Time : 3 hrs. M.M. : 300

Answers & Solutions

for

JEE (MAIN)-2021 (Online) Phase-2

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS :

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper A, B, C consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part has two sections.

(i) Section-I : This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) Section-II : This section contains 10 questions. In Section-II, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and there is no negative marking for wrong answer.

PART–A : PHYSICS

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. An object is located at 2 km beneath the surface of the water. If the fractional compression $\frac{\Delta V}{V}$ is 1.36%, the ratio of hydraulic stress to the corresponding hydraulic strain will be ________.

   [Given: density of water is 1000 kgm^{-3} and g = 9.8 ms^{-2}]

   (1) 1.44×10^7 Nm^{-2} (2) 2.26×10^9 Nm^{-2}

   (3) 1.96×10^7 Nm^{-2} (4) 1.44×10^9 Nm^{-2}

Answer (4)

Sol. $\frac{\text{Stress}}{\text{Strain}} = \frac{\Delta P}{\left(\frac{\Delta V}{V}\right)}$

$= \frac{\rho g h}{\left(\frac{\Delta V}{V}\right)}$

$= \frac{10^3 \times 9.8 \times 2 \times 10^3}{\left(\frac{1.36}{100}\right)}$

$= 1.44 \times 10^9$ N/m^2

2. What happens to the inductive reactance and the current in a purely inductive circuit if the frequency is halved?

   (1) Inductive reactance will be doubled and current will be halved

   (2) Both, inducting reactance and current will be doubled

   (3) Inductive reactance will be halved and current will be doubled

   (4) Both, inductive reactance and current will be halved

Answer (3)

Sol. $X_L = \omega L$

$I = \frac{\varepsilon}{X_L}$

∴ Reactance gets halved and current gets doubled.

3. A rubber ball is released from a height of 5 m above the floor. It bounces back repeatedly, always rising to $\frac{81}{100}$ of the height through which it falls. Find the average speed of the ball.

   (Take g = 10 ms^{-2})

   (1) 3.50 ms^{-1} (2) 2.0 ms^{-1}

   (3) 2.50 ms^{-1} (4) 3.0 ms^{-1}

Answer (3)

Sol. $h = h_0 e^2$

$\Rightarrow \frac{81}{100} = e^2 \quad \Rightarrow \quad e = 0.9$

$S_{total} = h_0 + 2h_1 + 2h_2 + 2h_3 + \ldots$

$= h_0 [1 + 2(e^2 + e^4 + e^6 + \ldots)]$

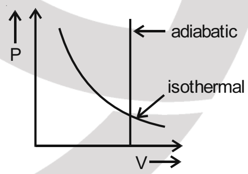
$= h_0 \left[1 + 2 \times e^2 \times \left(\frac{1}{1-e^2}\right)\right] = h_0 \left(\frac{1+e^2}{1-e^2}\right)$

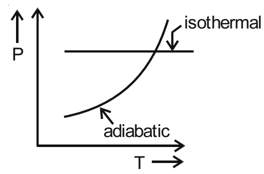
$T_{total} = \sqrt{\frac{2h_0}{g}} [1 + 2 \times e + 2e^2 + \ldots]$

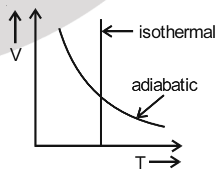
$= \sqrt{\frac{2 \times 5}{10}} \left[1 + 2e\left(\frac{1}{1-e}\right)\right] = 1 \times \left(\frac{1+e}{1-e}\right)$

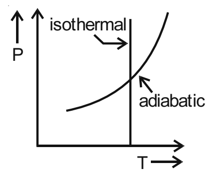
$\therefore V_{av} = \frac{5 \times (1 + 0.9^2)}{(1 - 0.9^2)} \times \left(\frac{1-0.9}{1+0.9}\right) = 2.50$ m/s

4. Which one is the correct option for the two different thermodynamic processes?

(a)


(b)


(c)


(d)


   (1) (c) and (d)

   (2) (a) only

   (3) (c) and (a)

   (4) (b) and (c)

Answer (1)

Sol. For adiabatic, TV^{r-1} = constant

$\Rightarrow$ (c) is correct

and, $P \times \left(\frac{T}{P}\right)^{\gamma}$ = constant

$\Rightarrow$ $P^{1-\gamma}T^{\gamma}$ = constant

$\Rightarrow$ $T^{\gamma} \propto P^{\gamma-1}$

$\Rightarrow$ (d) is correct

5. If one mole of the polyatomic gas is having two vibrational modes and β is the ratio of molar specific heats for polyatomic gas $\left(\beta = \frac{C_P}{C_V}\right)$ then the value of β is:

(1) 1.35 (2) 1.2

(3) 1.25 (4) 1.02

Answer (3)

Sol. $\beta = \frac{C_P}{C_V} = 1 + \frac{2}{f}$

f = 3 + 3 + 2 = 8

$\therefore \beta = 1 + \frac{2}{8}$

$= \frac{5}{4}$

= 1.25

6. The velocity of a particle is v = v_0 + gt + Ft^2. Its position is x = 0 at t = 0; then its displacement after time (t = 1) is :

(1) $v_0 + \frac{g}{2} + F$

(2) $v_0 + g + F$

(3) $v_0 + \frac{g}{2} + \frac{F}{3}$

(4) $v_0 + 2g + 3F$

Answer (3)

Sol. $v = v_0 + gt + Ft^2$

$dx = \int_0^1 (v_0 + gt + Ft^2)dt$

$= \left[v_0 t + g\frac{t^2}{2} + \frac{Ft^3}{3}\right]_0^1$

$= v_0 + \frac{g}{2} + \frac{F}{3}$

7. Two particles A and B of equal masses are suspended from two massless springs of spring constants K_1 and K_2 respectively. If the maximum velocities during oscillations are equal, the ratio of the amplitude of A and B is

(1) $\sqrt{\frac{K_2}{K_1}}$ (2) $\frac{K_2}{K_1}$

(3) $\frac{K_1}{K_2}$ (4) $\sqrt{\frac{K_1}{K_2}}$

Answer (1)

Sol. $V_1 = W_1 \times A_1$

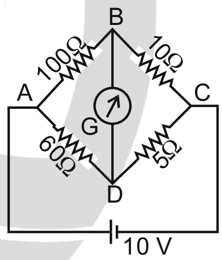
$V_2 = W_2 \times A_2$

$\Rightarrow \frac{A_1}{A_2} = \frac{V_1}{V_2} \times \frac{W_2}{W_1}$

$= 1 \times \sqrt{\frac{K_2}{m} \times \frac{m}{K_1}}$

$= \sqrt{\frac{K_2}{K_1}}$

8. The four arms of a Wheatstone bridge have resistances as shown in the figure. A galvanometer of 15Ω resistance is connected across BD. Calculate the current through the galvanometer when a potential difference of 10 V is maintained across AC.

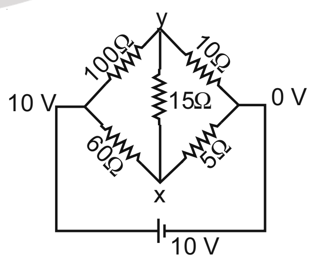


(1) 2.44 μA (2) 4.87 μA

(3) 2.44 mA (4) 4.87 mA

Answer (4)

Sol.



$\frac{10-x}{60} + \frac{0-x}{5} + \frac{y-x}{15} = 0$

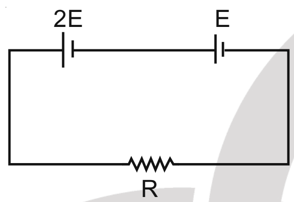
and, $\dfrac{10-y}{100}+\dfrac{0-y}{10}+\dfrac{x-y}{15}=0$

$\Rightarrow x = 0.79\,V,\ y = 0.865$

$\therefore i_{PB} = \left(\dfrac{y-x}{15}\right)$

$= 4.87 \times 10^{-3}$A

9. Two cells of emf 2E and E with internal resistance r_1 and r_2 respectively are connected in series to an external resistor R (see figure). The value of R, at which the potential difference across the terminals of the first cell becomes zero is

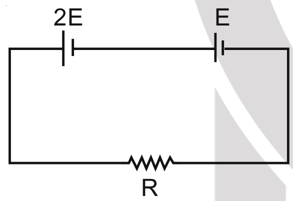


(1) $r_1 + r_2$ (2) $r_1 - r_2$

(3) $\dfrac{r_1}{2}+r_2$ (4) $\dfrac{r_1}{2}-r_2$

Answer (4)

Sol.



$i = \dfrac{3E}{R+r_1+r_2}$

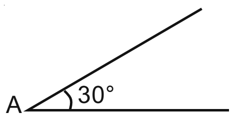
$V_T = 2E - ir_1 = 0$

$\Rightarrow 2E = \dfrac{3E}{R+r_1+r_2}\times r_1$

$\Rightarrow R+r_1+r_2 = \dfrac{3r_1}{2}$

$\Rightarrow R = \dfrac{r_1}{2}-r_2$

10. A sphere of mass 2 kg and radius 0.5 m is rolling with an initial speed of 1 ms^{-1} goes up an inclined plane which makes an angle of 30° with the horizontal plane, without slipping. How long will the sphere take to return to the starting point A?

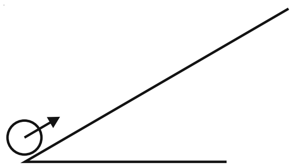


(1) 0.60 s (2) 0.80 s

(3) 0.57 s (4) 0.52 s

Answer (3)

Sol.



