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Time : 3 hrs.

Answers & Solutions

M.M. : 300

for

JEE (MAIN)-2021 (Online) Phase-2

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS :

- (1) The test is of **3 hours** duration.
- (2) The Test Booklet consists of 90 questions. The maximum marks are 300.
- (3) There are **three** parts in the question paper A, B, C consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part has two sections.
 - (i) Section-I : This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **-1 mark** for wrong answer.
 - (ii) Section-II : This section contains 10 questions. In Section-II, attempt any **five questions out of 10**. The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and there is no negative marking for wrong answer.

4. The time taken for the magnetic energy to reach 25% of its maximum value, when a solenoid of resistance R , inductance L is connected to a battery, is:

- (1) $\frac{L}{R} \ln 5$ (2) Infinite
 (3) $\frac{L}{R} \ln 10$ (4) $\frac{L}{R} \ln 2$

Answer (4)

Sol. $U = \frac{1}{2} LI^2$

if $U = \frac{U_{\max}}{4} \Rightarrow I = \frac{I_{\max}}{2}$

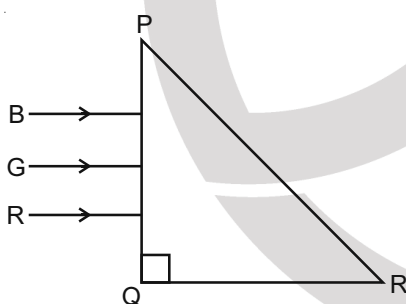
$I = I_0 \left(1 - e^{-\frac{Rt}{L}}\right)$

$\Rightarrow \frac{1}{2} = 1 - e^{-\frac{Rt}{L}}$

$\Rightarrow e^{-\frac{Rt}{L}} = \frac{1}{2} \Rightarrow (t) \frac{R}{L} = \ln 2$

$\Rightarrow t = \frac{L}{R} \ln 2$

5. Three rays of light, namely red (R), green (G) and blue (B) are incident on the face PQ of a right angled prism PQR as shown in the figure.



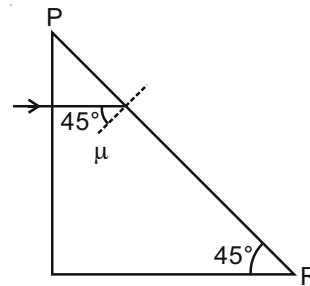
The refractive indices of the material of the prism for red, green and blue wavelength are 1.27, 1.42 and 1.49 respectively. The colour of the ray(s) emerging out of the face PR is:

- (1) Blue
 (2) Green
 (3) Red
 (4) Blue and Green

Answer (3)

Sol. * Data incomplete

*Assuming angle at R = 45°



$\sin \theta_c = \frac{1}{\mu} \Rightarrow \mu = \sqrt{2}$

$\mu = 1.41$

$\mu_{\text{Red}} = 1.27 < 1.41$

\Rightarrow Only red will emerge

6. A proton and an α -particle, having kinetic energies K_p and K_α respectively, enter into a magnetic field at right angles.

The ratio of the radii of trajectory of proton to that of α -particle is 2 : 1. The ratio of $K_p : K_\alpha$ is :

- (1) 1 : 8 (2) 8 : 1
 (3) 1 : 4 (4) 4 : 1

Answer (4)

Sol. $R = \frac{mV}{qB} = \frac{\sqrt{2mK}}{qB}$

$\frac{R_p}{R_\alpha} = \frac{\sqrt{m_p K_p} q_\alpha}{\sqrt{m_\alpha K_\alpha} q_p}$

$\Rightarrow \frac{K_p}{K_\alpha} = 4 : 1$

7. In a series LCR circuit, the inductive reactance (X_L) is 10Ω and the capacitive reactance (X_C) is 4Ω . The resistance (R) in the circuit is 6Ω .

The power factor of the circuit is :

- (1) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{2}$
 (3) $\frac{1}{2\sqrt{2}}$ (4) $\frac{\sqrt{3}}{2}$

Answer (1)

Sol. $\cos \phi = \frac{R}{Z} = \frac{6}{\sqrt{(10-4)^2 + (6)^2}}$
 $= \frac{6}{\sqrt{36 \times 2}} = \frac{1}{\sqrt{2}}$

8. Consider a uniform wire of mass M and length L . It is bent into a semicircle. Its moment of inertia about a line perpendicular to the plane of the wire passing through the centre is:

- (1) $\frac{2 ML^2}{5 \pi^2}$ (2) $\frac{1 ML^2}{4 \pi^2}$
 (3) $\frac{1 ML^2}{2 \pi^2}$ (4) $\frac{ML^2}{\pi^2}$

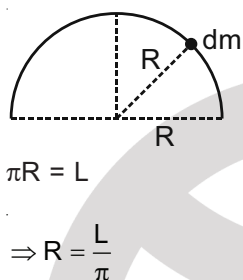
Answer (4)

Sol. $I = \int (dm)r^2$

$I = (R^2) \int dm$

$= MR^2$

$= \frac{ML^2}{\pi^2}$



9. Consider a sample of oxygen behaving like an ideal gas. At 300 K, the ratio of root mean square (rms) velocity to the average velocity of gas molecule would be :

(Molecular weight of oxygen is 32 g/mol; $R = 8.3 \text{ J K}^{-1} \text{ mol}^{-1}$)

- (1) $\sqrt{\frac{8\pi}{3}}$ (2) $\sqrt{\frac{3}{3}}$
 (3) $\sqrt{\frac{8}{3}}$ (4) $\sqrt{\frac{3\pi}{8}}$

Answer (4)

Sol. $V_{rms} = \sqrt{\frac{3RT}{M}}$

$V_{avg} = \sqrt{\frac{8RT}{\pi M}}$

$\Rightarrow \frac{V_{rms}}{V_{avg}} = \sqrt{\frac{3\pi}{8}}$

10. For an adiabatic expansion of an ideal gas, the fractional change in its pressure is equal to (where γ is the ratio of specific heats) :

- (1) $-\gamma \frac{V}{dV}$ (2) $-\gamma \frac{dV}{V}$
 (3) $-\frac{1}{\gamma} \frac{dV}{V}$ (4) $\frac{dV}{V}$

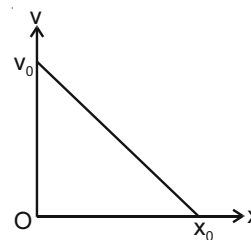
Answer (2)

Sol. $PV^\gamma = C$

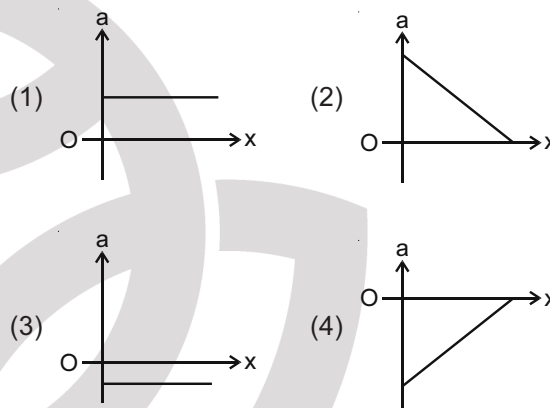
$\Rightarrow (P)(\gamma)V^{\gamma-1}dV + V^\gamma dP = 0$

$\Rightarrow dP = \frac{-\gamma P}{V} dV \Rightarrow \frac{dP}{P} = -\frac{\gamma dV}{V}$

11. The velocity-displacement graph of a particle is shown in the figure.



The acceleration-displacement graph of the same particle is represented by:



Answer (4)

Sol. $a = v \frac{dv}{dx}$

$\frac{dv}{dx} < 0$ and v is decreasing with x

12. The decay of a proton to neutron is:

- (1) Always possible as it is associated only with β^+ decay
 (2) Not possible but neutron to proton conversion is possible
 (3) Not possible as proton mass is less than the neutron mass
 (4) Possible only inside the nucleus

Answer (4)

Sol. β^+ decay is possible only inside the nucleus.

13. The speed of electrons in a scanning electron microscope is $1 \times 10^7 \text{ ms}^{-1}$. If the protons having the same speed are used instead of electrons, then the resolving power of scanning proton microscope will be changed by a factor of

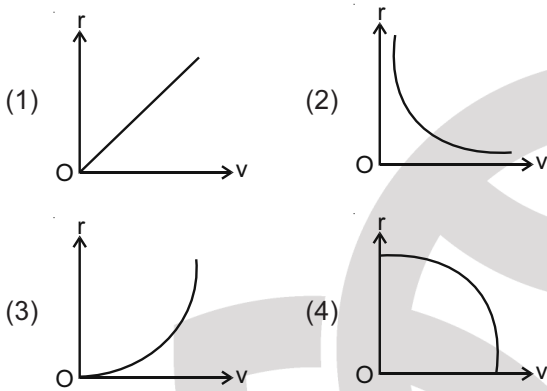
- (1) $\frac{1}{\sqrt{1837}}$ (2) $\frac{1}{1837}$
 (3) $\sqrt{1837}$ (4) 1837

Answer (4)

Sol. $R.P \propto \frac{1}{\lambda}$, $\lambda \propto \frac{1}{P}$

14. A particle of mass m moves in a circular orbit under the central potential field, $U(r) = -\frac{C}{r}$, where C is a positive constant.

The correct radius – velocity graph of the particle's motion is



Answer (2)

Sol. $F_r = -\frac{dU}{dr}$
 $= -\frac{d}{dr}\left(-\frac{C}{r}\right) = -\frac{C}{r^2}$
 $\Rightarrow \frac{mv^2}{r} = \frac{C}{r^2}$
 $\Rightarrow v^2 = \frac{C}{mr}$

15. Which of the following statements are correct?
- (A) Electric monopoles do not exist whereas magnetic monopoles exist.
 - (B) Magnetic field lines due to a solenoid at its ends and outside cannot be completely straight and confined.
 - (C) Magnetic field lines are completely confined within a toroid.
 - (D) Magnetic field lines inside a bar magnet are not parallel.
 - (E) $x = -1$ is the condition for a perfect diamagnetic material, where x is its magnetic susceptibility.
- Choose the correct answer from the options given below:
- (1) (C) and (E) only (2) (B) and (C) only
 (3) (B) and (D) only (4) (A) and (B) only

Answer (3)

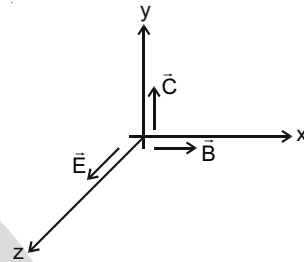
Sol. Theory based.

