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■ India's First Medical and Engineering Entrance Coaching centre by Prof. **P.C THOMAS** now in UAE ■

Time : 3 hrs.

## Answers & Solutions

M.M. : 300

*for*

## JEE (MAIN)-2021 (Online) Phase-3

(Physics, Chemistry and Mathematics)

### IMPORTANT INSTRUCTIONS :

- (1) The test is of **3 hours** duration.
- (2) The Test Booklet consists of 90 questions. The maximum marks are 300.
- (3) There are **three** parts in the question paper A, B, C consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part has two sections.
  - (i) Section-I : This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **-1 mark** for wrong answer.
  - (ii) Section-II : This section contains 10 questions. In Section-II, attempt any **five questions out of 10**. There will be **no negative marking for Section-II**. The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and there is no negative marking for wrong answer.

## PART-A : PHYSICS

### SECTION - I

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer :**

1. Two vectors  $\vec{P}$  and  $\vec{Q}$  have equal magnitudes. If the magnitude of  $\vec{P} + \vec{Q}$  is  $n$  times the magnitude of  $\vec{P} - \vec{Q}$ , then angle between  $\vec{P}$  and  $\vec{Q}$  is

(1)  $\cos^{-1}\left(\frac{n^2-1}{n^2+1}\right)$       (2)  $\sin^{-1}\left(\frac{n-1}{n+1}\right)$   
 (3)  $\sin^{-1}\left(\frac{n^2-1}{n^2+1}\right)$       (4)  $\cos^{-1}\left(\frac{n-1}{n+1}\right)$

**Answer (1)**

**Sol.**  $|\vec{P} + \vec{Q}|^2 = n^2 |\vec{P} - \vec{Q}|^2$   
 $\Rightarrow P^2 + Q^2 + 2PQ\cos\theta = n^2 (P^2 + Q^2 - 2PQ\cos\theta)$   
 $\Rightarrow 1 + 1 + 2\cos\theta = n^2 (1 + 1 - 2\cos\theta)$   
 $\Rightarrow 1 + \cos\theta = n^2 (1 - \cos\theta)$   
 $\Rightarrow \frac{1 + \cos\theta}{1 - \cos\theta} = \frac{n^2}{1}$   
 $\Rightarrow \frac{1}{\cos\theta} = \frac{n^2 + 1}{n^2 - 1}$   
 $\Rightarrow \theta = \cos^{-1}\left(\frac{n^2 - 1}{n^2 + 1}\right)$

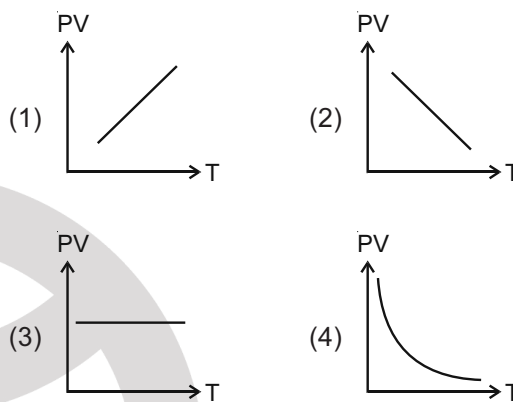
2. Two small drops of mercury each of radius  $R$  coalesce to form a single large drop. The ratio of total surface energy before and after the change is

(1) 2 : 1      (2) 1 : 2  
 (3)  $2^{\frac{1}{3}}$  : 1      (4) 1 :  $2^{\frac{1}{3}}$

**Answer (3)**

**Sol.**  $\frac{4}{3}\pi R^3 \times 2 = \frac{4}{3}\pi R_2^3$   
 $\Rightarrow R_2 = R \times 2^{\frac{1}{3}}$   
 $\therefore \frac{E_1}{E_2} = \frac{T \times 4\pi R^2 \times 2}{T \times 4\pi R_2^2 \times 2^{\frac{2}{3}}}$   
 $= \frac{2}{2^{\frac{2}{3}}} = 2^{\frac{1}{3}}$

3. Which of the following graphs represent the behavior of an ideal gas? Symbols have their usual meaning.



**Answer (1)**

**Sol.**  $PV = nRT$   
 $\Rightarrow PV \propto T$   
 $\Rightarrow$  straight line with positive slope.

4. The correct relation between the degrees of freedom  $f$  and the ratio of specific heat  $\gamma$  is

(1)  $f = \frac{1}{\gamma + 1}$       (2)  $f = \frac{2}{\gamma - 1}$   
 (3)  $f = \frac{2}{\gamma + 1}$       (4)  $f = \frac{\gamma + 1}{2}$