$a = \dfrac{mg\sin\theta \times r^2}{\left(\dfrac{7}{5}mr^2\right)}$

$= \dfrac{5}{7}g\sin\theta$

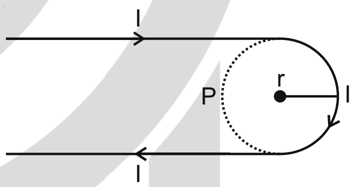
$= \dfrac{5}{7}\times 10\times\dfrac{1}{2} = \dfrac{25}{7}$ m/s^2

$\therefore \Delta t = 2\times\left(\dfrac{u}{a}\right)$

$= 2\times\dfrac{1}{\left(\dfrac{25}{7}\right)}$

$\simeq 0.57$ s

11. A hairpin like shape as shown in figure is made by bending a long current carrying wire. What is the magnitude of a magnetic field at point P which lies on the centre of the semicircle?



(1) $\dfrac{\mu_0 I}{4\pi r}(2+\pi)$ (2) $\dfrac{\mu_0 I}{4\pi r}(2-\pi)$

(3) $\dfrac{\mu_0 I}{2\pi r}(2-\pi)$ (4) $\dfrac{\mu_0 I}{2\pi r}(2+\pi)$

Answer (1)

Sol. Field due to straight section $= \dfrac{\mu_0 I}{4\pi r}$

Field due to circular section $= \dfrac{\mu_0 I}{4r}$

Net field $= \dfrac{\mu_0 I}{2\pi r}+\dfrac{\mu_0 I}{4r}$

12. A carrier signal C(t) = 25 sin(2.512 × 10^{10}t) is amplitude modulated by a message signal m(t) = 5 sin(1.57 × 10^{8}t) and transmitted through an antenna. What will be the bandwidth of the modulated signal?

(1) 50 MHz (2) 2.01 GHz

(3) 1987.5 MHz (4) 8 GHz

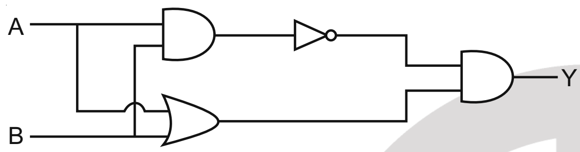
Answer (1)

Sol. Bandwidth $= \frac{2\omega_m}{2\pi} = \frac{\omega_m}{\pi}$

$= \frac{1.57 \times 10^8}{\pi} = \frac{10^8}{2}$

= 50 MHz

13. Which one of the following will be the output of the given circuit?



(1) NAND Gate (2) XOR Gate

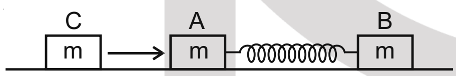
(3) NOR Gate (4) AND Gate

Answer (2)

Sol. $X = (A + B) \cdot \overline{AB}$

$= (A + B) \cdot (\overline{A} + \overline{B})$

14. Two identical blocks A and B each of mass m resting on the smooth horizontal floor are connected by a light spring of natural length L and spring constant K. A third block C of mass m moving with a speed v along the line joining A and B collides with A. The maximum compression in the spring is



(1) $\sqrt{\frac{mv}{2K}}$

(2) $\sqrt{\frac{mv}{K}}$

(3) $v\sqrt{\frac{m}{2K}}$

(4) $\sqrt{\frac{m}{2K}}$

Answer (3)

Sol. Maximum compression occurs when velocities of A and B are equal.

$$\frac{1}{2}mv_0^2 - \frac{1}{4}mv_0^2 = \frac{1}{2}Kx^2$$

$$v_0\sqrt{\frac{m}{2K}} = x$$

15. Two identical photocathodes receive the light of frequencies f_1 and f_2 respectively. If the velocities of the photo-electrons coming out are v_1 and v_2 respectively, then

(1) $v_1^2 + v_2^2 = \frac{2h}{m}[f_1 + f_2]$

(2) $v_1 - v_2 = \left[\frac{2h}{m}(f_1 - f_2)\right]^{\frac{1}{2}}$

(3) $v_1 + v_2 = \left[\frac{2h}{m}(f_1 + f_2)\right]^{\frac{1}{2}}$

(4) $v_1^2 - v_2^2 = \frac{2h}{m}[f_1 - f_2]$

Answer (4)

Sol. $hf_1 = \phi + \frac{1}{2}mv_1^2$

$hf_2 = \phi + \frac{1}{2}mv_2^2$

$v_1^2 - v_2^2 = \frac{2h}{m}(f_1 - f_2)$

16. A block of mass 1 kg attached to a spring is made to oscillate with an initial amplitude of 12 cm. After 2 minutes the amplitude decreases to 6 cm. Determine the value of the damping constant for this motion. (take ln 2 = 0.693)

(1) 0.69×10^2 kg s^{-1} (2) 3.3×10^2 kg s^{-1}

(3) 5.7×10^3 kg s^{-1} (4) 1.16×10^2 kg s^{-1}

Answer (*)

Sol. $A = A_0 e^{-bt/2m}$

$$b = \frac{2m\ln(2)}{t} = 2 \times \frac{0.693}{120} = 1.14 \times 10^{-2}$$

(*) None of the option is correct.

17. A sound wave of frequency 245 Hz travels with the speed of 300 ms^{-1} along the positive x-axis. Each point of the wave moves to and fro through a total distance of 6 cm. What will be the mathematical expression of this travelling wave?

(1) Y(x, t) = 0.06 [sin 0.8x – (0.5×10^3)t]

(2) Y(x, t) = 0.03 [sin 5.1x – (1.5×10^3)t]

(3) Y(x, t) = 0.03 [sin 5.1x – (0.2×10^3)t]

(4) Y(x, t) = 0.06 [sin 5.1x – (1.5×10^3)t]

Answer (2)

Sol. 2A = 0.06

A = 0.03

$$\lambda = \frac{300}{245}, k = \frac{2\pi}{300} \times 245 = 5.1$$

ω = 1540

So, option (2) matches.

18. A geostationary satellite is orbiting around an arbitrary planet 'P' at a height of 11R above the surface of 'P', R being the radius of 'P'. The time period of another satellite in hours at a height of 2R from the surface of 'P' is ______. 'P' has the time period of 24 hours.

(1) 5

(2) 3

(3) $6\sqrt{2}$

(4) $\frac{6}{\sqrt{2}}$

Answer (2)

Sol. $T^2 \propto R^3$

$$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3} = \frac{1}{64}$$

$$T_1 = \frac{(24)}{(64)^{1/2}} = 3 \text{ hrs.}$$

19. The atomic hydrogen emits a line spectrum consisting of various series. Which series of hydrogen atomic spectra is lying in the visible region?

(1) Balmer series (2) Brackett series

(3) Lyman series (4) Paschen series

Answer (1)

Sol. Balmer series lie in visible region.

20. Match List-I with List-II

	List-I		List-II
(a)	Phase difference between current and voltage in a purely resistive AC circuit	(i)	$\frac{\pi}{2}$; current leads voltage
(b)	Phase difference between current and voltage in a pure inductive AC circuit	(ii)	Zero
(c)	Phase difference between current and voltage in a pure capacitive AC circuit	(iii)	$\frac{\pi}{2}$; current lags voltage
(d)	Phase difference between current and voltage in an LCR series circuit	(iv)	$\tan^{-1}\left(\frac{X_C - X_L}{R}\right)$

Choose the most appropriate answer from the options given below:

(1) (a)-(i), (b)-(iii), (c)-(iv), (d)-(ii)

(2) (a)-(ii), (b)-(iv), (c)-(iii), (d)-(i)

(3) (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i)

(4) (a)-(ii), (b)-(iii), (c)-(i), (d)-(iv)

Answer (4)

Sol. For purely resistive circuit, current is in phase with voltage.

Whereas for inductive and capacitive circuit phase difference between voltage and current is $\frac{\pi}{2}$.

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. Seawater at a frequency f = 9 × 10^2 Hz, has permittivity $\varepsilon = 80\varepsilon_0$ and resistivity ρ = 0.25 Ωm. Imagine a parallel plate capacitor is immersed in seawater and is driven by an alternating voltage source $V(t) = V_0 \sin(2\pi ft)$. Then the conduction current density becomes 10^x times the displacement current density after time $t = \frac{1}{800}$ s. The value of x is ___________.

$$\left(\text{Given}: \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ Nm}^2\text{C}^{-2}\right)$$

Answer (6)

Sol. $i_c = \frac{V_0}{R}\text{Sin}\,\omega t$

$i_d = k\varepsilon_0 \frac{d}{dt}\left(\frac{v_0}{d}\text{Sin}\,\omega t\,A\right)$

$= \frac{k\varepsilon_0 A\ v_0\omega\cos\omega t}{d}$

$= k\omega c v_0 \cos\omega t$

$\frac{j_c}{j_d} = \frac{i_{c/A}}{i_{d/A}} = \frac{\tan\omega t}{k\omega RC} = \frac{1}{80\times 2\pi\times 900\times\frac{1}{4}\times\varepsilon_0}$

$= 10^6$

2. The electric field in a region is given by $\vec{E} = \frac{2}{5}E_0\hat{i} + \frac{3}{5}E_0\hat{j}$ with $E_0 = 4.0\times 10^3\ \frac{N}{C}$. The flux of this field through a rectangular surface area 0.4 m^2 parallel to the Y – Z plane is _________ $Nm^2\ C^{-1}$.

Answer (640)

Sol. $\phi = \vec{E}\cdot\vec{A}$

$= \frac{E_0}{5}(2\hat{i}+3\hat{j})\cdot(0.4\hat{i})$

$= \frac{4000}{5}(2\times 0.4)$

$= 640\ Nm^2\ C^{-1}$

3. The electric field intensity produced by the radiation coming from a 100 W bulb at a distance of 3 m is E. The electric field intensity produced by the radiation coming from 60 W at the same distance is

$\sqrt{\frac{x}{5}}E$. Where the value of x = _________.