16. A plane electromagnetic wave propagating along y-direction can have the following pair of electric field (\vec{E}) and magnetic field (\vec{B}) components.

- (1) E_x, B_z or E_z, B_x (2) E_x, B_y or E_y, B_x
 (3) E_y, B_y or E_z, B_z (4) E_y, B_x or E_x, B_y

Answer (1)

Sol. $\vec{E} \times \vec{B} = \vec{C}$



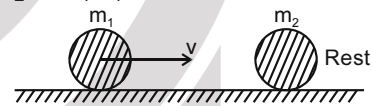
17. An object of mass m_1 collides with another object of mass m_2 , which is at rest. After the collision the objects move with equal speeds in opposite direction. The ratio of the masses $m_2 : m_1$ is:

- (1) 2 : 1 (2) 3 : 1
 (3) 1 : 2 (4) 1 : 1

Answer (2)

Sol. Assuming $e = 1$

$m_1 v = (m_2 - m_1)v_1$



$\Rightarrow v_1 = \frac{m_1 v}{(m_2 - m_1)} \dots(i)$

also $2v_1 = v$

$\Rightarrow v_1 = \frac{v}{2}$

$\Rightarrow \frac{1}{2} = \frac{m_1}{m_2 - m_1}$

$\Rightarrow \frac{m_2}{m_1} = 3$

18. The angular momentum of a planet of mass M moving around the sun in an elliptical orbit is \vec{L} . The magnitude of the areal velocity of the planet is:

- (1) $\frac{L}{2M}$ (2) $\frac{L}{M}$
 (3) $\frac{4L}{M}$ (4) $\frac{2L}{M}$

Answer (1)

Sol. $\frac{dA}{dt} = \frac{L}{2M}$ standard result

19. The function of time representing a simple harmonic motion with a period of $\frac{\pi}{\omega}$ is:

- (1) $3 \cos\left(\frac{\pi}{4} - 2\omega t\right)$
- (2) $\cos(\omega t) + \cos(2\omega t) + \cos(3\omega t)$
- (3) $\sin^2(\omega t)$
- (4) $\sin(\omega t) + \cos(\omega t)$

Answer (1)

Sol. $T = \frac{2\pi}{\omega'} = \frac{\pi}{\omega}$
 $\Rightarrow \omega' = 2\omega$

$\Rightarrow y = 3 \cos\left(\frac{\pi}{4} - 2\omega t\right)$

20. An ideal gas in a cylinder is separated by a piston in such a way that the entropy of one part is S_1 and that of the other part is S_2 . Given that $S_1 > S_2$. If the piston is removed then the total entropy of the system will be:

- (1) $S_1 - S_2$
- (2) $S_1 \times S_2$
- (3) $\frac{S_1}{S_2}$
- (4) $S_1 + S_2$

Answer (4)

Sol. Entropy will increase.

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. The radius of a sphere is measured to be (7.50 ± 0.85) cm. Suppose the percentage error in its volume is x.

The value of x, to the nearest x, is _____.

Answer (34)

Sol. $v = \frac{4}{3}\pi R^3$

$\Rightarrow \frac{\Delta v}{v} \times 100 = (3) \frac{\Delta R}{R} \times 100$
 $= (3) \times \frac{0.85}{7.5} \times 100$
 $= 34$

2. An infinite number of point charges, each carrying $1 \mu\text{C}$ charge, are placed along the y-axis at $y = 1 \text{ m}, 2 \text{ m}, 4 \text{ m}, 8 \text{ m}$ _____.

The total force on a 1 C point charge, placed at the origin, is $x \times 10^3 \text{ N}$.

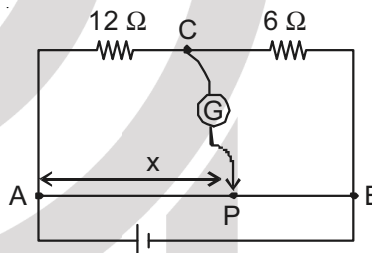
The value of x, to the nearest integer, is _____.

[Take $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2/\text{C}^2$]

Answer (12)

Sol. $F = (k)(10^{-6}) \left[\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{4^2} + \dots \right]$
 $= \frac{(k)10^{-6}}{1 - \frac{1}{4}} = \frac{(9 \times 10^9) \times 4 \times 10^{-6} \text{ N}}{3}$
 $= 12 \times 10^3 \text{ N}$

3. Consider a 72 cm long wire AB as shown in the figure. The galvanometer jockey is placed at P on AB at a distance x cm from A. The galvanometer shows zero deflection.



The value of x, to the nearest integer, is _____.

Answer (48)

Sol. $\frac{12}{6} = \frac{x}{(72 - x)}$
 $\Rightarrow x = 48 \text{ cm}$

4. A galaxy is moving away from the earth at a speed of 286 kms^{-1} . The shift in the wavelength of a redline at 630 nm is $x \times 10^{-10} \text{ m}$.

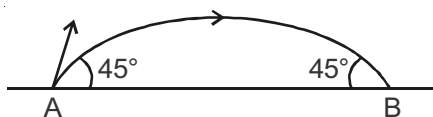
The value of x, to the nearest integer, is _____.

[Take the value of speed of light c, as $3 \times 10^8 \text{ ms}^{-1}$]

Answer (6)

Sol. $\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$
 $\Rightarrow \Delta\lambda = \frac{v}{c} \times \lambda$
 $= \frac{286 \times 10^3}{3 \times 10^8} \times 630 \times 10^{-9}$
 $= 6.006 \times 10^{-10}$

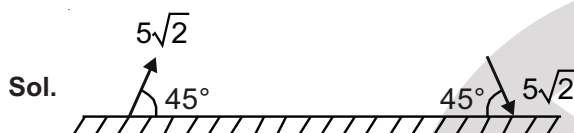
5. The projectile motion of a particle of mass 5 g is shown in the figure.



The initial velocity of the particle is $5\sqrt{2}$ ms⁻¹ and the air resistance is assumed to be negligible. The magnitude of the change in momentum between the points A and B is $x \times 10^{-2}$ kgms⁻¹.

The value of x, to the nearest integer, is _____.

Answer (5)



$$|\Delta \vec{P}| = 2 \times (5 \times 10^{-3}) (5) \\ = 5 \times 10^{-2} \text{ kg ms}^{-1}$$

6. Two wires of same length and thickness having specific resistances $6 \Omega \text{ cm}$ and $3 \Omega \text{ cm}$ respectively are connected in parallel. The effective resistivity is $\rho \Omega \text{ cm}$. The value of ρ , to the nearest integer, is _____.

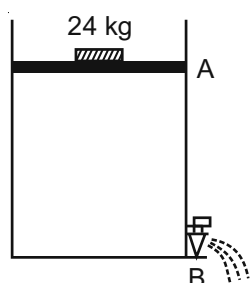
Answer (4)

$$\text{Sol. } R = \frac{R_1 R_2}{R_1 + R_2} \\ \rho(l) = \frac{\left(\frac{\rho_1 l}{A}\right) \left(\frac{\rho_2 l}{A}\right)}{2A \left(\frac{\rho_1 l}{A} + \frac{\rho_2 l}{A}\right)} \\ \frac{\rho}{2} = \frac{\rho_1 \rho_2}{\rho_1 + \rho_2} \Rightarrow \frac{\rho}{2} = \frac{\rho_1 \rho_2}{(\rho_1 + \rho_2)} = 2 \\ \Rightarrow \rho = 4$$

7. Consider a water tank as shown in the figure. It's cross-sectional area is 0.4 m^2 . The tank has an opening B near the bottom whose cross-section area is 1 cm^2 . A load of 24 kg is applied on the water at the top when the height of the water level is 40 cm above the bottom, the velocity of water coming out the opening B is $v \text{ ms}^{-1}$.

The value of v, to the nearest integer, is _____.

[Take value of g to be 10 ms^{-2}]



Answer (3)

$$\text{Sol. } \frac{mg}{A} + \rho g H = \frac{1}{2} \rho v^2 \\ \Rightarrow \frac{240}{0.4} + 1000 \times 10 \times 0.4 = \frac{1}{2} \times 1000 \times v^2 \\ \Rightarrow v = 3 \text{ m/s}$$

8. A ball of mass 4 kg, moving with a velocity of 10 ms^{-1} , collides with a spring of length 8 m and force constant 100 Nm^{-1} . The length of the compressed spring is x m. The value of x, to the nearest integer, is _____.

Answer (6)

$$\text{Sol. } \frac{1}{2} \times 4 \times (10)^2 = \frac{1}{2} \times 100 \times (\Delta x)^2 \\ \Rightarrow \Delta x = 2 \text{ m} \\ \Rightarrow x = 8 - 2 = 6 \text{ m}$$

9. A TV transmission tower antenna is at a height of 20 m. Suppose that the receiving antenna is at.

- (i) ground level
(ii) a height of 5 m

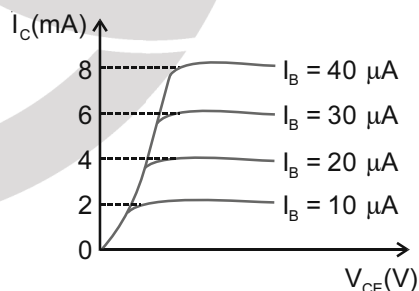
The increase in antenna range in case (ii) relative to case (i) is n%.

The value of n, to the nearest integer, is _____.