**Answer (2)**

**Sol.**  $\therefore \gamma = 1 + \frac{2}{f}$   
 $\Rightarrow \frac{2}{f} = \gamma - 1$   
 $\Rightarrow f = \frac{2}{\gamma - 1}$

5. The magnetic susceptibility of a material of a rod is 499. Permeability in vacuum is  $4\pi \times 10^{-7}$  H/m. Absolute permeability of the material of the rod is:

(1)  $\pi \times 10^{-4}$  H/m      (2)  $4\pi \times 10^{-4}$  H/m  
 (3)  $2\pi \times 10^{-4}$  H/m      (4)  $3\pi \times 10^{-4}$  H/m

**Answer (3)**

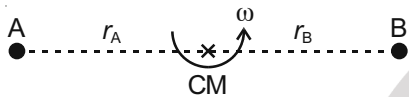
**Sol.**  $\chi = 499$   
 $\therefore \mu = (1 + \chi) \times \mu_0$   
 $= 500 \times 4\pi \times 10^{-7}$   
 $= 2\pi \times 10^{-4}$  H/m

6. Consider a binary star system of star A and star B with masses  $m_A$  and  $m_B$  revolving in a circular orbit of radii  $r_A$  and  $r_B$ , respectively. If  $T_A$  and  $T_B$  are the time period of star A and star B, respectively, then:

- (1)  $T_A = T_B$                       (2)  $\frac{T_A}{T_B} = \left(\frac{r_A}{r_B}\right)^{\frac{3}{2}}$   
 (3)  $T_A > T_B$  (if  $r_A > r_B$ )    (4)  $T_A > T_B$  (if  $m_A > m_B$ )

**Answer (1)**

**Sol.**  $T_A = \frac{2\pi}{\omega}$



$$T_B = \frac{2\pi}{\omega}$$

$$\Rightarrow T_A = T_B$$

7. A body rolls down an inclined plane without slipping. The kinetic energy of rotation is 50% of its translational kinetic energy. The body is:

- (1) Solid cylinder                      (2) Hollow cylinder  
 (3) Ring                                      (4) Solid sphere

**Answer (1)**

**Sol.**  $\frac{k_T}{k_R} = \frac{k_T}{\frac{1}{2}k_T} = 2$

$$\frac{MR^2}{I_{CM}} = 2$$

$$\Rightarrow I_{CM} = \frac{MR^2}{2}$$

8. At an angle of  $30^\circ$  to the magnetic meridian, the apparent dip is  $45^\circ$ . Find the true dip:

- (1)  $\tan^{-1} \frac{2}{\sqrt{3}}$                                   (2)  $\tan^{-1} \sqrt{3}$   
 (3)  $\tan^{-1} \frac{1}{\sqrt{3}}$                                   (4)  $\tan^{-1} \frac{\sqrt{3}}{2}$

**Answer (4)**

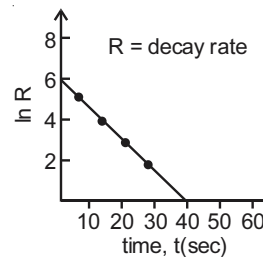
**Sol.**  $\tan \theta = \frac{B_V}{B_H}$

$$\tan 45 = \frac{B_V}{B_H \cos 30} = \frac{2B_V}{B_H \sqrt{3}}$$

$$\Rightarrow B_V = \frac{\sqrt{3}}{2} B_H$$

$$\Rightarrow \theta = \tan^{-1} \left( \frac{\sqrt{3}}{2} \right)$$

9. For a certain radioactive process the graph between  $\ln R$  and  $t(\text{sec})$  is obtained as shown in the figure. Then the value of half life for the unknown radioactive material is approximately:



- (1) 4.62 sec                                  (2) 6.93 sec  
 (3) 9.15 sec                                  (4) 2.62 sec

**Answer (1)**

**Sol.**  $R = R_0 e^{-\lambda t}$

$$\Rightarrow \ln R = \ln R_0 - \lambda t$$

$$\text{at } t = 0, \ln R = 6$$

$$\Rightarrow 6 = \ln R_0 \quad \dots(1)$$

$$\text{at } t = 40 \text{ s, } \ln R = 0$$

$$\Rightarrow \lambda = \frac{\ln R_0}{t} = \frac{6}{40}$$

$$\Rightarrow t_{\frac{1}{2}} = \frac{\ln 2}{\lambda} = \frac{0.693 \times 40}{6} = 4.62 \text{ s}$$

