid x^2 + y^2 = 1\}$

(3) $S = \{(x, y) \mid x^2 + y^2 = \sqrt{2}\}$

(4) $S = \{(x, y) \mid x^2 + y^2 = 4\}$

Answer (1)

Sol. $\therefore$ R = {(P, Q) | P and Q are at the same distance from the origin}.

Then equivalence class of (1, –1) will contain all such points which lies on circumference of the circle of centre at origin and passing through point (1, –1).

i.e., radius of circle $= \sqrt{1^2+1^2} = \sqrt{2}$

$\therefore$ Required equivalence class of (S)

$= \{(x, y) \mid x^2 + y^2 = 2\}$.

16. The rate of growth of bacteria in a culture is proportional to the number of bacteria present and the bacteria count is 1000 at initial time t = 0. The number of bacteria is increased by 20% in 2 hours. If the population of bacteria is 2000 after $\frac{k}{\log_e\left(\frac{6}{5}\right)}$ hours, then $\left(\frac{k}{\log_e 2}\right)^2$ is equal to :

(1) 16 (2) 4

(3) 8 (4) 2

Answer (2)

Sol. At t = 0 $B_\circ = 1000$

$$\frac{dB}{dt} \propto B$$

$$\Rightarrow \int_{B_\circ}^{1\cdot 2B_\circ}\frac{dB}{B} = \int_0^2 kt \qquad \text{[Given]}$$

$$\ln\left(\frac{1\cdot 2B_\circ}{B_\circ}\right) = 2k$$

$$\Rightarrow k = \frac{1}{2}\ln(1\cdot 2)$$

To find time when B = 2000

$$\Rightarrow \int_{B_\circ}^{2B_\circ}\frac{dB}{B} = \frac{1}{2}\ln(1\cdot 2)\int_0^t dt$$

$$\ln 2 = \frac{1}{2}\ln(1\cdot 2)t$$

$$\Rightarrow t = \frac{\ln 4}{\ln\left(\frac{6}{5}\right)}\text{hrs.}$$

$\therefore$ R = ln = 4

Thus $\left(\frac{K}{\ln}\right)^2 = 2^2 = 4$

17. Let f be any function defined on R and let it satisfy the condition :

$$|f(x)-f(y)| \le |(x-y)^2|, \forall (x,y)\in R$$

If $f(0) = 1$, then :

(1) $f(x)$ can take any value in R

(2) $f(x) < 0, \forall x \in R$

(3) $f(x) = 0, \forall x \in R$

(4) $f(x) > 0, \forall x \in R$

Answer (4)

Sol. $|f(x)-f(y)| \le |(x-y)^2|$

$\Rightarrow \left|\frac{f(x)-f(y)}{x-y}\right| \le |x-y|$

$\Rightarrow \left|\lim_{x\to y}\frac{f(x)-f(y)}{x-y}\right| \le \left|\lim_{x\to y}(x-y)\right|$

$\Rightarrow |f'(x)| \le 0$

$\Rightarrow f'(x) = 0$

$\Rightarrow$ f(x) is constant function.

$\because$ f(0) = 1 then f(x) = 1

18. If $\frac{\sin^{-1}x}{a} = \frac{\cos^{-1}x}{b} = \frac{\tan^{-1}y}{c}$; 0 < x < 1, then the value of $\cos\left(\frac{\pi c}{a+b}\right)$ is :

(1) $1 - y^2$ (2) $\frac{1-y^2}{y\sqrt{y}}$

(3) $\frac{1-y^2}{1+y^2}$ (4) $\frac{1-y^2}{2y}$

Answer (3)

Sol. $\because \frac{\sin^{-1}x}{a} = \frac{\cos^{-1}x}{b} = \frac{\tan^{-1}y}{c} = k$ (say)

$\therefore \sin^{-1}x = ak$, $\cos^{-1}x = bk$ and $\tan^{-1}y = ck$

Now,

$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

$(a+b)x = \frac{\pi}{2}$

$\therefore k = \frac{\pi}{2(a+b)}$

Now $\tan^{-1}y = \frac{\pi c}{2(a+b)}$

$\therefore \cos\left(\frac{\pi c}{ab}\right) = \cos(2\tan^{-1}y)$

$= \cos\left(\cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right)$ [if y > 0]

$= \frac{1-y^2}{1+y^2}$

19. The intersection of three lines x – y = 0, x + 2y = 3 and 2x + y = 6 is a :

(1) None of the above

(2) Isosceles triangle

(3) Right angled triangle

(4) Equilateral triangle

Answer (2)

Sol. The given three lines are x – y = 0, x + 2y = 3 and 2x + y = 6 then point of intersection

lines x – y = 0 and x + 2y = 3 is (1, 1)

lines x – y = 0 and 2x + y = 6 is (2, 2)

and lines x + 2y = 3 and 2x + y = 0 is (3, 0)