Answer (50)

$$\text{Sol. } d = \sqrt{2hR_e} \\ d_1 = \sqrt{2 \times 20 \times R_e} \\ d_2 = \sqrt{2 \times 20 \times R_e} + \sqrt{2 \times 5 \times R_e} \\ \Delta d = \frac{(d_2 - d_1)}{d_1} = \frac{\sqrt{10 R_e}}{\sqrt{40 R_e}} = \frac{1}{2} \\ \% \text{ increase} = 50$$

10. The typical output characteristics curve for a transistor working in the common-emitter configuration is shown in the figure.



The estimated current gain from the figure is _____.

Answer (200)

$$\text{Sol. } \beta = \frac{\Delta I_C}{\Delta I_B} \\ = \frac{2 \times 10^{-3}}{10 \times 10^{-6}} = 200$$

PART-B : CHEMISTRY

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Given below are two statements :

Statement I : Non-biodegradable wastes are generated by the thermal power plants.

Statement II : Bio-degradable detergents leads to eutrophication.

In the light of the above statements, choose the most appropriate answer from the options given below :

- (1) Both statement I and statement II are false
- (2) Statement I is true but statement II is false
- (3) Statement I is false but statement II is true
- (4) Both statement I and statement II are true

Answer (4)

Sol. NCERT, Page No. – 415, 417 – Part-II, Class-XI

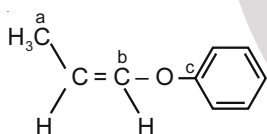
2. The oxide that shows magnetic property is :

- (1) MgO
- (2) Na₂O
- (3) SiO₂
- (4) Mn₃O₄

Answer (4)

Sol. Mn₃O₄ is magnetic in nature.

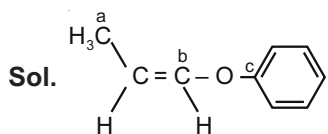
3. In the following molecule,



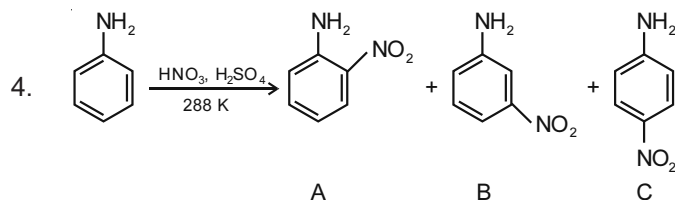
Hybridisation of Carbon a, b and c respectively are :

- (1) sp³, sp, sp
- (2) sp³, sp², sp²
- (3) sp³, sp, sp²
- (4) sp³, sp², sp

Answer (2)



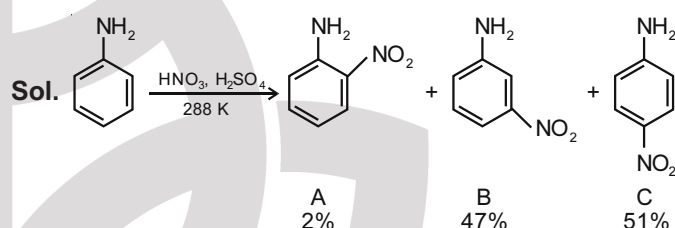
a – sp³
b – sp³
c – sp²



Consider the given reaction, percentage yield of :

- (1) C > B > A
- (2) C > A > B
- (3) B > C > A
- (4) A > C > B

Answer (1)



In strongly acidic medium aniline is protonated to form anilinium ion making it m-directing but p-derivative is also formed in significant amount which is a major product.

5. Deficiency of vitamin K causes :

- (1) Cheilosis
- (2) Increase in blood clotting time
- (3) Increase in fragility of RBC's
- (4) Decrease in blood clotting time

Answer (2)

Sol. Deficiency of vitamin K causes increase in blood clotting time.

6. Match List-I with List-II :

List-I (Class of Chemicals)	List-II (Example)
(a) Antifertility drug	(i) Meprobamate
(b) Antibiotic	(ii) Alitame
(c) Tranquilizer	(iii) Norethindrone
(d) Artificial Sweetener	(iv) Salvarsan

Choose the most appropriate match :

- (1) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)
- (2) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)
- (3) (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i)
- (4) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)

Answer (1)

- Sol.** (a) Antifertility drug — Norethindrone
 (b) Antibiotic — Salvarsan
 (c) Tranquilizer — Meprobamate
 (d) Artificial Sweetener — Alitame
 (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)

7. Given below are two statements :

Statement I : C_2H_5OH and $AgCN$ both can generate nucleophile.

Statement II : KCN and $AgCN$ both will generate nitrile nucleophile with all reaction conditions.

Choose the **most appropriate** option.

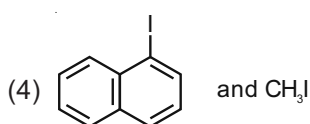
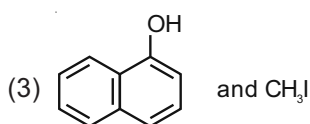
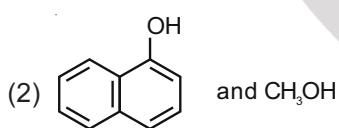
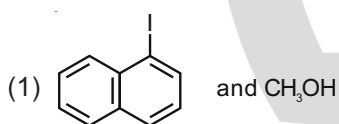
- (1) Statement I is true but statement II is false
- (2) Statement I is false but statement II is true
- (3) Both statement I and statement II are true
- (4) Both statement I and statement II are false

Answer (1)

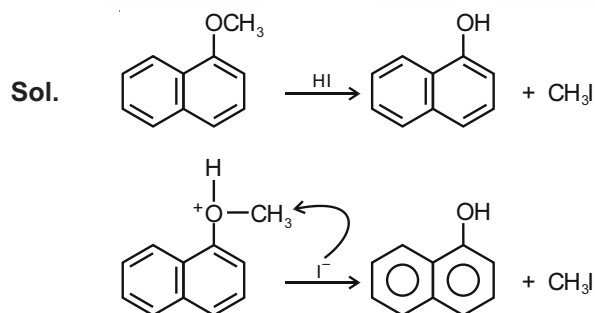
Sol. Both C_2H_5OH and $AgCN$ can generate nucleophile.

KCN generates nitriles on substitution reactions with haloalkanes where $AgCN$ generates isonitriles on substitution reactions with haloalkanes. Because KCN is ionic and has 'C' nucleophilic centre whereas $AgCN$ is covalent and has 'N' nucleophilic centre.

8. Main Products formed during a reaction of 1-methoxy naphthalene with hydroiodic acid are :



Answer (3)



9. The first ionization energy of magnesium is smaller as compared to that of elements X and Y, but higher than that of Z. The elements X, Y and Z, respectively, are

- (1) chlorine, lithium and sodium
- (2) argon, lithium and sodium
- (3) argon, chlorine and sodium
- (4) neon, sodium and chlorine

Answer (3)

Sol. First ionisation energy of Mg is small than Argon and chlorine but higher than Na.

So $X \rightarrow$ Argon

$Y \rightarrow$ Chlorine

$Z \rightarrow$ Sodium

10. The charges on the colloidal CdS sol and TiO_2 sol are, respectively

- (1) positive and positive
- (2) positive and negative
- (3) negative and positive
- (4) negative and negative

Answer (3)

Sol. CdS is negatively charged sol whereas TiO_2 is positively charged sol.

11. In basic medium, H_2O_2 exhibits which of the following reactions ?

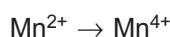
- (A) $Mn^{2+} \rightarrow Mn^{4+}$
- (B) $I_2 \rightarrow I^-$
- (C) $PbS \rightarrow PbSO_4$

Choose the **most appropriate** answer from the options given below.

- (1) (A), (C) only
- (2) (A), (B) only
- (3) (A) only
- (4) (B) only

Answer (2)

Sol. Possible reactions in basic medium are



$PbS \rightarrow PbSO_4$ is possible in acidic medium

12. Given below are two statements :

Statement I : Bohr's theory accounts for the stability and line spectrum of Li^+ ion.

Statement II : Bohr's theory was unable to explain the splitting of spectral lines in the presence of a magnetic field.

In the light of the above statements, choose the **most appropriate** answer from the options given below.

- (1) Both statement I and statement II are true
- (2) Statement I is false but statement II is true
- (3) Both statement I and statement II are false
- (4) Statement I is true but statement II is false

Answer (2)

Sol. Bohr's theory is applicable for unielectronic species only

Li^+ has two electrons

Bohr's theory could not explain the splitting of spectral lines in the presence of external magnetic field (Zeeman effect)

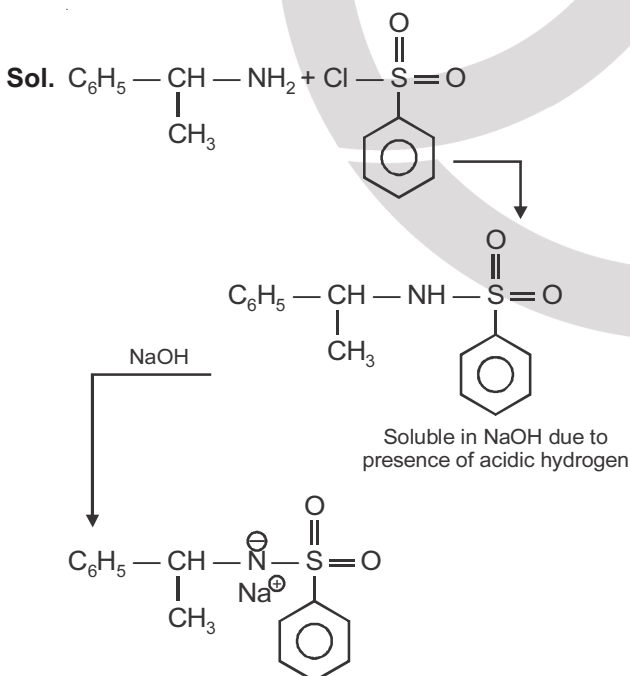
Statement I – false

Statement II – true

13. An organic compound "A" on treatment with benzene sulphonyl chloride gives compound B. B is soluble in dil. NaOH solution. Compound A is

- (1) $\text{C}_6\text{H}_5 - \text{N} - (\text{CH}_3)_2$
- (2) $\text{C}_6\text{H}_5 - \text{CH}_2\text{NHCH}_3$
- (3) $\text{C}_6\text{H}_5 - \underset{\text{CH}_3}{\text{CH}} - \text{NH}_2$
- (4) $\text{C}_6\text{H}_5 - \text{NHCH}_2\text{CH}_3$

Answer (3)



14. Match **List-I** with **List-II** :

List-I	List-II
(a) Mercury	(i) Vapour phase refining
(b) Copper	(ii) Distillation refining
(c) Silicon	(iii) Electrolytic refining
(d) Nickel	(iv) Zone refining

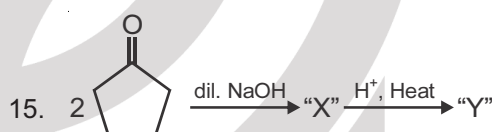
Choose the **most appropriate** answer from the option given below.