Answer (3)

Sol. $I = \frac{P}{4\pi r^2} = \frac{E_0^2}{2\mu_0 C}$

$E_0 \propto \frac{\sqrt{P}}{r}$

$E_2 = E_1\sqrt{\frac{P_2}{P_1}} = E_0\sqrt{\frac{3}{5}}$

4. The image of an object placed in air formed by a convex refracting surface is at a distance of 10 m behind the surface. The image is real and is at $\frac{2}{3}^{rd}$ of the distance of the object from the surface. The wavelength of light inside the surface is $\frac{2}{3}$ times the wavelength in air. The radius of the curved surface is $\frac{x}{13}$m. The value of 'x' is _________.

Answer (30)

Sol. $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2-\mu_1}{R}$

$\mu_1 = 1,\quad \mu_2 = 1.5$

$\frac{1.5}{+10} - \frac{1}{-15} = \frac{1.5-1}{+R}$

$R = \frac{30}{13}$m

5. Suppose you have taken a dilute solution of oleic acid in such a way that its concentration becomes 0.01 cm^3 of oleic acid per cm^3 of the solution. Then you make a thin film of this solution (monomolecular thickness) of area 4 cm^2 by considering 100 spherical drops of radius $\left(\frac{3}{40\pi}\right)^{\frac{1}{3}}\times 10^{-3}$ cm. Then the thickness of oleic acid layer will be x × 10^{-14} m. Where x is _________.

Answer (25)

Sol. Volume of oleic acid = $100\times\frac{4}{3}\pi r^3\times\frac{1}{100}$

$\frac{4}{3}\pi r^3 = A\times t$

$t = 25\times 10^{-14}$ m

$x = 25$

6. A particle of mass m moves in a circular orbit in a central potential field $U(r) = U_0 r^4$. If Bohr's quantization conditions are applied, radii of possible orbitals r_n vary with $n^{\frac{1}{\alpha}}$, where α is_____.

Answer (3)

Sol. $U(r) = U_0 r^4$

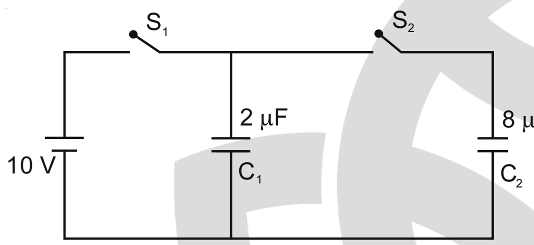
$F(r) = -\frac{dU(r)}{dr} = -4U_0 r^3$

$$\frac{mv_n^2}{r_n} = 4U_0 r_n^3 \qquad .....(1)$$

$$mvr_n = n\frac{h}{2\pi} \qquad .....(2)$$

$$r_n \propto n^{\frac{1}{3}}$$

7. A 2 μF capacitor C_1 is first charged to a potential difference of 10 V using a battery. Then the battery is removed and the capacitor is connected to an uncharged capacitor C_2 of 8 μF. The charge in C_2 on equilibrium condition is______μC. (Round off to the nearest Integer)



Answer (16)

Sol. $Q_1 = C_1V = 20\ \mu C$

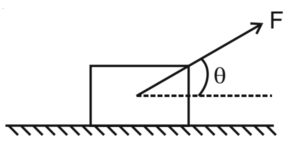
$$q_2 = \left(\frac{Q_1 + Q_2}{C_1 + C_2}\right).C_2 = 16\ \mu C$$

8. A body of mass 1 kg rests on a horizontal floor with which it has a coefficient of static friction $\frac{1}{\sqrt{3}}$. It is desired to make the body move by applying the minimum possible force F N. The value of F will be______. (Round off to the Nearest Integer)

[Take g = 10 ms^{-2}]

Answer (5)

Sol. Force required to pull the block

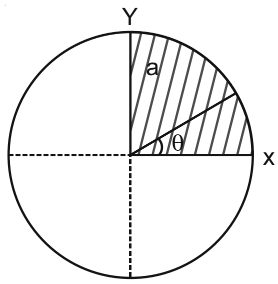


$$F = \frac{\mu Mg}{\cos\theta + \mu\sin\theta}$$

$$F_{min} = \frac{\mu Mg}{\sqrt{1+\mu^2}} = 5\ N$$

9. The disc of mass M with uniform surface mass density σ is shown in the figure. The centre of mass of the quarter disc (the shaded area) is at the position $\frac{x}{3}\frac{a}{\pi}, \frac{x}{3}\frac{a}{\pi}$ where x is______.(Round off to the Nearest Integer)

[a is an area as shown in the figure]

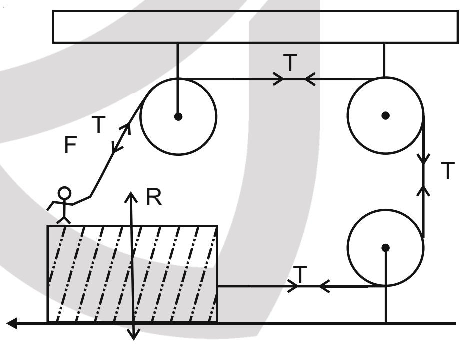


Answer (4)

Sol. $(X_{CM}, Y_{CM}) = \left(\frac{4a}{3\pi}, \frac{4a}{3\pi}\right)$

10. A boy of mass 4 kg is standing on a piece of wood having mass 5 kg. If the coefficient of friction between the wood and the floor is 0.5, the maximum force that the boy can exert on the rope so that the piece of wood does not move from its place is______N. (Round off to the Nearest Integer)

[Take g = 10 ms^{-2}]



Answer (30)

Sol. T = Mg – N

R = mg + N = (μ + m)g – T

For no movement of block

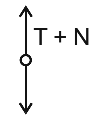
$$T \le \mu R$$

$$T \le \mu[(M+m)g - T]$$

$$T \le \frac{\mu(M+m)g}{1+\mu}$$

T_{max} = 30 N

For Man:



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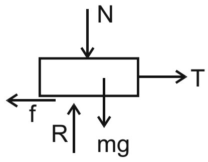


PART–B : CHEMISTRY

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Match List-I with List-II :

List-I	List-II
(a) Haematite	(i) $Al_2O_3 \cdot xH_2O$
(b) Bauxite	(ii) Fe_2O_3
(c) Magnetite	(iii) $CuCO_3 \cdot Cu(OH)_2$
(d) Malachite	(iv) Fe_3O_4

Choose the **correct** answer from the options given below.

(1) (a)-(i), (b)-(iii), (c)-(ii), (d)-(iv)

(2) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)

(3) (a)-(iv), (b)-(i), (c)-(ii), (d)-(iii)

(4) (a)-(ii), (b)-(iii), (c)-(i), (d)-(iv)

Answer (2)

Sol. (a) Haematite ⟶ (ii) Fe_2O_3

(b) Bauxite ⟶ (i) $Al_2O_3 \cdot xH_2O$

(c) Magnetite ⟶ (iv) Fe_3O_4

(d) Malachite ⟶ (iii) $CuCO_3 \cdot Cu(OH)_2$

2. $$\underset{}{C_{12}H_{22}O_{11}} + H_2O \xrightarrow{\text{Enzyme A}} \underset{\text{Glucose}}{C_6H_{12}O_6} + \underset{\text{Fructose}}{C_6H_{12}O_6}$$

$$\underset{\text{Glucose}}{C_6H_{12}O_6} \xrightarrow{\text{Enzyme B}} 2C_2H_5OH + 2CO_2$$

In the above reactions, the enzyme A and enzyme B respectively are

(1) Amylase and Invertase

(2) Invertase and Zymase

(3) Zymase and Invertase

(4) Invertase and Amylase

Answer (2)

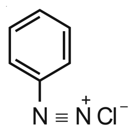
Sol. $$\underset{\text{Sucrose}}{C_{12}H_{22}O_{11}} + H_2O \xrightarrow{\text{Invertase}} \underset{\text{Glucose}}{C_6H_{12}O_6} + \underset{\text{Fructose}}{C_6H_{12}O_6}$$

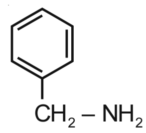
$$\underset{\text{Glucose}}{C_6H_{12}O_6} \xrightarrow{\text{Zymase}} 2C_2H_5OH + 2CO_2$$

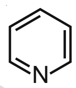
∴ Enzyme A is Invertase

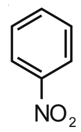
Enzyme B is Zymase

3. Nitrogen can be estimated by Kjeldahl's method for which of the following compound?

(1) 

(2) 

(3) 

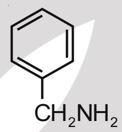
(4) 

Answer (2)

Sol. Kjeldahl's method can't be used for compounds containing nitro or azo groups.

Also for compounds containing nitrogen in ring, Kjeldahl's method can't be used.

(e.g. for pyridine or quinoline)

∴  Compound gives nitrogen estimation through Kjeldahl's method.

4. For the coagulation of a negative sol, the species below, that has the highest flocculating power is

(1) Ba^{2+} (2) PO_4^{3-}

(3) SO_4^{2-} (4) Na^+

Answer (1)

Sol. For flocculation of anion, cations are used. Greater the charge on cation, higher its flocculating power.

So, Ba^{2+} has the highest flocculating power.

5. During which of the following processes, does entropy decrease?

(A) Freezing of water to ice at 0°C

(B) Freezing of water to ice at –10°C

(C) $N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$

(D) Adsorption of CO(g) on lead surface

(E) Dissolution of NaCl in water

Choose the **correct** answer from the options given below.

(1) (A) and (E) only

(2) (A), (C) and (E) only

(3) (A), (B), (C) and (D) only

(4) (B) and (C) only

Answer (3)

Sol. For A, B, C and D entropy decreases

While in case of E, entropy increases.