The triangle ABC has vertices A(1, 1), B(2, 2) and C(3, 0)

$\therefore$ $AB = \sqrt{2}$, $BC = \sqrt{5}$ and $AC = \sqrt{5}$

$\therefore$ ΔABC is isosceles

20. If (1, 5, 35), (7, 5, 5), (1, λ, 7) and (2λ, 1, 2) are coplanar, then the sum of all possible values of λ is

(1) $\frac{44}{5}$

(2) $-\frac{44}{5}$

(3) $\frac{39}{5}$

(4) $-\frac{39}{5}$

Answer (1)

Sol. Four points (1, 5, 35), (7, 5, 5), (1, λ, 7) and (2λ, 1, 2) are coplanar then

$$\begin{vmatrix} 6 & 0 & -30 \\ 0 & \lambda-5 & -28 \\ 2\lambda-1 & -4 & -33 \end{vmatrix} = 0$$

$$\begin{vmatrix} 6 & 0 & 0 \\ 0 & \lambda-5 & -28 \\ 2\lambda-1 & -4 & 10\lambda-38 \end{vmatrix} (R_3 \to (C_3+5+C_1)) = 0$$

$6\big((\lambda-5)(10\lambda-38)-112\big) = 0$

$\therefore$ $10\lambda^2 - 88\lambda + 78 = 0$

$\Rightarrow$ $5\lambda^2 - 44\lambda + 39 = 0$

$\therefore$ Sum of all possible values of $\lambda = \frac{44}{5}$

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. The number of integral values of 'k' for which the equation $3\sin x + 4\cos x = k+1$ has a solution, $k \in R$ is _____.

Answer (11)

Sol. $3\sin x + 4\cos x = k + 1$ has a solution then

$k+1 \in [-5, 5]$

$\therefore \quad k \in [-6, 4]$

$\therefore$ Number of possible integral values of k = 11.

2. The value of the integral $\int_0^{\pi} |\sin 2x| dx$ is _____.

Answer (02)

Sol. $= \int_0^{\pi} |\sin 2x| dx$

$= \int_0^{\frac{\pi}{2}} \sin 2x dx + \int_{\frac{\pi}{2}}^{\pi} -\sin 2x dx$

$= \left[-\frac{\cos 2x}{2}\right]_0^{\frac{\pi}{2}} + \left[\frac{\cos 2x}{2}\right]_{\frac{\pi}{2}}^{\pi}$

$= \frac{1}{2} + \frac{1}{2} + \left(\frac{1}{2} + \frac{1}{2}\right)$

$= 2$

3. If $\sqrt{3}\left(\cos^2 x\right) = \left(\sqrt{3} - 1\right) \cos x + 1$, the number of solutions of the given equation when $x \in \left[0, \frac{\pi}{2}\right]$ is _____.

Answer (01)

Sol. $\sqrt{3}\cos^2 x = (\sqrt{3} - 1)\cos x + 1$

$\sqrt{3}\cos^2 x = \sqrt{3}\cos x + \cos x - 1 = 0$

$\sqrt{3}\cos x(\cos x - 1) + (\cos x - 1) = 0$

$(\cos x - 1)(\sqrt{3}\cos x + 1) = 0$

$\therefore \quad \cos x = 1 \text{ or } -\frac{1}{\sqrt{3}}$

$\therefore$ Number of solution in $x \in \left[0, \frac{\pi}{2}\right]$ is 1.

4. The sum of 162^{th} power of the roots of the equation $x^3 - 2x^2 + 2x - 1 = 0$ is _____.

Answer (3)

Sol. $x^3 - 1 + 2x - 2x^2 = 0$

$\Rightarrow (x - 1)[x^2 - x + 1] = 0$

$\Rightarrow x = 1, -\omega, -\omega^2$

$S = 1^{162} + (-\omega)^{162} + (-\omega^2)^{162}$

$= 1 + 1 + 1 = 3$

5. The difference between degree and order of a differential equation that represents the family of curves given by $y^2 = a\left(x + \frac{\sqrt{a}}{2}\right), a > 0$ is _____.

Answer (02)

Sol. $y^2 = a\left(x + \frac{\sqrt{a}}{2}\right)$ ...(1)

$2y.\frac{dy}{dx} = a$ ...(2)

From (1) and (2)

$y^2 = 2y\frac{dy}{dx}\left(x + \frac{1}{2}\sqrt{2y\frac{dy}{dx}}\right)$

$y - 2x\frac{dy}{dx} = y\frac{dy}{dx}.\sqrt{2y\frac{dy}{dx}}$

$\Rightarrow \left(y - 2x\frac{dy}{dx}\right)^2 = 2y^3\left(\frac{dy}{dx}\right)^3$

$\Rightarrow$ Order 1 and degree 3.