- (1) (a)-(ii), (b)-(iv), (c)-(iii), (d)-(i)
- (2) (a)-(ii), (b)-(iii), (c)-(i), (d)-(iv)
- (3) (a)-(i), (b)-(iv), (c)-(ii), (d)-(iii)
- (4) (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i)



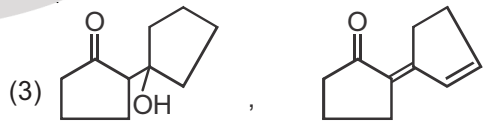

Answer (4)

Sol.	Element	Refining method
(a)	Mercury	Distillation refining
(b)	Copper	Electrolytic refining
(c)	Silicon	Zone refining
(d)	Nickel	Vapour phase refining

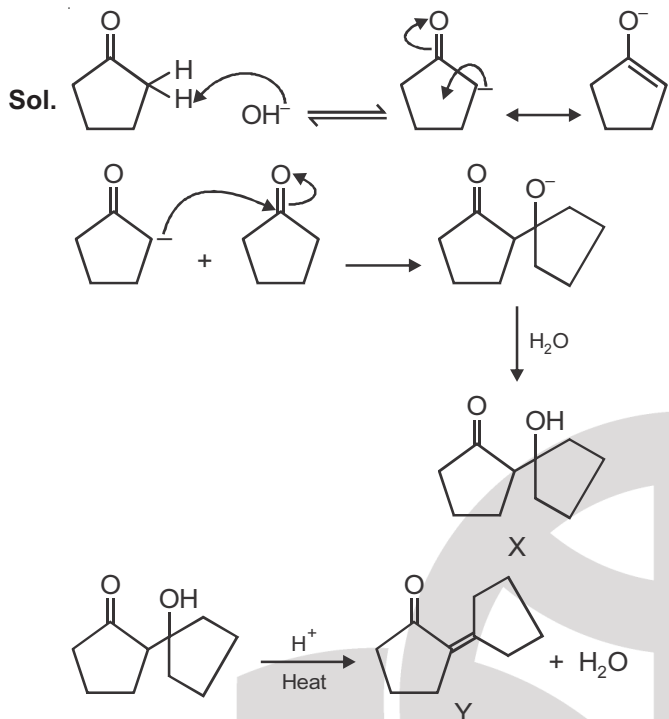
So, (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i).



Consider the above reaction, the product 'X' and 'Y' respectively are

- (1) 
- (2) 
- (3) 
- (4) 

Answer (4)



16. The secondary valency and the number of hydrogen bonded water molecule(s) in $\text{CuSO}_4 \cdot 5\text{H}_2\text{O}$, respectively, are
- (1) 5 and 1 (2) 6 and 4
 (3) 6 and 5 (4) 4 and 1

Answer (4)

Sol. Out of five H_2O molecules, four H_2O are the part of secondary valency and one H_2O is H-bonded.

17. Match List-I with List-II :

List-I	List-II
(a) Be	(i) treatment of cancer
(b) Mg	(ii) extraction of metals
(c) Ca	(iii) incendiary bombs and signals
(d) Ra	(iv) windows of X-ray tubes
	(v) bearings for motor engines

Choose the **most appropriate** answer from the option given below.

- (1) (a)-(iii), (b)-(iv), (c)-(ii), (d)-(v)
 (2) (a)-(iv), (b)-(iii), (c)-(i), (d)-(ii)
 (3) (a)-(iii), (b)-(iv), (c)-(v), (d)-(ii)
 (4) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)

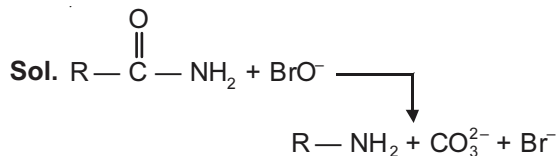
Answer (4)

Sol. Be – used in making windows of X-ray tube
 Mg – incendiary bombs and signals
 Ca – extraction of metals
 Ra – treatment of cancer

18. In the reaction of hypobromite with amide, the carbonyl carbon is lost as

- (1) CO_3^{2-} (2) HCO_3^-
 (3) CO_2 (4) CO

Answer (1)



Carbonyl carbon is lost as CO_3^{2-} .

19. A hard substance melts at high temperature and is an insulator in both solid and in molten state. This solid is most likely to be a/an:

- (1) Metallic solid (2) Ionic solid
 (3) Molecular solid (4) Covalent solid

Answer (4)

Sol. Covalent solid have high melting point due to network like structure and they are insulator e.g. diamond, SiO_2

20. The oxidation states of nitrogen in NO, NO_2 , N_2O and NO_3^- are in the order of :

- (1) $\text{NO}_2 > \text{NO}_3^- > \text{NO} > \text{N}_2\text{O}$
 (2) $\text{NO}_3^- > \text{NO}_2 > \text{NO} > \text{N}_2\text{O}$
 (3) $\text{N}_2\text{O} > \text{NO}_2 > \text{NO} > \text{NO}_3^-$
 (4) $\text{NO} > \text{NO}_2 > \text{N}_2\text{O} > \text{NO}_3^-$

Answer (2)

Sol.

	O.S of N
NO_3^-	+5
NO_2	+4
NO	+2
N_2O	+1

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

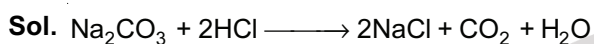
1. 10.0 mL of Na_2CO_3 solution is titrated against 0.2 M HCl solution. The following titre values were obtained in 5 readings:

4.8 mL, 4.9 mL, 5.0 mL, 5.0 mL and 5.0 mL

Based on these readings and convention of titrimetric estimation the concentration of Na_2CO_3 solution is _____ mM

(Round off the Nearest integer).

Answer (50)



equivalents of Na_2CO_3 = equivalents of HCl

$$2 \times M \times 10 = 1 \times 0.2 \times 5$$

$$M = \frac{0.2 \times 5}{20} = 0.05 \text{ M}$$

$$= 5 \times 10^{-2} \text{ M}$$

$$= 50 \times 10^{-3} \text{ M} = 50 \text{ mM}$$

2. A reaction has a half life of 1 min. The time required for 99.9% completion of the reaction is _____ min. (Round off to the Nearest integer).

[Use : $\ln 2 = 0.69$; $\ln 10 = 2.3$]

Answer (10)**

Sol. Order is not mentioned, question is incomplete. If we assume, the given reaction to be first order, then

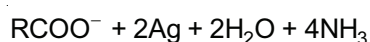
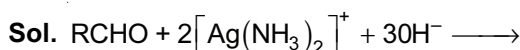
$$\ln(1000) = \frac{\ln 2}{t_{1/2}} t$$

$$t = \frac{1 \ln(1000)}{\ln 2}$$

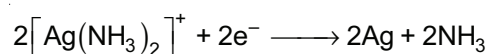
$$= 10 \text{ min}$$

3. In Tollen's test for aldehyde, the overall number of electron(s) transferred to the Tollen's reagent formula $[\text{Ag}(\text{NH}_3)_2]^+$ per aldehyde group to form silver mirror is _____. (Round off to the Nearest Integer).

Answer (2)

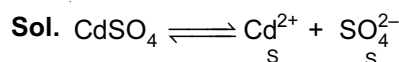


for 1 mol of R-CHO, 2 mol of $[\text{Ag}(\text{NH}_3)_2]^+$ is reduced.



4. The solubility of CdSO_4 in water is $80 \times 10^{-4} \text{ mol L}^{-1}$. Its solubility in 0.01 M H_2SO_4 solution is _____ $\times 10^{-6} \text{ mol L}^{-1}$. (Round off to the Nearest Integer). (Assume that solubility is much less than 0.01 M)

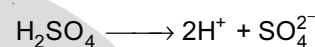
Answer (64)



$$k_{\text{sp}} = \text{S}^2$$

$$= (8 \times 10^{-4})^2$$

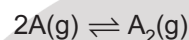
$$k_{\text{sp}} = 64 \times 10^{-8}$$



$$64 \times 10^{-8} = \text{S}' \times 0.01$$

$$\text{S}' = \frac{64 \times 10^{-8}}{0.01} = 64 \times 10^{-6} \text{ mol/L}$$

5. The gas phase reaction



at 400 K has $\Delta G^\circ = +25.2 \text{ kJ mol}^{-1}$

The equilibrium constant K_C for this reaction is _____ $\times 10^{-2}$. (Round off to the Nearest Integer).

[Use : $R = 8.3 \text{ J mol}^{-1} \text{ K}^{-1}$, $\ln 10 = 2.3 \log_{10}$, $2 = 0.30$, $1 \text{ atm} = 1 \text{ bar}$]

[antilog (-0.3) = 0.501]

Answer (166)



$$\Delta G^\circ = 25.2 \text{ kJ mol}^{-1} \text{ at } 400 \text{ K}$$

$$\Delta G^\circ = -RT \ln K_p$$

$$25.2 \times 10^3 = -8.3 \times 400 \times 2.3 \log K_p$$

$$\log K_p = \frac{-25.2 \times 10^3}{8.3 \times 400 \times 2.3}$$

$$\log K_p = -3.3$$

$$K_p = 5 \times 10^{-4}$$

$$K_p = k_c (\text{RT})^{-1}$$

$$K_c = 5 \times 10^{-4} \times 8.3 \times 400 = 1.66$$

$$= 166 \times 10^{-2}$$

6. The molar conductivities at infinite dilution of barium chloride, sulphuric acid and hydrochloric acid are 280, 860 and 426 $\text{S cm}^2 \text{ mol}^{-1}$ respectively. The molar conductivity at infinite dilution of barium sulphate is _____ $\text{S cm}^2 \text{ mol}^{-1}$. (Round off to the Nearest Integer).