6. Choose the **correct** statement regarding the formation of carbocations A and B given.

$$CH_3-CH_2-CH=CH_2+HBr \rightarrow CH_3-CH_2-CH_2-\overset{+}{C}H_2+Br^- \text{ ("A")}$$
$$CH_3-CH_2-CH=CH_2+HBr \rightarrow CH_3-CH_2-\overset{+}{C}H-CH_3+Br^- \text{ ("B")}$$

(1) Carbocation A is more stable and formed relatively at slow rate

(2) Carbocation A is more stable and formed relatively at faster rate

(3) Carbocation B is more stable and formed relatively at slow rate

(4) Carbocation B is more stable and formed relatively at faster rate

Answer (4)

Sol. Carbocation B is more stable as it is secondary carbocation having more number of α-hydrogens and having greater +I effect.

∴ Carbocation B formed at a faster rate than carbocation A.

7. The common positive oxidation states for an element with atomic number 24, are

(1) +1 and +3 to +6

(2) +1 to +6

(3) +2 to +6

(4) +1 and +3

Answer (3)

Sol. Z = 24 represents chromium

Common positive oxidation state of Cr are from +2 to +6

where +3 and +6 are the most common ones.

8. The functional groups that are responsible for the ion-exchange property of cation and anion exchange resins, respectively, are

(1) $-SO_3H$ and $-NH_2$

(2) $-NH_2$ and $-COOH$

(3) $-SO_3H$ and $-COOH$

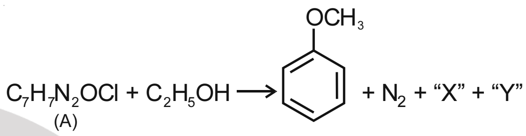
(4) $-NH_2$ and $-SO_3H$

Answer (1)

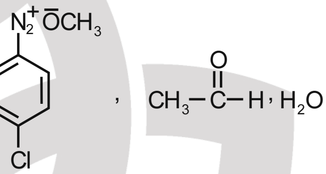
Sol. Cation exchange resins contain large organic molecule with $-SO_3H$ group.

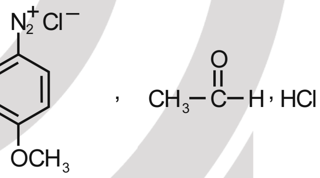
In cation exchange process H^+ exchanges for Na^+, Ca^{2+}, Mg^{2+} and other cations present in water.

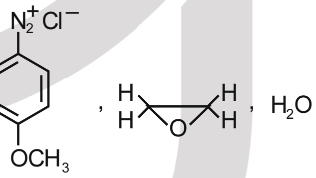
While anion exchange resins contain $-NH_2$ in form of $-NH_3^+ OH^-$ where OH^- exchanges for anions like Cl^-, HCO_3^-, SO_4^{2-}, etc.

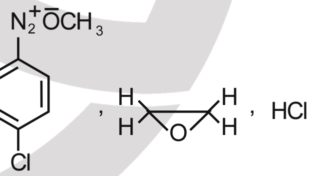
9. 

In the above reaction, the structural formula of (A), "X" and "Y" respectively are

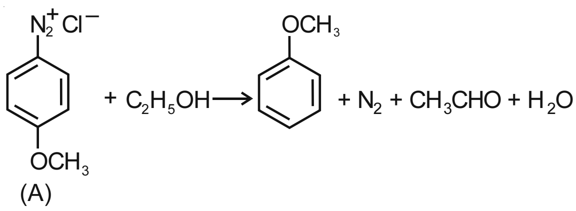
(1) 

(2) 

(3) 

(4) 

Answer (2)

Sol. 

C_2H_5OH behaves as mild reducing agent and itself gets oxidised to ethanal.

10. Match **List-I** with **List-II**.

List-I **Chemical Compound**	**List-II** **Used as**
(a) Sucralose	(i) Synthetic detergent
(b) Glyceryl ester of stearic acid	(ii) Artificial sweetener
(c) Sodium benzoate	(iii) Antiseptic
(d) Bithionol	(iv) Food preservative

Choose the correct match.

(1) (a)-(i), (b)-(ii), (c)-(iv), (d)-(iii)

(2) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)

(3) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)

(4) (a)-(iii), (b)-(ii), (c)-(iv), (d)-(i)

Answer (3)

Sol. (a) Sucralose $\rightarrow$ (ii) Artificial sweetener

(b) Glyceryl ester of stearic acid $\rightarrow$ (i) Synthetic detergent

(c) Sodium benzoate $\rightarrow$ (iv) food preservative

(d) Bithionol $\rightarrow$ (iii) Antiseptic

11. Match **List-I** with **List-II** :

List-I	**List-II**
(a) $[Co(NH_3)_6][Cr(CN)_6]$	(i) Linkage isomerism
(b) $[Co(NH_3)_3(NO_2)_3]$	(ii) Solvate isomerism
(c) $[Cr(H_2O)_6]Cl_3$	(iii) Co-ordination isomerism
(d) *cis*-$[CrCl_2(ox)_2]^{3-}$	(iv) Optical isomerism

Choose the correct answer from the options given below.

(1) (a)-(iv), (b)-(ii), (c)-(iii), (d)-(i)

(2) (a)-(iii), (b)-(i), (c)-(ii), (d)-(iv)

(3) (a)-(i), (b)-(ii), (c)-(iii), (d)-(iv)

(4) (a)-(ii), (b)-(i), (c)-(iii), (d)-(iv)

Answer (2)

Sol. (a) $[Co(NH_3)_6][Cr(CN)_6] \rightarrow$ (iii) Coordination isomerism

(b) $[Co(NH_3)_3(NO_2)_3] \rightarrow$ (i) Linkage isomerism

(c) $[Cr(H_2O)_6]Cl_3 \rightarrow$ (ii) Solvate isomerism

(d) *cis*-$[CrCl_2(ox)_2]^{3-}$ (iv) Optical isomerism

12. The set that represents the pair of neutral oxides of nitrogen is

(1) NO and NO_2 (2) N_2O and N_2O_3

(3) NO and N_2O (4) N_2O and NO_2

Answer (3)

Sol. Among nitrogen oxides

$\underbrace{NO\ \&\ N_2O}_{\text{Neutral}}$ $\underbrace{N_2O_3,\ NO_2,\ N_2O_5}_{\text{Acidic}}$

13. Amongst the following, the linear species is

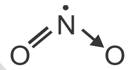
(1) N_3^- (2) NO_2

(3) O_3 (4) Cl_2O

Answer (1)

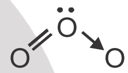
Sol. N_3^- : $\bar{N}=\overset{+}{N}=\bar{N}$ Linear

NO_2 :



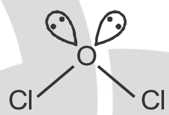
Bent

O_3 :



Bent

Cl_2O:



Bent

14. Given below are two statements :

Statement I : 2-methylbutane on oxidation with $KMnO_4$ gives 2-methylbutan-2-ol.

Statement II : n-alkanes can be easily oxidised to corresponding alcohols with $KMnO_4$.

Choose the **correct** option.

(1) Both **statement I** and **statement II** are incorrect

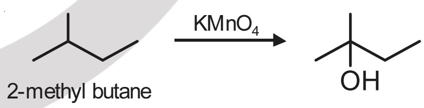
(2) **Statement I** is correct but **statement II** is incorrect

(3) **Statement I** is incorrect but **statement II** is correct

(4) Both **statement I** and **statement II** are correct

Answer (2)

Sol. Alkanes having tertiary H can be oxidised to corresponding alcohols by $KMnO_4$.



whereas ordinary alkanes resist oxidation.

15. Primary, secondary and tertiary amines can be separated using

(1) para-Toluene sulphonyl chloride

(2) Acetyl amide

(3) Chloroform and KOH

(4) Benzene sulphonic acid

Answer (1)

Sol. Hinsberg test is used to distinguish 1°, 2° and 3° amines.

Reagent used is ($PhSO_2Cl$).

So, para-toluene sulphonyl chloride can be used.

16. Which of the following statement(s) is(are) **incorrect** reason for eutrophication?

(A) Excess usage of fertilisers

(B) Excess usage of detergents

(C) Dense plant population in water bodies

(D) Lack of nutrients in water bodies that prevent plant growth

Choose the **most appropriate** answer from the options given below.

(1) (D) Only (2) (A) only

(3) (B) and (D) only (4) (C) only

Answer (1)

Sol. Nutrient enriched water bodies supports dense plant population. This is because of the excess usage of fertilisers and detergents. This process is known as eutrophication.

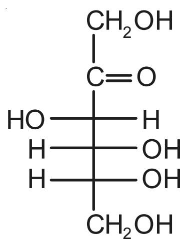
17. Fructose is an example of

(1) Pyranose (2) Heptose

(3) Aldohexose (4) Ketohexose

Answer (4)

Sol. Fructose is a ketohexose.



18. The set of elements that differ in mutual relationship from those of the other sets is

(1) B – Si (2) Li – Na

(3) Be – Al (4) Li – Mg

Answer (2)

Sol. Li ↘ Mg, Be ↘ Al, B ↘ Si (Shows diagonal relationship)

Li Be B

Na Mg Al Si

Li – Na belongs to same group.

19. One of the by-products formed during the recovery of NH_3 from solvay process is

(1) $CaCl_2$ (2) $Ca(OH)_2$

(3) $NaHCO_3$ (4) NH_4Cl

Answer (1)

Sol. Recovery of NH_3 :

$$2NH_4Cl + Ca(OH)_2 \longrightarrow 2NH_3 + CaCl_2 + H_2O.$$

By product : $CaCl_2$

20. The correct pair(s) of the ambident nucleophiles is (are):

(A) AgCN/KCN

(B) RCOOAg/RCOOK

(C) $AgNO_2/KNO_2$

(D) AgI/KI

(1) (B) and (C) only

(2) (B) only

(3) (A) only

(4) (A) and (C) only

Answer (4)

Sol. AgCN/KCN → ambident nucleophile

Nucleophilic site is N and C in the above two case.