10. The length of a metal wire is  $l_1$ , when the tension in it is  $T_1$  and is  $l_2$  when the tension is  $T_2$ . The natural length of the wire is:

- (1)  $\sqrt{l_1 l_2}$                                   (2)  $\frac{l_1 T_2 - l_2 T_1}{T_2 - T_1}$   
 (3)  $\frac{l_1 + l_2}{2}$                                   (4)  $\frac{l_1 T_2 + l_2 T_1}{T_2 + T_1}$

**Answer (2)**

**Sol.**  $T_1 = k(l_1 - l_0) \quad \dots(i)$

$$T_2 = k(l_2 - l_0) \quad \dots(ii)$$

From (i) and (ii)

$$l_0 = \frac{l_1 T_2 - l_2 T_1}{T_2 - T_1}$$

11. In an electromagnetic wave the electric field vector and magnetic field vector are given as  $\vec{E} = E_0 \hat{i}$  and  $\vec{B} = B_0 \hat{k}$  respectively. The direction of propagation of electromagnetic wave is along:

- (1)  $(-\hat{k})$                                   (2)  $(-\hat{j})$   
 (3)  $\hat{j}$     (4)  $(\hat{k})$

**Answer (2)**

**Sol.** Direction of propagation of wave = Direction of  $\vec{E} \times \vec{B}$   
 = Direction of  $E_0 \hat{i} \times B_0 \hat{k}$   
 = Direction of  $E_0 B_0 (-\hat{j})$   
 =  $-\hat{j}$

12. If the Kinetic energy of a moving body becomes four times its initial Kinetic energy, then the percentage change in its momentum will be:

- (1) 200%                      (2) 100%  
 (3) 400%                      (4) 300%

**Answer (2)**

**Sol.** Momentum,  $P = \sqrt{2km}$

$$\% \text{ change in } P = \frac{P_2 - P_1}{P_1} \times 100\%$$

$$= \frac{\sqrt{2k_2 m} - \sqrt{2k_1 m}}{\sqrt{2k_1 m}} \times 100\%$$

$$= \frac{\sqrt{k_2} - \sqrt{k_1}}{\sqrt{k_1}} \times 100\%$$

$$= \left( \sqrt{\frac{k_2}{k_1}} - 1 \right) \times 100\%$$

$$= \left( \sqrt{\frac{4k_1}{k_1}} - 1 \right) \times 100\%$$

$$= 100\%$$

13. With what speed should a galaxy move outward with respect to earth so that the sodium-D line at wavelength 5890 Å is observed at 5896 Å?

- (1) 322 km/sec                      (2) 306 km/sec  
 (3) 336 km/sec                      (4) 296 km/sec

**Answer (2)**

**Sol.**  $\lambda_{\text{obs}} = \lambda_{\text{actual}} \sqrt{\frac{1 + \frac{V}{C}}{1 - \frac{V}{C}}}$

$$5896 = 5890 \left( 1 + \frac{V}{C} \right) \quad [\text{for } V \ll C]$$

$$V = \frac{C \times 6}{5890} = 306 \text{ km/s}$$

14. A body at rest is moved along a horizontal straight line by a machine delivering a constant power. The distance moved by the body in time 't' is proportional to

- (1)  $t^2$                                       (2)  $t^4$   
 (3)  $t^{\frac{3}{2}}$                                       (4)  $t^{\frac{3}{4}}$

**Answer (3)**

**Sol.**  $P = C$

So, K.E. = Pt

$$\frac{1}{2}mv^2 = Pt$$

$$\Rightarrow v = \sqrt{\frac{2P}{m}} \sqrt{t}$$

$$\Rightarrow v = C\sqrt{t}$$

$$\frac{ds}{dt} = C\sqrt{t}$$

$$\int ds = \int C\sqrt{t} dt$$

$$\text{Distance} = C t^{3/2}$$

15. A satellite is launched into a circular orbit of radius R around earth, while a second satellite is launched into a circular orbit of radius 1.02R. The percentage difference in the time periods of the two satellites is

- (1) 1.5                                      (2) 2.0  
 (3) 0.7                                      (4) 3.0