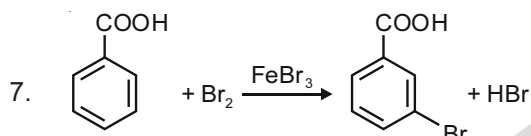
Answer (288)

Sol. $\wedge^\infty (\text{BaCl}_2) = 280 \text{ S cm}^2 \text{ mol}^{-1}$

$\wedge^\infty (\text{H}_2\text{SO}_4) = 860 \text{ S cm}^2 \text{ mol}^{-1}$

$\wedge^\infty (\text{HCl}) = 426 \text{ S cm}^2 \text{ mol}^{-1}$

$$\begin{aligned} \wedge^\infty (\text{BaSO}_4) &= \wedge^\infty (\text{BaCl}_2) + \wedge^\infty (\text{H}_2\text{SO}_4) - 2\wedge^\infty (\text{HCl}) \\ &= 280 + 860 - 2 \times 426 \\ &= 288 \text{ S cm}^2 \text{ mol}^{-1} \end{aligned}$$

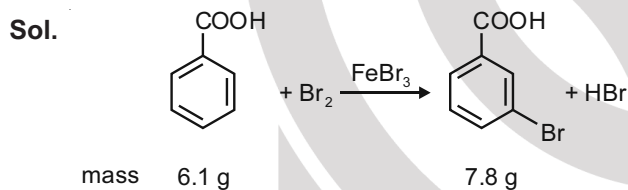


Consider the above reaction where 6.1 g of Benzoic acid is used to get 7.8 g of m-bromobenzoic acid. The percentage yield of the product is _____.

(Round off to Nearest Integer).

[Given : Atomic masses : C : 120 u, H : 1.0 u, O : 16.0 u, Br : 80.0 u]

Answer (78)



mass in gram 6.1 g

moles $\frac{6.1}{122} = 0.05$

1 mol of benzoic acid give 1 mol of m-bromobenzoic acid. 0.05 mol of benzoic acid will give 0.05 mol of m-bromobenzoic acid.

So, percentage yield is

$$\% \text{ yield} = \frac{7.8 \times 100}{0.05 \times 201} = 77.61\%$$

$\approx 78\%$

8. The number of species below that have two lone pairs of electrons in their central atom is _____. (Round off to the Nearest Integer).

SF_4 , BF_4^- , ClF_3 , AsF_3 , PCl_5 , XeF_4 , SF_6

Answer (2)

Sol. Species	No. of lone pair of electron present on central atom
SF_4	1
BF_4^-	0
ClF_3	2

AsF_3 1

PCl_5 0

BrF_5 1

XeF_4 2

SF_6 0

9. A solute A dimerizes in water. The boiling point of a 2 molal solution of A is 100.52°C . The percentage association of A is _____. (Round off to the Nearest Integer).

[Use : K_b for water = $0.52 \text{ K kg mol}^{-1}$

Boiling point of water = 100°C]

Answer (100)

Sol. $2A \rightarrow A_2$
 $1 - \alpha \quad \quad \frac{\alpha}{2}$

$i = 1 - \alpha + \frac{\alpha}{2}$

Also

$\Delta T_b = iK_b m$

$0.52 = i \times 0.52 \times 2$

$i = \frac{1}{2}$

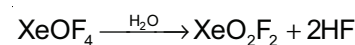
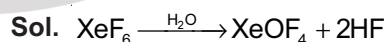
$\frac{1}{2} = 1 - \alpha + \frac{\alpha}{2}$

$\Rightarrow 1 - \frac{\alpha}{2} = \frac{1}{2}$

% of association of A is 100%

10. A xenon compound 'A' upon partial hydrolysis gives XeO_2F_2 . The number of lone pair of electrons present in compound A is _____. (Round off to Nearest Integer).

Answer (1)**



A can be both XeF_6 and XeOF_4 . Total number of lone pair in XeF_6 is 19 and total number of lone pair in XeOF_4 is 15.

Lone pair present on central atom is XeF_6 and XeOF_4 is 1.

PART-C : MATHEMATICS

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function defined as

$$f(x) = \begin{cases} \frac{\sin(a+1)x + \sin 2x}{2x}, & \text{if } x < 0 \\ b, & \text{if } x = 0 \\ \frac{\sqrt{x+bx^3} - \sqrt{x}}{bx^{5/2}}, & \text{if } x > 0 \end{cases}$$

If f is continuous at $x = 0$, then the value of $a + b$ is equal to :

- (1) $-\frac{5}{2}$ (2) $-\frac{3}{2}$
 (3) -3 (4) -2

Answer (1)

Sol. $\therefore f(x)$ is continuous at $x = 0$

$$\begin{aligned} \therefore \lim_{x \rightarrow 0^-} f(x) &= \lim_{x \rightarrow 0^+} f(x) = f(0) \\ \Rightarrow \lim_{h \rightarrow 0} \frac{\sin(a+1)(-h) - \sin 2h}{-2h} &= \lim_{h \rightarrow 0} \frac{\sqrt{h+bh^3} - \sqrt{h}}{bh^{5/2}} = b \\ \Rightarrow \frac{a+1}{2} + 1 &= \lim_{h \rightarrow 0} \frac{bh^2}{bh^2(\sqrt{4+bh^2} + 1)} = b \\ \Rightarrow \frac{a+3}{2} &= \frac{1}{2} = b \\ \Rightarrow a &= -3, b = \frac{1}{2} \\ \therefore a+b &= -3 + \frac{1}{2} = -\frac{5}{2}. \end{aligned}$$

2. If P and Q are two statements, then which of the following compound statement is a tautology?

- (1) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow (P \wedge Q)$
 (2) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow P$
 (3) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow \sim P$
 (4) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow Q$

Answer (3)

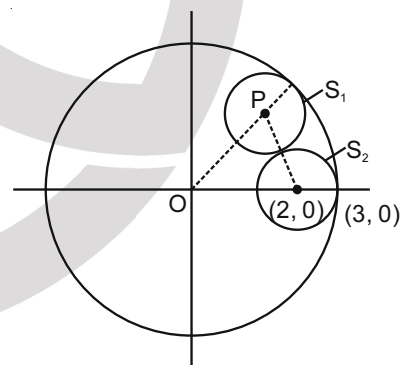
Sol. $((P \rightarrow Q) \wedge \sim Q) \rightarrow \sim P$
 $= ((\sim P \vee Q) \wedge \sim Q) \rightarrow \sim P$
 $= ((\sim P \wedge \sim Q) \vee C) \rightarrow \sim P$
 $= (\sim P \wedge \sim Q) \rightarrow \sim P$
 $= \sim(\sim P \wedge \sim Q) \vee \sim P$
 $= (P \vee Q) \vee \sim P$
 $= (P \vee \sim P) \vee Q$
 $= t \vee Q$
 $= t$ (tautology)

3. Let $S_1 : x^2 + y^2 = 9$ and $S_2 : (x - 2)^2 + y^2 = 1$. Then the locus of center of a variable circle S which touches S_1 internally and S_2 externally always passes through the points:

- (1) $\left(\frac{1}{2}, \pm \frac{\sqrt{5}}{2}\right)$ (2) $(0, \pm\sqrt{3})$
 (3) $\left(2, \pm \frac{3}{2}\right)$ (4) $(1, \pm 2)$

Answer (3)

Sol. Let variable centre of required circle (S) be (x_1, y_1) and its radius be r units.



$\therefore S$ touches S_1 internally.
 $\therefore OP = 3 - r$
 $\Rightarrow \sqrt{x_1^2 + y_1^2} = 3 - r$... (i)
 and S touches S_2 externally.
 $\therefore \sqrt{(x_1 - 2)^2 + y_1^2} = 1 + r$... (ii)

from eq. (i) and (ii), required locus of centre is

$$\sqrt{x^2 + y^2} + \sqrt{(x-2)^2 + y^2} = 4$$

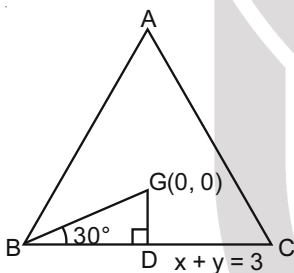
Clearly point $\left(2, \pm \frac{3}{2}\right)$ lies on the locus.