$AgNO_2/KNO_2$ → ambident nucleophiles

N and O are the nucleophilic site in these two cases

While in B and D, there is only one nucleophilic site.

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. Consider the reaction $N_2O_4(g) \rightleftharpoons 2NO_2(g)$.

The temperature at which K_C = 20.4 and K_P = 600.1, is ________ K. (Round off to the Nearest Integer). [Assume all gases are ideal and R = 0.0831 L bar K^{-1} mol^{-1}]

Answer (354)

Sol. $N_2O_4(g) \rightleftharpoons 2NO_2(g)$

$\Delta n_g = 2 - 1 = 1$

$K_P = K_C (RT)^{\Delta n_g}$

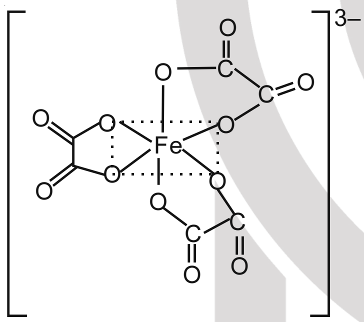
$600.1 = 20.4\ (0.0831 \times T)^1$

$T = \dfrac{600.1}{20.4 \times 0.0831} = 354\ K$

2. On complete reaction of $FeCl_3$ with oxalic acid in aqueous solution containing KOH, resulted in the formation of product A. The secondary valency of Fe in the product A is ______. (Round off to the Nearest Integer).

Answer (6)

Sol. $FeCl_3 + \text{oxalic acid} \xrightarrow{KOH} \left[Fe(C_2O_4)_3\right]^{3-}$



Coordination number = 6

Secondary valency is 6

3. The number of chlorine atoms in 20 mL of chlorine gas at STP is _______ 10^{21}. (Round off to the Nearest integer).

[Assume chlorine is an ideal gas at STP

$R = 0.083$ L bar $mol^{-1}K^{-1}$, $N_A = 6.023 \times 10^{23}$]

Answer (1)

Sol. Volume of Cl_2 at STP = 20 mL

Moles of chlorine gas = $\dfrac{20}{22400}$

Molecules of Cl_2 gas = $\dfrac{20}{22400} \times 6.023 \times 10^{23}$

Atoms of Cl = $2 \times \dfrac{20}{22400} \times 6.023 \times 10^{23}$

$\approx 1 \times 10^{23}$

4. The reaction $2A + B_2 \rightarrow 2AB$ is an elementary reaction. For a certain quantity of reactants. If the volume of the reaction vessel is reduced by a factor of 3, the rate of the reaction increases by a factor of _____. (Round off to the Nearest Integer).

Answer (27)

Sol. $2A + B_2 \longrightarrow 2B$

rate $\propto [A]^2\ [B_2]\ \propto\ (P_A)^2 \left(P_{B_2}\right)$

Now, if volume is reduced by a factor of 3, then P is increased by a factor of 3.

$\therefore \left(V \propto \dfrac{1}{P}\right)$

rate′ $\propto \left(3P_A\right)^2 \left(3P_{B_2}\right)$

$\propto 27P_A^2.P_{B_2}$

Rate increases by 27 times the previous rate.

5. KBr is doped with 10^{-5} mole percent of $SrBr_2$. The number of cationic vacancies in 1 g of KBr crystal is _____ 10^{14}. (Round off the Nearest Integer).

[Atomic Mass : K : 39.1 u, Br : 79.9 u $N_A = 6.023 \times 10^{23}$]

Answer (5)

Sol. 1 Sr^{2+} replaces two K^+. It occupies 1 of the position and 1 void is created.

Number of vacancies in 1 mole of KBr = $\dfrac{10^{-5}}{100} = 6.023 \times 10^{23}$

Moles of KBr given $= \dfrac{1}{119}$

$\therefore$ Total vacancies in 1 g of KBr $= \dfrac{1}{119} \times \dfrac{1}{10^7} \times 6.023 \times 10^{23}$

$= 5.06 \times 10^{14}$

$\approx 5 \times 10^{14}$

6. A KCl solution of conductivity 0.14 Sm^{-1} shows a resistance of 4.19 Ω in a conductivity cell. If the same cell is filled with an HCl solution, the resistance drops of 1.03 Ω. The conductivity of the HCl solution is _____ $\times 10^{-2}$ S m^{-1}. (Round off to the Nearest Integer).

Answer (57)

Sol. For KCl

$k_1 = 0.14\ Sm^{-1}$

$R_1 = 4.19\ \Omega$

For HCl

$k_2 = x\ Sm^{-1}$ (Let)

$R_2 = 1.03\ \Omega$

$$R = \rho\left(\frac{l}{a}\right) \Rightarrow kR = \underbrace{\left(\frac{l}{a}\right)}_{\text{Cell constant}}$$

$k_1R_1 = k_2R_2$

$0.14 \times 4.19 = 1.03 \times k_2$

$k_2 = 0.5695\ Sm^{-1}$

$\approx 57 \times 10^{-2}\ Sm^{-1}$

7. A 1 molal $K_4Fe(CN)_6$ solution has a degree of dissociation of 0.4. Its boiling point is equal to that of another solution which contains 18.1 weight percent of a non electrolytic solute A. The molar mass of A is _______ u. (Round off to the Nearest Integer).

[Density of water = 1.0 g cm^{-3}]

Answer (85)

Sol. i for $K_4[Fe(CN)_6] = 1 + (5 - 1)\ 0.4 = 2.6$

$$\Delta T_{b_1} = 2.6 \times K_f \times 1$$

i for A = 1.

$$(\Delta T_b)_A = 1 \times \frac{18.1 \times 1000}{A \times 81.9} \times K_f$$

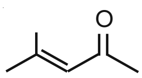
Equating these two

A = 85

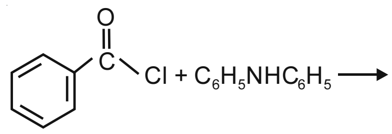
8. The total number of C – C sigma bond/s in mesityl oxide ($C_6H_{10}O$) is _______. (Round off to the Nearest Integer).

Answer (5)

Sol. Structure of mesityl oxide is



Number of C–C sigma bonds = 5

9. 

0.140 g   0.388 g

$C_6H_5 - \overset{O}{\overset{||}{C}} - N - (C_6H_5)_2$

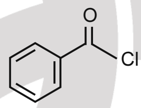
0.210 g

Consider the above reaction. The percentage yield of amide product is ____. (Round off to the Nearest Integer).

(Given : Atomic mass : C : 12.0 u, H : 1.0 u, N : 14.0 u, O : 16.0 u, Cl : 35.5 u)

Answer (77)

Sol.



$+ C_6H_5NHC_6H_5 \longrightarrow C_6H_5 - \overset{O}{\overset{||}{C}} - N(C_6H_5)_2$

Mass	0.140 g	0.388 g	0.210 g (Observed yield)
Moles	$\approx 10^{-3}$ (Limiting Reagent)	2.3×10^{-3}	Theoretical yield $= 10^{-3} \times 273$ = 0.273 g

$$\therefore \%\ \text{yield} = \frac{0.210}{0.273} \times 100 \approx 77\%$$

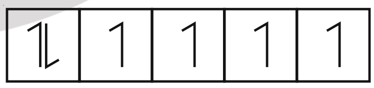
10. In the ground state of atomic Fe(Z = 26), the spin-only magnetic moment is ______ × 10^{-1} BM. (Round off to the Nearest Integer).

[Given : $\sqrt{3} = 1.73$, $\sqrt{2} = 1.41$]

Answer (49)

Sol. Fe(Z = 26)

Electronic configuration : $4s^2 3d^6$



n = 4

$$\therefore \mu = \sqrt{4(4+2)} = \sqrt{24}\ \text{BM}$$

= 4.89

$\approx 49 \times 10^{-1}$ BM

PART–C : MATHEMATICS

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. The number of solutions of the equation x + 2 tanx $= \frac{\pi}{2}$ in the interval [0, 2π] is :

(1) 2

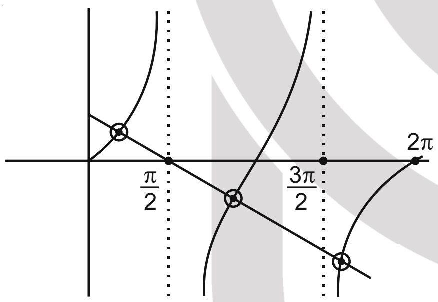
(2) 4

(3) 3

(4) 5

Answer (3)

Sol. $\because \quad 2\tan x = \frac{\pi}{2} - x$



3 solutions.