6. If y = y(x) is the solution of the equation $e^{\sin y}\cos y\frac{dy}{dx} + e^{\sin y}\cos x = \cos x,\ y(0) = 0;$

then $1 + y\left(\frac{\pi}{6}\right) + \frac{\sqrt{3}}{2}y\left(\frac{\pi}{3}\right) + \frac{1}{\sqrt{2}}y\left(\frac{\pi}{4}\right)$ is equal to _____.

Answer (1)

Sol. $e^{\sin y} \cdot \cos x\frac{dy}{dx} + e^{\sin y} \cdot \cos x = \cos x$

Let $e^{siny} = Y$

$\Rightarrow \frac{dY}{dx} + Y\cos x = \cos x$

$\Rightarrow I.F = e^{\sin x}$

$\Rightarrow Y \cdot e^{\sin x} = \int e^{\sin x} \cdot \cos dx + c$

$\Rightarrow e^{siny} \cdot e^{\sin x} = e^{\sin x} + c$

When x = 0, y = 0 then c = 0

$\Rightarrow e^{\sin x + siny} = e^{\sin x} \Rightarrow siny = 0$

$\Rightarrow$ y = 0

$\Rightarrow$ y(x) = 0

hence, $1 + y\left(\frac{\pi}{6}\right) + \frac{\sqrt{3}}{2}y\left(\frac{\pi}{3}\right) + \frac{1}{\sqrt{2}}y\left(\frac{\pi}{4}\right) = 1$

7. Let $(\lambda, 2, 1)$ be a point on the plane which passes through the point (4, –2, 2). If the plane is perpendicular to the line joining the points (–2, –21, 29) and (–1, –16, 23), then $\left(\frac{\lambda}{11}\right)^2 - \frac{4\lambda}{11} - 4$ is equal to _____.

Answer (8)

Sol. Vector perpendicular to the plane is

$\bar{n} = \hat{i} + 5\hat{j} - 6\hat{k}$.

Given $A(\lambda, 2, 1)$ and $B(4, -2, 2)$

$\because \overline{AB} \perp \bar{n}$, so

$(\lambda - 4) + 5 \times 4 - 6(-1) = 0$

$\Rightarrow \lambda - 4 + 20 + 6 = 0$

$\Rightarrow \lambda = -22$

$\Rightarrow \frac{\lambda}{11} = -2$

hence $\left(\frac{\lambda}{11}\right)^2 - 4\left(\frac{\lambda}{11}\right) - 4 = 8$

8. The number of solutions of the equation $\log_4(x-1) = \log_2(x-3)$ is _____.

Answer (1)

Sol. Domain : x – 1 > 0 and x – 3 > 0

$\Rightarrow x \in (3, \infty)$

$\because \log_4(x-1) = \log_2(x-3)$

$\Rightarrow x - 1 = (x-3)^2$

$\Rightarrow x^2 - 7x + 8 = 0$

$\Rightarrow x = \frac{7 \pm \sqrt{17}}{2}$

but only $\frac{7+\sqrt{17}}{2}$ is the correct answer.

9. Let m, n $\in$ N and gcd (2, n) = 1, If

$30\binom{30}{0} + 29\binom{30}{1} + \ldots\ldots + 2\binom{30}{28} + 1\binom{30}{29} = n \cdot 2^m$,

then n + m is equal to _____.

$\left(\text{Here } \binom{n}{k} = {}^nC_k\right)$

Answer (45)

Sol. $\sum_{r=0}^{29} (30-r) \cdot {}^{30}C_r$

$= \sum_{r=1}^{30} r \cdot {}^{30}C_{30-r} = \sum_{r=1}^{30} r \cdot {}^{30}C_r$

$= 30\sum_{r=1}^{30} {}^{29}C_{r-1} = 30 \cdot 2^{29}$

$= 15 \cdot 2^{30}$

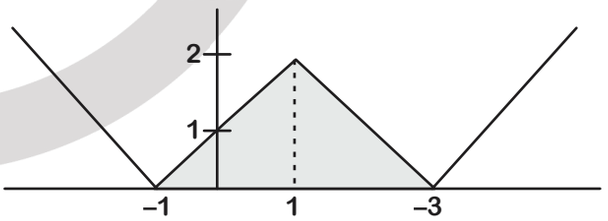
Clearly n = 15, m = 30

and m + n = 45

10. The area bounded by the lines $y = ||x-1|-2|$ is _____.

Answer (*)

Sol.



Area of the shaded region $= \frac{1}{2}(4 \times 2) = 4$

* As per given answer key the equation in the question should be $|y| = ||x-1|-2|$

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