4. Let the centroid of an equilateral triangle ABC be at the origin. Let one of the sides of the equilateral triangle be along the straight line $x + y = 3$. If R and r be the radius of circumcircle and incircle respectively of ΔABC , then $(R + r)$ is equal to :

- (1) $3\sqrt{2}$
 (2) $2\sqrt{2}$
 (3) $\frac{9}{\sqrt{2}}$
 (4) $7\sqrt{2}$

Answer (3)

Sol. Here $GD = \frac{3}{\sqrt{2}}$



$$\therefore \frac{GD}{BD} = \tan 30^\circ$$

$$\therefore BD = \frac{3\sqrt{3}}{\sqrt{2}}$$

$$\therefore \text{side length} = a = 3\sqrt{6}$$

$$\text{Circumradius} = R = \frac{a}{2\sin A} = \frac{3\sqrt{6}}{2 \cdot \frac{\sqrt{3}}{2}} = 3\sqrt{2}$$

$$\text{and inradius} = r = GD = \frac{3}{\sqrt{2}}$$

$$\therefore R + r = 3\sqrt{2} + \frac{3}{\sqrt{2}} = \frac{9}{\sqrt{2}}$$

5. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} = (y+1)\left((y+1)e^{x^2/2} - x\right)$, $0 < x < 2.1$, with $y(2) = 0$. Then the value of $\frac{dy}{dx}$ at $x = 1$ is equal to :

- (1) $\frac{e^{5/2}}{(1+e^2)^2}$ (2) $\frac{-e^{3/2}}{(e^2+1)^2}$
 (3) $\frac{5e^{1/2}}{(e^2+1)^2}$ (4) $-\frac{2e^2}{(1+e^2)^2}$

Answer (2)

Sol. $\frac{dy}{dx} = (y+1)^2 e^{x^2/2} - x(y+1) \dots(A)$

$$\frac{dy}{dx} + x(y+1) = (y+1)^2 e^{x^2/2}$$

$$\frac{1}{(y+1)^2} \frac{dy}{dx} + x \times \frac{1}{y+1} = e^{x^2/2} \dots(i)$$

$$\text{Let } \frac{1}{y+1} = z \Rightarrow \frac{-1}{(y+1)^2} \frac{dy}{dx} = \frac{dz}{dx}$$

$$-\frac{dz}{dx} + xz = e^{x^2/2}$$

$$\frac{dz}{dx} - x \cdot z = -e^{x^2/2} \dots(ii)$$

$$\text{IF} = e^{\int -x dx} = e^{-x^2/2}$$

$$z \cdot e^{-x^2/2} = \int -e^{-x^2/2} \times e^{-x^2/2} dx = \int -1 dx = -x + C$$

$$\frac{1}{(y+1)e^{x^2/2}} = -x + C \dots(iii)$$

$$y(2) = 0 \Rightarrow \frac{1}{1 \times e^2} = -2 + C$$

$$C = 2 + \frac{1}{e^2}$$

$$(iii) \Rightarrow \frac{1}{(y+1)e^{x^2/2}} = -x + 2 + \frac{1}{e^2}$$

at $x = 1$,

$$\frac{1}{(y+1)e^{\frac{1}{2}}} = 1 + \frac{1}{e^2} = \frac{e^2+1}{e^2}$$

$$(y+1)e^{\frac{1}{2}} = \frac{e^2}{e^2+1}$$

$$y+1 = \frac{e^{3/2}}{e^2+1}$$

$$(A) \Rightarrow \frac{dy}{dx} = \frac{e^{3/2}}{e^2+1} \left(\frac{e^{3/2}}{e^2+1} e^{1/2} - 1 \right)$$

$$= \frac{e^{3/2}}{e^2+1} \left(\frac{e^2}{e^2+1} - 1 \right)$$

$$= \frac{-e^{3/2}}{(e^2+1)^2}$$

6. Let in a series of $2n$ observations, half of them are equal to a and remaining half are equal to $-a$. Also by adding a constant b in each of these observations, the mean and standard deviation of new set become 5 and 20, respectively. Then the value of $a^2 + b^2$ is equal to :

- (1) 925 (2) 650
(3) 425 (4) 250

Answer (3)

Sol. Old mean = $\frac{\sum x_i}{n} = 0$

New mean = $0 + b = 5$

$\Rightarrow b = 5$

Old S.D = $\sqrt{\frac{\sum (x_i - \bar{x})^2}{n}} = \sqrt{\frac{\sum a^2}{n}} = a$

New S.D = old S.D = $a = 20$

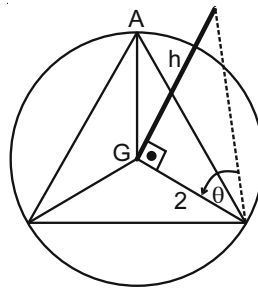
$a^2 + b^2 = 425$

7. A pole stands vertically inside a triangular park ABC. Let the angle of elevation of the top of the pole from each corner of the park be $\frac{\pi}{3}$. If the radius of the circumcircle of ΔABC is 2, then the height of the pole is equal to :

- (1) $\frac{2\sqrt{3}}{3}$ (2) $\frac{1}{\sqrt{3}}$
(3) $2\sqrt{3}$ (4) $\sqrt{3}$

Answer (3)

Sol.



Triangle is equilateral

Pole is at circumcenter

$$h = 2 \tan \frac{\pi}{3}$$

$$h = 2\sqrt{3}$$

8. Let $g(x) = \int_0^x f(t)dt$, where f is continuous function

in $[0, 3]$ such that $\frac{1}{3} \leq f(t) \leq 1$ for all $t \in [0, 1]$ and

$0 \leq f(t) \leq \frac{1}{2}$ for all $t \in [1, 3]$. The largest possible interval in which $g(3)$ lies is :

- (1) $[1, 3]$ (2) $\left[\frac{1}{3}, 2\right]$
(3) $\left[-1, -\frac{1}{2}\right]$ (4) $\left[-\frac{3}{2}, -1\right]$

Answer (2)

Sol. $g(3) = \int_0^3 f(t)dt$

$$g(3) = \int_0^1 f(t)dt + \int_1^2 f(t)dt \quad \dots(i)$$

$$\int_0^1 \frac{1}{3} dt \leq \int_0^1 f(t)dt \leq \int_0^1 1 dt$$

$$\frac{1}{3} \leq \int_0^1 f(t)dt \leq 1 \quad \dots(ii)$$

$$\int_1^2 0 dt \leq \int_1^2 f(t)dt \leq \int_1^2 \frac{1}{2} dt$$

$$0 \leq \int_1^2 f(t)dt \leq \frac{1}{2} \times 2 = 1 \quad \dots(iii)$$

$$(i), (ii), (iii) \Rightarrow \frac{1}{3} \leq g(3) \leq 2$$

9. Let $f : \mathbf{R} - \{3\} \rightarrow \mathbf{R} - \{1\}$ be defined by

$$f(x) = \frac{x-2}{x-3}$$

Let $g : \mathbf{R} \rightarrow \mathbf{R}$ be given as $g(x) = 2x - 3$. Then, the sum of all the values of x for which

$$f^{-1}(x) + g^{-1}(x) = \frac{13}{2} \text{ is equal to.}$$

- (1) 3 (2) 5
(3) 7 (4) 2

Answer (2)

Sol. Finding inverse of $f(x)$

$$y = \frac{x-2}{x-3} \Rightarrow xy - 3y = x - 2 \Rightarrow x(y-1) = 3y - 2$$

$$\therefore f^{-1}(x) = \frac{3x-2}{x-1}$$

Similarly for $g^{-1}(x)$

$$y = 2x - 3 \Rightarrow x = \frac{y+3}{2} \Rightarrow g^{-1}(x) = \frac{x+3}{2}$$

$$\therefore \frac{3x-2}{x-1} + \frac{x+3}{2} = \frac{13}{2}$$

$$\Rightarrow 6x - 4 + x^2 + 2x - 3 = 13x - 13$$

$$\Rightarrow x^2 - 5x + 6 = 0$$

$$\Rightarrow (x-2)(x-3) = 0$$

$$\Rightarrow x = 2 \text{ or } 3$$

10. If $15\sin^4\alpha + 10\cos^4\alpha = 6$, for some $\alpha \in \mathbf{R}$, then the value of $27\sec^6\alpha + 8\operatorname{cosec}^6\alpha$ is equal to :

- (1) 350 (2) 250
(3) 400 (4) 500

Answer (2)

Sol. $15\sin^2\alpha + 10(1 - \sin^2\alpha)^2 = 6$

$$\Rightarrow 25\sin^2\alpha - 20\sin^2\alpha + 4 = 0$$

$$\Rightarrow 25\sin^2\alpha - 10\sin^2\alpha - 10\sin^2\alpha + 4 = 0$$

$$\Rightarrow (5\sin^2\alpha - 2)^2 = 0 \Rightarrow \sin^2\alpha = \frac{2}{5}$$

$$\therefore \cos^2\alpha = \frac{3}{5}$$

$$\therefore 27\sec^6\alpha + 8\operatorname{cosec}^6\alpha = 27\left(\frac{5}{3}\right)^3 + 8\left(\frac{5}{2}\right)^3$$

$$= 125 + 125 = 250$$

11. Let a complex number be $w = 1 - \sqrt{3}i$. Let another complex number z be such that $|zw| = 1$ and

$$\arg(z) - \arg(w) = \frac{\pi}{2}. \text{ Then the area of the triangle}$$

with vertices origin, z and w is equal to :

(1) $\frac{1}{2}$ (2) 2

(3) 4 (4) $\frac{1}{4}$

Answer (1)

Sol. $|wz| = 1 \Rightarrow |w||z| = 1$ and $|w| = 2$

$$\Rightarrow |z| = \frac{1}{2}$$

$$\text{Also } \arg(z) - \arg(w) = \frac{\pi}{2}$$

$$\Rightarrow z = \frac{1}{2} \cdot \frac{(1 - i\sqrt{3})}{2} \cdot i$$

$$\text{Area of triangle} = \frac{1}{2} \cdot 2 \times \frac{1}{2} = \frac{1}{2}$$

12. Let the system of linear equations

$$4x + \lambda y + 2z = 0$$

$$2x - y + z = 0$$

$$\mu x + 2y + 3z = 0, \lambda, \mu \in \mathbf{R}.$$

has a non-trivial solution. Then which of the following is true?

- (1) $\mu = -6, \lambda \in \mathbf{R}$ (2) $\lambda = 3, \mu \in \mathbf{R}$
(3) $\mu = 6, \lambda \in \mathbf{R}$ (4) $\lambda = 2, \mu \in \mathbf{R}$

Answer (3)

$$\text{Sol. } \begin{vmatrix} 4 & \lambda & 2 \\ 2 & -1 & 1 \\ \mu & 2 & 3 \end{vmatrix} = 0$$

$$\Rightarrow \mu\lambda + 2\mu - 6\lambda - 12 = 0$$

$$\Rightarrow (\lambda + 2)(\mu - 6) = 0$$

$$\lambda = -2 \text{ or } \mu = 6$$

13. Define a relation R over a class of $n \times n$ real matrices A and B as " ARB iff there exists a non-singular matrix P such that $PAP^{-1} = B$ ". Then which of the following is true?