2. If the integral $\int_0^{10} \frac{[\sin 2\pi x]}{e^{x-[x]}} dx = \alpha e^{-1} + \beta e^{-\frac{1}{2}} + \gamma$, where α, β, γ are integers and [x] denotes the greatest integer less than or equal to x, then the value of α + β + γ is equal to :

(1) 25

(2) 10

(3) 0

(4) 20

Answer (3)

Sol. $\int_0^{10} \frac{[\sin 2\pi x]}{e^{\{x\}}} dx = 10\int_0^1 \frac{[\sin 2\pi x]}{e^x} dx$

$= -10\int_{1/2}^{1} \frac{dx}{e^x} = 10e^{-x}\Big|_{1/2}^{1} = 10 = (e^{-1} - e^{-1/2})$

$\Rightarrow \quad \alpha + \beta + \gamma = 0$

3. If x, y, z are in arithmetic progression with common difference d, x ≠ 3d, and the determinant of the matrix $\begin{bmatrix} 3 & 4\sqrt{2} & x \\ 4 & 5\sqrt{2} & y \\ 5 & k & z \end{bmatrix}$ is zero, then the value of k^2 is :

(1) 12 (2) 36

(3) 72 (4) 6

Answer (3)

Sol. $\because \begin{vmatrix} 3 & 4\sqrt{2} & x \\ 4 & 5\sqrt{2} & y \\ 5 & k & z \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 3 & 4\sqrt{2} & x \\ 1 & \sqrt{2} & d \\ 1 & k-5\sqrt{2} & d \end{vmatrix} = 0$

$\begin{vmatrix} 3 & 4\sqrt{2} & x \\ 1 & \sqrt{2} & d \\ 0 & k-6\sqrt{2} & 0 \end{vmatrix} = 0$

$\Rightarrow \quad (k - 6\sqrt{2})(3d - x) = 0$

$\Rightarrow \quad k = 6\sqrt{2} \Rightarrow k^2 = 72$

4. Consider the function f : R → R defined by

$f(x) = \begin{cases} \left(2 - \sin\left(\frac{1}{x}\right)\right)|x|, & x \neq 0 \\ 0 & , x = 0 \end{cases}$. Then f is :

(1) Monotonic on (0, ∞) only

(2) Monotonic on (– ∞, 0) ∪ (0, ∞)

(3) Monotonic on (– ∞, 0) only

(4) Not monotonic on (– ∞, 0) and (0, ∞)

Answer (4)

Sol. $f'(x) = \begin{cases} 2 - \sin\left(\frac{1}{x}\right) + \frac{1}{x}\cos\left(\frac{1}{x}\right) & \text{if } x > 0 \\ -2 + \sin\left(\frac{1}{x}\right) - \frac{1}{x}\cos\left(\frac{1}{x}\right) & \text{if } x < 0 \end{cases}$

$\because \quad 2 - \sin\left(\frac{1}{x}\right) + \frac{1}{x}\cos\left(\frac{1}{x}\right)$ is continuous on either sides of origin.

Also $f'\left(\frac{3}{\pi}\right)$ is +ve and $f'\left(\frac{1}{\pi}\right)$ is –ve, hence f′(x) is changing its sign.

So f(x) is non monotonic in (0, ∞) and (–∞, 0)

5. If the equation of plane passing through the mirror image of a point (2, 3, 1) with respect to line $\frac{x+1}{2}=\frac{y-3}{1}=\frac{z+2}{-1}$ and containing the line $\frac{x-2}{3}=\frac{1-y}{2}=\frac{z+1}{1}$ is $\alpha x + \beta y + \gamma z = 24$, then $\alpha+\beta+\gamma$ is equal to :

(1) 20 (2) 18

(3) 19 (4) 21

Answer (3)

Sol. Let foot of perpendicular from P(2, 3, 1) on the line.

$L:\frac{x+1}{2}=\frac{y-3}{1}=\frac{z+2}{-1}$ be Q(2λ –1, λ + 3, –λ –2)

∴ PQ is perpendicular to L, then

2(2λ –3) + λ – (–λ –3) = 0 ⇒ $\lambda=\frac{1}{2}$

$Q\left(0,\frac{7}{2},\frac{-5}{2}\right)$

So image of P in L is R(–2, 4 –6)

Equation of required plane,

$$\begin{vmatrix} x+2 & y-4 & z+6 \\ 3 & -2 & 1 \\ 4 & -3 & 5 \end{vmatrix}=0$$

⇒ 7x + 11y + z = 24

6. Let L be a tangent line to the parabola $y^2 = 4x - 20$ at (6, 2). If L is also a tangent to the ellipse $\frac{x^2}{2}+\frac{y^2}{b}=1$, then the value of b is equal to :

(1) 11

(2) 16

(3) 14

(4) 20

Answer (3)

Sol. $y^2 = 4x - 20 \Rightarrow \frac{dy}{dx}=\frac{2}{y}\Rightarrow \frac{dy}{dx_{(6,2)}}=1$

Equation of tangent,

T : y – 2 = 1(x – 6) ⇒ y = x – 4

∵ T is tangent to given ellipse,

$T: y = x \pm \sqrt{2+b}$

Clearly $\sqrt{2+b}=4 \Rightarrow b=14$

7. Let $y = y(x)$ be the solution of the differential equation

cosx(3sinx + cosx + 3)dy = (1 + ysinx(3sinx + cosx + 3))dx, $0\le x\le\frac{\pi}{2}$, y(0) = 0. Then, $y\left(\frac{\pi}{3}\right)$ is equal to

(1) $2\log_e\left(\frac{2\sqrt{3}+10}{11}\right)$ (2) $2\log_e\left(\frac{3\sqrt{3}-8}{4}\right)$

(3) $2\log_e\left(\frac{2\sqrt{3}+9}{6}\right)$ (4) $2\log_e\left(\frac{\sqrt{3}+7}{2}\right)$

Answer (1)

Sol. (3sinx + cosx + 3) (cosx dy – ysinx dx) = dx

$\Rightarrow d(\cos x\cdot y)=\frac{dx}{3\sin x+\cos x+3}$

$\Rightarrow d(y\cdot\cos x)=\frac{\sec^2\frac{x}{2}}{4+6\tan\frac{x}{2}+2\tan^2\frac{x}{2}}dx$

$\Rightarrow y\cos x=\int\frac{dt}{t^2+3t+2}$ where $t=\tan\frac{x}{2}$

$y\cos x=\ln\left|\frac{\tan\frac{x}{2}+1}{\tan\frac{x}{2}+2}\right|+C$

∵ y(0) = 0 ⇒ C = ln2

Then $y\left(\frac{\pi}{3}\right)=2\left[\ln\left(\frac{\sqrt{3}+1}{2\sqrt{3}+1}\right)+\ln 2\right]$

$=2\ln\left(\frac{2\sqrt{3}+2}{2\sqrt{3}+1}\right)=2\ln\left(\frac{10+2\sqrt{3}}{11}\right)$

8. Let S_1, S_2 and S_3 be three sets defined as

$S_1=\{z\in\mathbb{C}:|z-1|\le\sqrt{2}\}$

$S_2=\{z\in\mathbb{C}:\mathrm{Re}((1-i)z)\ge 1\}$

$S_3=\{z\in\mathbb{C}:\mathrm{Im}(z)\le 1\}$

Then the set $S_1\cap S_2\cap S_3$

(1) Has infinitely many elements

(2) Is a singleton

(3) Has exactly three elements

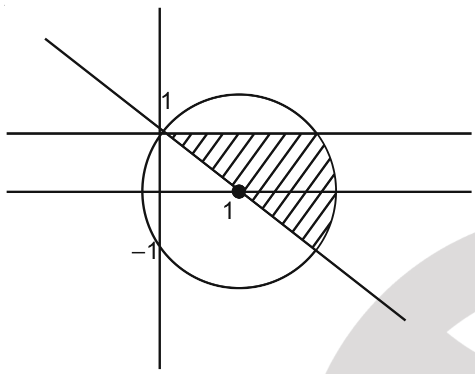
(4) Has exactly two elements

Answer (1)

Sol. S_1 is interior of the circle having centre 1 and radius $\sqrt{2}$.

For S_2, let z = x + iy

$\Rightarrow$ x + y ≥ 1



Clearly there will be infinitely many elements in set $S_1 \cap S_2 \cap S_3$.

9. Let O be the origin. Let $\overrightarrow{OP} = x\hat{i} + y\hat{j} - \hat{k}$ and $\overrightarrow{OQ} = -\hat{i} + 2\hat{j} + 3x\hat{k}, x, y \in R,\ x > 0$, be such that $|\overrightarrow{PQ}| = \sqrt{20}$ and the vector $\overrightarrow{OP}$ is perpendicular to $\overrightarrow{OQ}$. If $\overrightarrow{OR} = 3\hat{i} + z\hat{j} - 7\hat{k}, z \in R$, is coplanar with $\overrightarrow{OP}$ and $\overrightarrow{OQ}$, then the value of $x^2 + y^2 + z^2$ is equal to

(1) 7 (2) 9

(3) 2 (4) 1

Answer (2)

Sol. $\because\ \overrightarrow{OP} \cdot \overrightarrow{OQ} = 0 \Rightarrow -x + 2y - 3x = 0 \Rightarrow 2x = y$

and $|\overrightarrow{OQ} - \overrightarrow{OP}|^2 = 20 \Rightarrow (x + 1)^2 + (y - 2)^2 + (3x + 1)^2 = 20$

$\Rightarrow 14x^2 = 14 \Rightarrow x = 1$

$\because\ \left[\overrightarrow{OP}\ \overrightarrow{OQ}\ \overrightarrow{OR}\right] = 0$

$$\Rightarrow \begin{vmatrix} 1 & 2 & -1 \\ -1 & 2 & 3 \\ 3 & z & -7 \end{vmatrix} = 0 \Rightarrow z = -2$$

So $x^2 + y^2 + z^2 = 9$

10. The value of $\sum_{r=0}^{6} \left({}^6C_r \cdot {}^6C_{6-r}\right)$ is equal to :

(1) 924 (2) 1124

(3) 1024 (4) 1324

Answer (1)

Sol. $\sum_{r=0}^{6} {}^6C_r \cdot {}^6C_{6-r}$ = Coeff. of x^6 in the expansion of $(1 + x)^6(x + 1)^6$

$= {}^{12}C_6$

= 924

11. Let a computer program generate only the digits 0 and 1 to form a string of binary numbers with probability of occurrence of 0 at even places be $\frac{1}{2}$ and probability of occurrence of 0 at the odd place be $\frac{1}{3}$. Then the probability that '10' is followed by '01' is equal to :

(1) $\frac{1}{18}$ (2) $\frac{1}{3}$

(3) $\frac{1}{6}$ (4) $\frac{1}{9}$

Answer (4)

Sol. '10' is followed by '01' can be if

...1 0 0 1 (1 at odd position, 0 at even position) or ...1 0 0 1 (1 at even position, 0 at odd position)

$$\Rightarrow \left(\frac{2}{3} \cdot \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{2}\right) + \left(\frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{2} \cdot \frac{2}{3}\right)$$

$$\Rightarrow \frac{4}{9 \cdot 4} = \frac{1}{9}$$

12. The number of solutions of the equation $\sin^{-1}\left[x^2 + \frac{1}{3}\right] + \cos^{-1}\left[x^2 - \frac{2}{3}\right] = x^2$, for $x \in [-1, 1]$ and [x] denotes the greatest integer less than or equal to x, is :

(1) Infinite (2) 2

(3) 4 (4) 0

Answer (4)

Sol. $\sin^{-1}\left(\left[x^2 + \frac{1}{3}\right]\right) + \cos^{-1}\left(\left[x^2 + \frac{1}{3}\right] - 1\right) = x^2$

$\because\ x^2 + \frac{1}{3} \in \left[\frac{1}{3}, \frac{4}{3}\right]$; so $\left[x^2 + \frac{1}{3}\right] = 0$ or 1

Hence L.H.S. is always equal to π.

and $x^2 = \pi$ has no solution in [–1, 1].