- (1) R is reflexive, symmetric but not transitive
(2) R is an equivalence relation
(3) R is symmetric, transitive but not reflexive
(4) R is reflexive, transitive but not symmetric

Answer (2)

Sol. For reflexive,

$$PAP^{-1} = A \text{ is true if } P = I$$

For symmetric,

If $PAP^{-1} = B$ then $PBP^{-1} = A$ must be true

$$\therefore PAP^{-1} = B \Rightarrow A = P^{-1}BP$$

and $PBP^{-1} = P(PAP^{-1})P^{-1} = P^2A(P^{-1})^2$ is equal to A

if P is involutory matrix (i.e. $P^2 = I$)

For transitive,

If $PAP^{-1} = B$ and $PBP^{-1} = C$ then $PAP^{-1} = C$ must be true

$\therefore C = PBP^{-1} = P^2AP^{-1}$ will be equal to PAP^{-1} if P is idempotent matrix (i.e. $P^2 = P$)

Hence relation R is equivalence relation.

14. In a triangle ABC, if $|\overline{BC}| = 8$, $|\overline{CA}| = 7$, $|\overline{AB}| = 10$,

then the projection of the vector \overline{AB} on \overline{AC} is equal to :

- (1) $\frac{115}{16}$ (2) $\frac{25}{4}$
 (3) $\frac{127}{20}$ (4) $\frac{85}{14}$

Answer (4)

Sol. Projection of \overline{AB} on $\overline{AC} = \frac{(\overline{AB}) \cdot (\overline{AC})}{|\overline{AC}|} = p$ (say)

$$= \frac{|\overline{AB}| |\overline{AC}| \cos \theta}{|\overline{AC}|}$$

$$\text{where } \cos \theta = \frac{10^2 + 7^2 - 8^2}{2 \cdot 10 \cdot 7}$$

$$\Rightarrow p = \frac{10 \cdot 85}{2 \cdot 10 \cdot 7} = \frac{85}{14}$$

15. Let S_1 be the sum of first 2n terms of an arithmetic progression. Let S_2 be the sum of first 4n terms of the same arithmetic progression. If $(S_2 - S_1)$ is 1000, then the sum of the first 6n terms of the arithmetic progression is equal to :

- (1) 7000 (2) 5000
 (3) 3000 (4) 1000

Answer (3)

$$\text{Sol. } S_1 = \frac{2n}{2} [2a + (2n-1)d]$$

(where a = T_1 and d is common difference)

$$S_2 = \frac{4n}{2} [2a + (4n-1)d]$$

$$\begin{aligned} S_2 - S_1 &= 2n[2a + (4n-1)d] - n[2a + (2n-1)d] = 1000 \\ &= n[2a + d(8n-2-2n+1)] = 1000 \\ &= n[2a + (6n-1)d] = 1000 \end{aligned}$$

$$S_6 = \frac{6n}{2} [2a + (6n-1)d] = 3(S_2 - S_1) = 3000$$

16. The area bounded by the curve $4y^2 = x^2(4-x)$ ($x-2$) is equal to :

- (1) $\frac{3\pi}{8}$ (2) $\frac{3\pi}{2}$
 (3) $\frac{\pi}{8}$ (4) $\frac{\pi}{16}$

Answer (2)

Sol. As RHS is always positive $x \in [2, 4]$ only

$$\text{Area} = \int_2^4 x(6x-8-x^2)^{\frac{1}{2}} dx = A$$

$$= \int_2^4 x\sqrt{1-(x-3)^2} dx \quad \dots(i)$$

$$A = \int_2^4 (6-x)\sqrt{1-(x-3)^2} dx \quad \dots(ii)$$

$$\Rightarrow 2A = \int_2^4 6\sqrt{1-(x-3)^2} dx$$

$$\Rightarrow A = \left(x-3\sqrt{1-(x-3)^2} + \frac{1}{2}\sin^{-1}(x-3) \right) \Big|_2^4$$

$$= 3 \left(3(0) + \frac{1}{2}\pi/2 - 0 - \frac{1}{2}(-\pi/2) \right)$$

$$= 3 \frac{\pi}{2} = \frac{3\pi}{2}$$

17. Let a tangent be drawn to the ellipse $\frac{x^2}{27} + y^2 = 1$ at

$(3\sqrt{3}\cos\theta, \sin\theta)$ where $\theta \in \left(0, \frac{\pi}{2}\right)$. Then the value

of θ such that the sum of intercepts on axes made by this tangent is minimum is equal to :

- (1) $\frac{\pi}{4}$ (2) $\frac{\pi}{8}$
 (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{3}$

Answer (3)

Sol. Tangent = $\frac{x}{3\sqrt{3}} \cos\theta + y \sin\theta = 1$

x-intercept = $3\sqrt{3} \sec\theta$

y-intercept = $\operatorname{cosec}\theta$

sum = $3\sqrt{3} \sec\theta + \operatorname{cosec}\theta = f(\theta) \quad \theta \in \left(0, \frac{\pi}{2}\right)$

$\Rightarrow f'(\theta) = 3\sqrt{3} \sec\theta \tan\theta - \operatorname{cosec}\theta \cot\theta = 0$

$\Rightarrow \frac{3\sqrt{3} \sin\theta}{\cos^2\theta} = \frac{\cos\theta}{\sin\theta}$

$\Rightarrow \tan^3\theta = \left(\frac{1}{\sqrt{3}}\right)^3$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \theta = \frac{\pi}{6}$

also $f(\theta)$ changes sign – to + hence minimum.

18. Let in a Binomial distribution, consisting of 5 independent trials, probabilities of exactly 1 and 2 successes be 0.4096 and 0.2048 respectively. Then the probability of getting exactly 3 successes is equal to :

(1) $\frac{80}{243}$

(2) $\frac{32}{625}$

(3) $\frac{40}{243}$

(4) $\frac{128}{625}$

Answer (2)

Sol. $n = 5$ (given)

also $5c_1 p q^4 = 0.4096$

and $5c_2 p^2 q^3 = 0.2048$

$\frac{5q}{10p} = \frac{0.4096}{0.2048}$

$\Rightarrow q = 4p$

also $p + q = 1$

$\Rightarrow p = \frac{1}{5}$ and $q = \frac{4}{5}$

$p(\text{exactly 3 successes}) = 5c_3 \cdot p^3 q^2$

$= 10 \frac{1}{5^3} \cdot \frac{16}{5^2} = \frac{32}{625}$

19. Let \vec{a} and \vec{b} be two non-zero vectors perpendicular to each other and $|\vec{a}| = |\vec{b}|$. If $|\vec{a} \times \vec{b}| = |\vec{a}|$, then the angle between the vectors $(\vec{a} + \vec{b} + (\vec{a} \times \vec{b}))$ and \vec{a} is equal to :

(1) $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

(2) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(3) $\sin^{-1}\left(\frac{1}{\sqrt{6}}\right)$

(4) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Answer (4)

Sol. Angle required is say θ

$\Rightarrow \cos\theta = \frac{\vec{a} \cdot (\vec{a} + \vec{b} + (\vec{a} \times \vec{b}))}{|\vec{a}| |\vec{a} + \vec{b} + (\vec{a} \times \vec{b})|}$

$= \frac{|\vec{a}|^2 + 0 + 0}{|\vec{a}| |\vec{a} + \vec{b} + (\vec{a} \times \vec{b})|}$

$= \frac{|\vec{a}|^2}{|\vec{a}| \sqrt{3} |\vec{a}|}$

(as \vec{a}, \vec{b} and $\vec{a} \times \vec{b}$ are mutually perpendicular to each other)

$\cos\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

20. Consider a hyperbola $H : x^2 - 2y^2 = 4$. Let the tangent at a point $P(4, \sqrt{6})$ meet the x-axis at Q and latus rectum at $R(x_1, y_1)$, $x_1 > 0$. If F is a focus of H which is nearer to the point P, then the area of ΔQFR is equal to.

(1) $\sqrt{6} - 1$

(2) $4\sqrt{6}$

(3) $4\sqrt{6} - 1$

(4) $\frac{7}{\sqrt{6}} - 2$

Answer (4)

Sol. Hyperbola $x^2 - 2y^2 = 4 \Rightarrow \frac{x^2}{4} - \frac{y^2}{2} = 1$

$e = \sqrt{1 + \frac{2}{4}} = \frac{\sqrt{3}}{\sqrt{2}}$

$S \equiv (\pm\sqrt{6}, 0)$

Directrix $\equiv x = \sqrt{6}$

Tangent at $(4, \sqrt{6})$ is $4x - 2\sqrt{6}y = 4$

$\therefore Q \equiv (1, 0), R \equiv \left(\sqrt{6}, \frac{4\sqrt{6} - 4}{2\sqrt{6}}\right)$

Area of $\Delta QFR = \frac{1}{2}(\sqrt{6} - 1) \frac{4(\sqrt{6} - 1)}{2\sqrt{6}} = \frac{7}{\sqrt{6}} - 2$

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. Let ${}^n C_r$ denote the binomial coefficient of x^r in the expansion of $(1+x)^n$.

If $\sum_{k=0}^{10} (2^2 + 3k) {}^n C_k = \alpha 3^{10} + \beta 2^{10}$, $\alpha, \beta \in \mathbb{R}$, then $\alpha + \beta$ equal to _____.

Answer (19*)

Sol. Take $n = 10$

$$\begin{aligned} \sum_{k=0}^{10} (3k+4) {}^{10} C_k &= 30 \sum_{k=1}^{10} {}^9 C_{k-1} + 4 \sum_{k=0}^{10} {}^{10} C_k \\ &= 30 \cdot 2^9 + 4 \cdot 2^{10} \\ &= 19 \cdot 2^{10} \\ \Rightarrow \alpha &= 0 \text{ and } \beta = 19 \end{aligned}$$

2. The term independent of x in the expansion of

$$\left[\frac{x+1}{x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1} - \frac{x-1}{x-x^{\frac{1}{2}}} \right]^{10}, \quad x \neq 1, \text{ is equal to}$$

Answer (210)

Sol.

$$\begin{aligned} &\left(\frac{x+1}{x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1} - \frac{x-1}{x-x^{\frac{1}{2}}} \right)^{10} \\ &= \left(\frac{\frac{1}{x^{\frac{2}{3}} + 1} - \frac{2}{x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1}}{\frac{2}{x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1} - \frac{1}{x^2(x^{\frac{1}{2}} - 1)}} - \frac{\frac{1}{x^2 - 1} - \frac{1}{x^2 + 1}}{\frac{1}{x^2(x^{\frac{1}{2}} - 1)}} \right)^{10} \\ &= \left(\frac{1}{x^{\frac{2}{3}} + 1} - (1+x^{\frac{1}{2}}) \right)^{10} \\ &= \left(x^{\frac{1}{3}} - x^{\frac{1}{2}} \right)^{10} \\ T_{r+1} &= {}^{10} C_r \left(x^{\frac{1}{3}} \right)^{(10-r)} \left(x^{-\frac{1}{2}} \right)^r \end{aligned}$$

For being independent of x : $\frac{10-r}{3} - \frac{r}{2} = 0 \Rightarrow r = 4$

Term independent of $x = {}^{10} C_4 = 210$

3. Let the mirror image of the point $(1, 3, a)$ with respect to the plane $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) - b = 0$ be $(-3, 5, 2)$. Then, the value of $|a + b|$ is equal to _____.