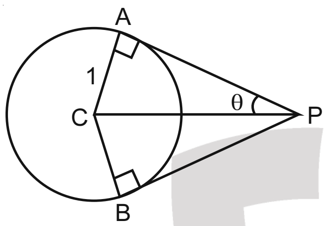
13. Two tangents are drawn from a point P to the circle $x^2 + y^2 - 2x - 4y + 4 = 0$, such that the angle between these tangents is $\tan^{-1}\left(\frac{12}{5}\right)$, where $\tan^{-1}\left(\frac{12}{5}\right) \in (0, \pi)$. If the centre of the circle is denoted by C and these tangents touch the circle at points A and B, then the ratio of the area of ΔPAB and ΔCAB is :

(1) 2 : 1 (2) 3 : 1

(3) 11 : 4 (4) 9 : 4

Answer (4)

Sol.



$$\tan 2\theta = \frac{12}{5}$$

$$\Rightarrow \quad \tan\theta = \frac{2}{3}$$

$$\frac{[\Delta PAB]}{[\Delta CAB]} = \frac{\frac{1}{2}PA^2 \cdot \sin 2\theta}{\frac{1}{2}CA^2 \sin(\pi - 2\theta)}$$

$$\left(\frac{PA}{CA}\right)^2 = \cot^2\theta = \frac{9}{4}$$

14. Let $f : R \to R$ be defined as $f(x) = e^{-x} \sin x$. If $F : [0, 1] \to R$ is a differentiable function such that $F(x) = \int_0^x f(t)dt$, then the value of $\int_0^1 (F'(x) + f(x))e^x dx$ lies in the interval

(1) $\left[\frac{335}{360}, \frac{336}{360}\right]$ (2) $\left[\frac{327}{360}, \frac{329}{360}\right]$

(3) $\left[\frac{330}{360}, \frac{331}{360}\right]$ (4) $\left[\frac{331}{360}, \frac{334}{360}\right]$

Answer (3)

Sol. $I = \int_0^1 e^x f'(x)dx + \int_0^1 e^x \cdot f(x)dx$

$$2\int_0^1 e^x f(x)dx = 2\int_0^1 \sin x \, dx$$

$$2\int_0^1 \left(x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{720} + ...\right)dx$$

$$\Rightarrow \quad 2\left[\frac{1}{2} - \frac{1}{4.6}\right] < I < 2\left[\frac{1}{2} - \frac{1}{4.6} + \frac{1}{6.120}\right]$$

$$\Rightarrow \quad \frac{11}{12} < I < \frac{331}{360}$$

15. The value of

$$\lim_{n\to\infty} \frac{[r] + [2r] + ... + [nr]}{n^2}$$

where r is a non-zero real number and [r] denotes the greatest integer less than or equal to r is equal to :

(1) 0 (2) $\frac{r}{2}$

(3) 2r (4) r

Answer (2)

Sol.
$r - 1 < [r] \le r$
$2r - 1 < [2r] \le 2r$
– – – – –
– – – – –
$nr - 1 < [nr] \le nr$

$$\frac{n(n+1)}{2n^2} r - \frac{1}{n} < \frac{[r] + [2r] + ... + [nr]}{n^2} \le \frac{n(n+1)}{2n^2} r$$

$$\Rightarrow \quad \lim_{n\to\infty} \frac{[r] + [2r] + ... + [nr]}{n^2} = \frac{r}{2}$$

16. Let the tangent to the circle $x^2 + y^2 = 25$ at the point R(3,4) meet x-axis and y-axis at points P and Q, respectively. If r is the radius of the circle passing through the origin O and having centre at the incentre of the triangle OPQ, then r^2 is equal to :

(1) $\frac{529}{64}$ (2) $\frac{585}{66}$

(3) $\frac{625}{72}$ (4) $\frac{125}{72}$

Answer (3)

Sol. T : $3x + 4y = 5$

So, $P\left(\frac{5}{3}, 0\right)$ and $Q\left(0, \frac{5}{4}\right)$

Incentre of ΔOPQ is $\left(\frac{25}{12}, \frac{25}{12}\right)$

So, $r^2 = \left(\frac{25}{12}\right)^2 + \left(\frac{25}{12}\right)^2 = 2\left(\frac{625}{144}\right) = \frac{625}{72}$

17. If the Boolean expression $(p \wedge q) \circledast (p \otimes q)$ is a tautology, then $\circledast$ and $\otimes$ are respectively given by :

(1) $\rightarrow, \rightarrow$ (2) $\wedge, \rightarrow$

(3) $\wedge, \vee$ (4) $\vee, \rightarrow$

Answer (1)

Sol. $\because$ $p \rightarrow q = \sim p \vee q$

$(p \wedge q) \rightarrow (p \rightarrow q) = \sim(p \wedge q) \vee (p \rightarrow q)$

$= (\sim p \vee \sim q) \vee (\sim p \vee q)$

$= \sim p \vee q \vee \sim q$ is a tautology.

18. If the sides AB, BC and CA of a triangle ABC have 3, 5 and 6 interior points respectively, then the total number of triangles that can be constructed using these points as vertices, is equal to :

(1) 240 (2) 364

(3) 360 (4) 333

Answer (4)

Sol. Total number of triangles $= {}^{14}C_3 - {}^3C_3 - {}^5C_3 - {}^6C_3$

$= 364 - 31 = 333$

19. If the curve $y = y(x)$ is the solution of the differential equation

$2(x^2 + x^{5/4})\,dy - y(x + x^{1/4})\,dx = 2x^{9/4}\,dx,\ x > 0$

which passes through the point $\left(1, 1 - \frac{4}{3}\log_e 2\right)$,

then the value of $y(16)$ is equal to :

(1) $\left(\frac{31}{3} - \frac{8}{3}\log_e 3\right)$ (2) $\left(\frac{31}{3} + \frac{8}{3}\log_e 3\right)$

(3) $4\left(\frac{31}{3} + \frac{8}{3}\log_e 3\right)$ (4) $4\left(\frac{31}{3} - \frac{8}{3}\log_e 3\right)$

Answer (4)

Sol. $\frac{dy}{dx} - \frac{y}{2x} = \frac{x}{1 + x^{3/4}}$

$\text{I.F.} = e^{-\int \frac{1}{2x}dx} = \frac{1}{\sqrt{x}}$

$\Rightarrow y \cdot \frac{1}{\sqrt{x}} = \int \frac{\sqrt{x}\,dx}{1 + x^{3/4}} + C$

$\Rightarrow \frac{y}{\sqrt{x}} = \int \frac{4t^5}{1 + t^3}dt + C$ where $t^4 = x$

$\Rightarrow \frac{y}{\sqrt{x}} = \frac{4}{3}\left[1 + x^{3/4} - \ln(1 + x^{3/4})\right] + C$

Put $x = 1$, $y = 1 - \frac{4}{3}\ln 2;\ 1 - \frac{4}{3}\ln 2 = \frac{4}{3}[2 - \ln 2] + C$

$\Rightarrow C = -\frac{5}{3}$

Put $x = 16$, $\frac{y}{4} = \frac{4}{3}[1 + 8 - \ln 9] - \frac{5}{3}$

$\Rightarrow y = 4\left[\frac{31}{3} - \frac{8}{3}\ln 3\right]$

20. The value of the limit $\lim_{\theta \to 0} \frac{\tan(\pi\cos^2\theta)}{\sin(2\pi\sin^2\theta)}$ is equal to :

(1) 0 (2) $-\frac{1}{2}$

(3) $\frac{1}{4}$ (4) $-\frac{1}{4}$

Answer (2)

Sol. $\lim_{\theta \to 0} \frac{\tan(\pi - \pi\sin^2\theta)}{\sin(2\pi\sin^2\theta)} = \lim_{\theta \to 0} - \frac{\tan(\pi\sin^2\theta)}{\sin(2\pi\sin^2\theta)}$

$= \lim_{\theta \to 0} - \frac{\tan(\pi\sin^2\theta)/\sin^2\theta}{\sin(2\pi\sin^2\theta)/\sin^2\theta}$

$= -\frac{\pi}{2\pi} = -\frac{1}{2}$

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. Let $f : [-1, 1] \rightarrow R$ be defined as $f(x) = ax^2 + bx + c$ for all $x \in [-1, 1]$, where $a, b, c \in R$ such that $f(-1) = 2$, $f'(-1) = 1$ and for $x \in (-1, 1)$ the maximum value of $f''(x)$ is $\frac{1}{2}$. If $f(x) \le \alpha$, $x \in [-1, 1]$, then the least value of α is equal to __________.

Answer (5)

Sol. $f(x) = ax^2 + bx + c$, $f'(x) = 2ax + b$, $f''(x) = 2a = \frac{1}{2}$

$a = \frac{1}{4}, -2a + b = 1 \Rightarrow b = \frac{3}{2}$ and $a - b + c = 2 \Rightarrow c = \frac{13}{4}$

$\because$ $f'(x) > 0\ \forall\ x \in [-1, 1]$

So, $f(x) \le f(1)$

$\Rightarrow$ $f(x) \le 5$

2. Let the coefficients of third, fourth and fifth terms in the expansion of $\left(x+\frac{a}{x^2}\right)^n$, $x \neq 0$, be in the ratio 12 : 8 : 3. Then the term independent of x in the expansion, is equal to ________.

Answer (60)

Sol. $\frac{^nC_2}{^nC_3}\cdot a = \frac{12}{8} \Rightarrow \frac{3a}{n-2} = \frac{3}{2}$ ...(i)

Similarly, $\frac{4a}{n-3} = \frac{8}{3}$ ...(ii)

From (i) and (ii), n = 6 and a = 2

The term independent of x = $^6C_4a^2$

= 15 × 4 = 60

3. If 1, $\log_{10}(4^x - 2)$ and $\log_{10}\left(4^x + \frac{18}{5}\right)$ are in arithmetic progression for a real number x, then the value of the determinant $\begin{vmatrix} 2\left(x-\frac{1}{2}\right) & x-1 & x^2 \\ 1 & 0 & x \\ x & 1 & 0 \end{vmatrix}$ is equal to :

Answer (2)

Sol. ∵ $\left(4^x - 2\right)^2 = 10\left(4^x + \frac{18}{5}\right)$

$\Rightarrow 4^{2x} - 14\cdot4^x - 32 = 0$

$\Rightarrow 4^x = 16 \Rightarrow x = 2$

$\begin{vmatrix} 2x-1 & x-1 & x^2 \\ 1 & 0 & x \\ x & 1 & 0 \end{vmatrix} = x\left(x^2-x\right)-\left(x^2-x\right)=(x-1)\left(x^2-x\right)$

= 2

4. Let tanα, tanβ and tanγ; α, β, γ ≠ $\frac{(2n-1)\pi}{2}$, $n \in N$ be the slopes of three line segments OA, OB and OC, respectively, where O is origin. If circumcentre of ΔABC coincides with origin and its orthocentre lies on y-axis, then the value of $\left(\frac{\cos3\alpha+\cos3\beta+\cos3\gamma}{\cos\alpha\cos\beta\cos\gamma}\right)^2$ is equal to ________.

Answer (144)

Sol. ∵ Origin is circumcentre, then let A(rcosα, rsinα) B(r cosβ, r sinβ) and C(r cosγ, r sinγ)

∵ Orthocentre lies on y-axis, then

cosα + cosβ + cosγ = 0

$\Rightarrow \cos^3\alpha + \cos^3\beta + \cos^3\gamma = 3\cos\alpha\cdot\cos\beta\cdot\cos\gamma$

Now, cos3α + cos3β + cos3γ

$= 4(\cos^3\alpha + \cos^3\beta + \cos^3\gamma)$

$\frac{\cos3\alpha+\cos3\beta+\cos3\gamma}{\cos\alpha\cos\beta\cos\gamma} = 12$

5. Let P be an arbitrary point having sum of the squares of the distances from the planes x + y + z = 0, lx – nz = 0 and x – 2y + z = 0, equal to 9. If the locus of the point P is $x^2 + y^2 + z^2 = 9$, then the value of l –n is equal to_____.

Answer (0)

Sol. ∵ $\left(\frac{x+y+z}{\sqrt{3}}\right)^2 + \left(\frac{lx-nz}{\sqrt{l^2+n^2}}\right)^2 + \left(\frac{x-2y+z}{\sqrt{6}}\right)^2 = 9$

$\Rightarrow x^2\left(\frac{1}{2}+\frac{l^2}{l^2+n^2}\right)+y^2(1)+z^2\left(\frac{1}{2}+\frac{n^2}{l^2+n^2}\right)+2xz\left(\frac{1}{3}-\frac{ln}{l^2+n^2}+\frac{1}{6}\right)=9$

Clearly $\frac{l^2}{l^2+n^2} = \frac{1}{2} \Rightarrow l = \pm n$ and $\frac{ln}{l^2+n^2} = \frac{1}{2}$

then l = n

6. Consider a set of 3n numbers having variance 4. In this set, the mean of first 2n numbers is 6 and the mean of the remaining n numbers is 3. A new set is constructed by adding 1 into each of first 2n numbers, and subtracting 1 from each of the remaining n numbers. If the variance of the new set is k, then 9k is equal to ______.

Answer (68)

Sol. Let $x_1, x_2, \ldots\ldots\ldots x_{3n}$ be the given numbers.

$\bar{x} = \frac{6.2n+3.n}{3n} = 5$

$4 = \frac{\sum x_i^2}{3n} - 25 \Rightarrow \frac{\sum x_i^2}{3n} = 29$ .....(i)

Let $y_i = x_i+1$ for $1 \le i \le 2n$

and $y_i = x_i-1$ for $2n+1 \le i \le 3n$

So $\bar{y} = \frac{\sum y_i}{3n} = \frac{\sum x_i + n}{3n} = 5 + \frac{1}{3} = \frac{16}{3}$

Now $k = \frac{\sum y_i^2}{3n} - \left(\frac{16}{3}\right)^2 = \frac{\sum x_i^2}{3n} + 2\left(\frac{\sum_{i=1}^{2n} x_i}{3n}\right) - 2\left(\frac{\sum_{i=2n+1}^{3n} x_i}{3n}\right) + 1 - \frac{256}{9}$

$k = 29+8-2+1-\frac{256}{9} = 36-\frac{256}{9} = \frac{68}{9}$

7. Let f : [– 3, 1] $\to$ R be given as

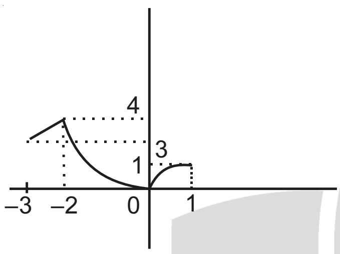
$$f(x)=\begin{cases}\min\{(x+6),x^2\}, & -3\le x\le 0\\ \max\{\sqrt{x},x^2\}, & 0\le x\le 1.\end{cases}$$

If the area bounded by y = f (x) and x-axis is A, then the value of 6A is equal to ______.

Answer (41)

Sol. Required Area

$$=\frac{1}{2}(3+4)+\frac{1}{3}(2\times 4)+\frac{2}{3}(1\times 1)$$



$$=\frac{7}{2}+\frac{8}{3}+\frac{2}{3}$$

$$=\frac{41}{6}$$

8. Let $I_n = \int_1^e x^{19}(\log|x|)^n\,dx$, where n $\in$ N. If $(20)I_{10} - \alpha I_9 + \beta I_8$, for natural numbers α and β, then $\alpha-\beta$ equals to ______.

Answer (1)

Sol. $I_n = \int_1^e x^{19}.(\ln x)^n\,dx$

$$\Rightarrow I_n = \left.\frac{(\ln x)^n\cdot x^{20}}{20}\right|_1^e - \int_1^e n(\ln x)^{n-1}\frac{x^{19}}{20}dx$$

$\Rightarrow 20I_n = e^{20} - nI_{n-1}$

So, $20I_{10} = e^{20} - 10I_9$

and $20I_9 = e^{20} - 9I_8$

$20I_{10} = 10I_9 + 9I_8$

9. Let $\vec{x}$ be a vector in the plane containing vectors $\vec{a} = 2\hat{i}-\hat{j}+\hat{k}$ and $\vec{b} = \hat{i}+2\hat{j}-\hat{k}$. If the vector $\vec{x}$ is perpendicular to $(3\hat{i}+2\hat{j}-\hat{k})$ and its projection on $\vec{a}$ is $\frac{17\sqrt{6}}{2}$, then the value of $|\vec{x}|^2$ is equal to ______.

Answer (486)

Sol. Let $\bar{x} = \lambda(\bar{a}\times\bar{b})\times\bar{c}$

$$\bar{a}\times\bar{b} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & 1\\ 1 & 2 & -1\end{vmatrix} = -\hat{i}+3\hat{j}+5\hat{k}$$

$$(\bar{a}\times\bar{b})\times\bar{c} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & 3 & 5\\ 3 & 2 & -1\end{vmatrix} = -13\hat{i}+14\hat{j}-11\hat{k}$$

$$\because \frac{\bar{x}\cdot\bar{a}}{|\bar{a}|} = \frac{17\sqrt{6}}{2} \Rightarrow \left|\frac{(-26-14-11)\lambda}{\sqrt{6}}\right| = \frac{17\sqrt{6}}{2}$$

$\Rightarrow \lambda = \pm 1$

$|\bar{x}|^2 = 13^2 + 14^2 + 11^2 = 486$

10. Let A = $\begin{bmatrix}a & b\\ c & d\end{bmatrix}$ and B = $\begin{bmatrix}\alpha\\ \beta\end{bmatrix} \ne \begin{bmatrix}0\\ 0\end{bmatrix}$ such that AB = B and a + d = 2021, then the value of ad – bc is equal to ____.

Answer (10)

Sol. $\because \begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}\alpha\\ \beta\end{bmatrix} = \begin{bmatrix}\alpha\\ \beta\end{bmatrix}$

$\Rightarrow a\alpha + b\beta = \alpha$ and $c\alpha + d\beta = \beta$

$$\Rightarrow \frac{\alpha}{\beta} = \frac{b}{1-a} = \frac{1-d}{c}$$

$\Rightarrow$ bc = ad – a – d + 1

$\Rightarrow$ ad – bc = a + d – 1

= 2020

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