Answer (1)

Sol. Equation of plane : $2x - y + z = b$

Midpoint of $(1, 3, a)$ & $(-3, 5, 2)$ lies on this plane

$$\Rightarrow 2 \left(\frac{1-3}{2} \right) - \left(\frac{3+5}{2} \right) + \left(\frac{2+a}{2} \right) = b$$

$$\Rightarrow a - 2b = 14 \quad \dots(i)$$

Also vector joining $(1, 3, a)$ & $(-3, 5, 2)$ is parallel to $2\hat{i} - \hat{j} + \hat{k}$

$$\text{i.e., } 4\hat{i} - 2\hat{j} + (a-2)\hat{k} = \lambda(2\hat{i} - \hat{j} + \hat{k})$$

$$\Rightarrow \lambda = 2 \Rightarrow a - 2 = 2 \Rightarrow a = 4$$

From (i), $b = -5$

Hence $|a + b| = 1$

4. Let P be a plane containing the line $\frac{x-1}{3} = \frac{y+6}{4} = \frac{z+5}{2}$ and parallel to the line $\frac{x-3}{4} = \frac{y-3}{-3} = \frac{z+5}{7}$. If the point $(1, -1, \alpha)$ lies on the plane P , then the value of $|5\alpha|$ is equal to _____.

Answer (38)

Sol. The plane is parallel to vectors

$$\vec{n}_1 = 3\hat{i} + 4\hat{j} + 2\hat{k} \quad \& \quad \vec{n}_2 = 4\hat{i} - 3\hat{j} + 7\hat{k}$$

Vector normal to plane = $\vec{n} = \vec{n}_1 \times \vec{n}_2$

$$\Rightarrow \vec{n} = 34\hat{i} - 13\hat{j} - 25\hat{k}$$

Point $(1, -6, -5)$ lies on the plane

Hence equation of plane is

$$((x-1)\hat{i} + (y+6)\hat{j} + (z+5)\hat{k}) \cdot (34\hat{i} - 13\hat{j} - 25\hat{k}) = 0$$

Passes through $(1, -1, \alpha)$

$$\Rightarrow (0\hat{i} + 5\hat{j} + (\alpha+5)\hat{k}) \cdot (34\hat{i} - 13\hat{j} - 25\hat{k}) = 0$$

$$\Rightarrow 0 - 65 - 25\alpha - 125 = 0$$

$$\Rightarrow 5\alpha = 38$$

$$|5\alpha| = 38$$

5. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfy the equation $f(x + y) = f(x) \cdot f(y)$ for all $x, y \in \mathbb{R}$ and $f(x) \neq 0$ for any $x \in \mathbb{R}$. If the function f is differentiable at $x = 0$ and $f'(0) = 3$, then

$$\lim_{h \rightarrow 0} \frac{1}{h} (f(h) - 1) \text{ is equal to } \underline{\hspace{2cm}}.$$

Answer (3)

Sol. $\therefore f(x + y) = f(x)f(y) \quad \forall x, y \in \mathbb{R}$

$$x = y = 0 \Rightarrow f(0) = (f(0))^2 \Rightarrow f(0) = 0 \text{ or } f(0) = 1$$

$$f(x) \neq 0 \text{ for any } x \in \mathbb{R} \Rightarrow f(0) = 1$$

$$\text{Given } f'(0) = 3$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{f(0+h) + f(0)}{h} = 3$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{f(h) - 1}{h} = 3$$

6. Let I be an identity matrix of order 2×2 and

$$P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}. \text{ Then the value of } n \in \mathbb{N} \text{ for which}$$

$$P_n = 5I - 8P \text{ is equal to } \underline{\hspace{2cm}}.$$

Answer (6)

Sol. $P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$

$$\begin{vmatrix} 2-\lambda & -1 \\ 5 & -3-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda^2 + \lambda - 1 = 0$$

$$\Rightarrow P^2 + P - I = 0$$

$$\Rightarrow P^2 = I - P$$

$$\Rightarrow P^4 = I + P^2 - 2P$$

$$\Rightarrow P^4 = 2I - 3P$$

$$\text{Now, } P^4 \cdot P^2 = (2I - 3P)(I - P) = 2I - 5P + 3P^2$$

$$\Rightarrow P^6 = 5I - 8P$$

$$\text{so } n = 6.$$

7. Let $P(x)$ be a real polynomial of degree 3 which vanishes at $x = -3$. Let $P(x)$ have local minima at

$$x=1, \text{ local maxima at } x = -1 \text{ and } \int_{-1}^1 P(x) dx = 18,$$

then the sum of all the coefficients of the polynomial $P(x)$ is equal to _____.

Answer (8)

Sol. $P'(x)$ is second degree polynomial and vanishes at

$$x = \pm 1$$

$$\text{let } P'(x) = k(x-1)(x+1) = kx^2 - k$$

$$\Rightarrow P(x) = \frac{k}{3}x^3 - kx + \lambda$$

$$\int_{-1}^1 p(x) dx = 18 \Rightarrow \int_{-1}^1 \left(\frac{k}{3}x^3 - kx + \lambda \right) dx = 2\lambda = 18$$

$$\Rightarrow \lambda = 9$$

$$\text{Also } P(-3) = 0$$

$$\Rightarrow \frac{k}{3}(-3)^3 - k(-3) + 9 = 0 \Rightarrow k = \frac{3}{2}$$

$$P(x) = \frac{1}{2}x^3 - \frac{3}{2}x + 9 \Rightarrow \text{sum of all the coefficients} = 8$$

8. Let $y = y(x)$ be the solution of the differential

$$\text{equation } xdy - ydx = \sqrt{x^2 - y^2} dx, x \geq 1, \text{ with}$$

$y(1) = 0$. If the area bounded by the line $x = 1$, $x = e^\pi$, $y = 0$ and $y = y(x)$ is $\alpha e^{2\pi} + \beta$, then the value of $10(\alpha + \beta)$ is equal to _____.

Answer (4)

Sol. $xdy - ydx = \sqrt{x^2 - y^2} dx$

$$\Rightarrow \frac{xdy - ydx}{x^2} = \frac{1}{x} \sqrt{1 - \left(\frac{y}{x}\right)^2} dx$$

$$\Rightarrow \frac{d\left(\frac{y}{x}\right)}{\sqrt{1 - \left(\frac{y}{x}\right)^2}} = \frac{dx}{x}$$

$$\Rightarrow \sin^{-1}\left(\frac{y}{x}\right) = \ln x + C$$

$$\Rightarrow y(1) = 0 \Rightarrow C = 0$$

$$\text{Here } y = x \cdot \sin(\ln x)$$

$A =$ The required area bounded is

$$= \int_1^{e^\pi} x \cdot \sin(\ln x) dx$$

$$\text{Put } x = e^t$$

$$\Rightarrow dx = e^t dt$$

$$A = \int_0^{\pi} e^t \cdot \sin t \cdot e^t dt$$

$$A = \int_0^{\pi} e^{2t} \cdot \sin t dt$$

$$= \frac{e^{2t}}{1^2 + 2^2} \cdot (2 \sin t - \cos t) \Big|_0^{\pi}$$

$$= \frac{1}{5} (e^{2\pi}(0+1) - 1(0-1))$$

$$= \frac{1}{5} \cdot e^{2\pi} + \frac{1}{5} = \alpha \cdot e^{2\pi} + \beta$$

$$\alpha = -\frac{1}{5}, \beta = \frac{1}{5}$$

$$10 (\alpha + \beta) = 4$$

9. If $f(x)$ and $g(x)$ are two polynomials such that the polynomial $P(x) = f(x^3) + x g(x^3)$ is divisible by $x^2 + x + 1$, then $P(1)$ is equal to _____.

Answer (Zero)

Sol. $x^2 + x + 1 = 0 \Rightarrow (x - \omega)(x - \omega^2) = 0$

where ω is complex cube root of unity

$P(x)$ is divisible by $x^2 + x + 1$

Here $P(\omega) = 0$ and $P(\omega^2) = 0$

$$\Rightarrow P(\omega) = f(\omega^3) + \omega g(\omega^3) = 0$$

$$0 = f(1) + \omega g(1) \quad \dots \text{eqn (i)}$$

$$\text{Also, } P(\omega^2) = f(\omega^6) + \omega^2 g(\omega^6) = 0$$

$$0 = f(1) + \omega^2 g(1) \quad \dots \text{eqn (ii)}$$

from (i) and (ii), $f(1) = g(1) = 0$

$$\text{Here } P(1) = f(1) + 1g(1) = 0$$

10. If $\sum_{r=1}^{10} r!(r^3 + 6r^2 + 2r + 5) = \alpha(11!)$,

then the value of α is equal to _____.

Answer (160)

Sol. $\sum_{r=1}^{10} r!(r^3 + 6r^2 + 2r + 5)$

$$= \sum_{r=1}^{10} ((r+3)! - (r+1)! - 8((r+1)! - r!))$$

$$= \sum_{r=1}^{10} ((r+3)! - (r+1)! - 8 \sum_{r=1}^{10} ((r+1)! - r!))$$

$$= 12! + 13! - 2! - 3! - 8(11! - 1!)$$

$$= 160 \cdot 11!$$

Hence $\alpha = 160$