**Answer (4)**

**Sol.**  $T^2 = CR^3$

$$2 \frac{dT}{T} = 3 \frac{dR}{R}$$

$$\frac{dT}{T} = \frac{3}{2} \times \frac{0.02R}{R}$$

$$= 0.03$$

So % difference in the time period

$$= 0.03 \times 100 = 3\%$$

16. For a series LCR circuit with  $R = 100 \Omega$ ,  $L = 0.5 \text{ mH}$  and  $C = 0.1 \text{ pF}$  connected across 220 V - 50 Hz AC supply, the phase angle between current and supplied voltage and the nature of the circuit is

- (1)  $\approx 90^\circ$ , predominantly capacitive circuit  
 (2)  $0^\circ$ , resonance circuit  
 (3)  $0^\circ$ , resistive circuit  
 (4)  $\approx 90^\circ$ , predominantly inductive circuit

**Answer (1)**

**Sol.**  $X_L = 2 \times \pi \times 50 \times 0.5 \times 10^{-3}$   
 $= 0.05 \ \Omega$

$$X_C = \frac{1}{\omega C} = \frac{10^{12}}{2\pi \times 50 \times 0.1}$$

$$= \frac{10^{11}}{\pi} \Omega$$

$X_C - X_L \gg R$

17. An electron having de-Broglie wavelength  $\lambda$  is incident on a target in a X-ray tube. Cut-off wavelength of emitted X-ray is:

- (1) 0                      (2)  $\frac{hc}{mc}$
- (3)  $\frac{2m^2c^2\lambda^2}{h^2}$           (4)  $\frac{2mc\lambda^2}{h}$

**Answer (4)**

**Sol.**  $p = \frac{h}{\lambda}$

$$\frac{p^2}{2m} = \frac{hc}{\lambda_0}$$

$$\frac{h^2}{2m\lambda^2} = \frac{hc}{\lambda_0}$$

$$\lambda_0 = \frac{2mc\lambda^2}{h}$$

18. If time ( $t$ ), velocity ( $v$ ), and angular momentum ( $l$ ) are taken as the fundamental units. Then the dimension of mass ( $m$ ) in terms of  $t$ ,  $v$ , and  $l$  is:

- (1)  $[t^{-1}v^1l^{-2}]$                       (2)  $[t^{-1}v^{-2}l^1]$
- (3)  $[t^1v^2l^{-1}]$                       (4)  $[t^2v^{-1}l^1]$

**Answer (2)**

**Sol.**  $l = [M^1L^2T^{-1}]$

$$M = l^\alpha t^\beta v^\gamma$$

$$M = [M^1L^2T^{-1}]^\alpha [T]^\beta [L^1T^{-1}]^\gamma$$

$$\alpha = 1$$

$$-\alpha + \beta - \gamma = 0$$

$$2\alpha + \gamma = 0$$

$$\gamma = -2$$

$$\beta = -1$$

19. A particle is making simple harmonic motion along the X-axis. If at a distances  $x_1$  and  $x_2$  from the mean position the velocities of the particle are  $v_1$  and  $v_2$  respectively. The time period of its oscillation is given as:

$$(1) T = 2\pi \sqrt{\frac{x_2^2 - x_1^2}{v_1^2 + v_2^2}}$$

$$(2) T = 2\pi \sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}$$

$$(3) T = 2\pi \sqrt{\frac{x_2^2 + x_1^2}{v_1^2 - v_2^2}}$$

$$(4) T = 2\pi \sqrt{\frac{x_2^2 + x_1^2}{v_1^2 + v_2^2}}$$

**Answer (2)**

**Sol.**  $v_1^2 = \omega^2(A^2 - x_1^2)$

$$v_2^2 = \omega^2(A^2 - x_2^2)$$

$$v_1^2 - v_2^2 = \omega^2(x_2^2 - x_1^2)$$

$$v_1^2 - v_2^2 = \left[\frac{2\pi}{T}\right]^2 (x_2^2 - x_1^2)$$

$$T = 2\pi \sqrt{\frac{(x_2^2 - x_1^2)}{(v_1^2 - v_2^2)}}$$

20. A boy reaches the airport and finds that the escalator is not working. He walks up the stationary escalator in time  $t_1$ . If he remains stationary on a moving escalator then the escalator takes him up in time  $t_2$ . The time taken by him to walk up on the moving escalator will be:

- (1)  $t_2 - t_1$                       (2)  $\frac{t_1 t_2}{t_2 - t_1}$
- (3)  $\frac{t_1 + t_2}{2}$                       (4)  $\frac{t_1 t_2}{t_2 + t_1}$

**Answer (4)**

**Sol.** Let  $l$  be the length of escalator,  $v_1$  be the velocity of man w.r.t. escalator,  $v_2$  be the velocity of escalator.

$$t_1 = \frac{l}{v_1}$$

$$t_2 = \frac{l}{v_2}$$

$$t = \frac{l}{v_1 + v_2} = \frac{l}{\frac{l}{t_1} + \frac{l}{t_2}}$$

$$t = \frac{t_1 t_2}{t_1 + t_2}$$

### SECTION - II

**Numerical Value Type Questions:** This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. A certain metallic surface is illuminated by monochromatic radiation of wavelength  $\lambda$ . The stopping potential for photoelectric current for this radiation is  $3V_0$ . If the same surface is illuminated with a radiation of wavelength  $2\lambda$ , the stopping potential is  $V_0$ . The threshold wavelength of this surface for photoelectric effect is \_\_\_\_\_  $\lambda$ .

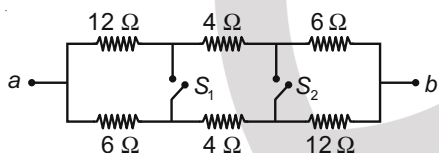
**Answer (4)**

$$\text{Sol. } e3V_0 = \frac{hc}{\lambda} - \frac{hc}{\lambda_0} \quad \dots(i)$$

$$eV_0 = \frac{hc}{2\lambda} - \frac{hc}{\lambda_0} \quad \dots(ii)$$

$$\Rightarrow \lambda_0 = 4\lambda$$

2. In the given figure switches  $S_1$  and  $S_2$  are in open condition. The resistance across  $ab$  when the switches  $S_1$  and  $S_2$  are closed is \_\_\_\_\_  $\Omega$ .



**Answer (10)**

$$\text{Sol. } R_{ab} = \frac{12 \times 6}{12 + 6} + \frac{4 \times 4}{4 + 4} + \frac{6 \times 12}{6 + 12} = 10 \Omega$$

3. Two bodies, a ring and a solid cylinder of same material are rolling down without slipping an inclined plane. The radii of the bodies are same. The ratio of velocity of the centre of mass at the bottom of the inclined plane of the ring to that of the cylinder is  $\frac{\sqrt{x}}{2}$ . Then, the value of  $x$  is \_\_\_\_\_.

**Answer (3)**

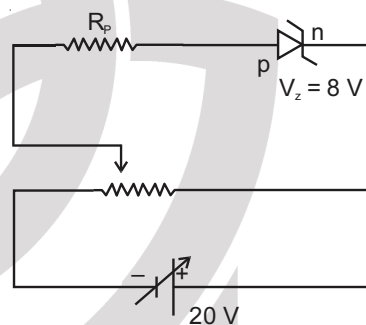
$$\text{Sol. } v = \sqrt{\frac{2gh}{1 + \frac{K^2}{R^2}}}$$

$$v_{\text{ring}} = \sqrt{gh}$$

$$v_{\text{cylinder}} = \sqrt{\frac{4}{3}gh}$$

$$\frac{v_{\text{ring}}}{v_{\text{cylinder}}} = \sqrt{\frac{3}{4}}$$

4. A zener diode having zener voltage 8 V and power dissipation rating of 0.5 W is connected across a potential divider arranged with maximum potential drop across zener diode is as shown in the diagram. The value of protective resistance  $R_p$  is \_\_\_\_\_  $\Omega$ .



**Answer (192)**

$$\text{Sol. } P = V_z I_z \Rightarrow I_z = \frac{1}{16} \text{ A}$$

$$I_z = \frac{V - V_z}{R_p}$$

$$R_p = (20 - 8) \times 16 = 192 \Omega$$

5. A body of mass ' $m$ ' is launched up on a rough inclined plane making an angle of  $30^\circ$  with the horizontal. The coefficient of friction between the body and plane is  $\frac{\sqrt{x}}{5}$  if the time of ascent is half of the time of descent. The value of  $x$  is \_\_\_\_\_.

**Answer (3)**

$$\text{Sol. } t_A = \sqrt{\frac{2\ell}{g(\sin\theta + \mu\cos\theta)}}$$

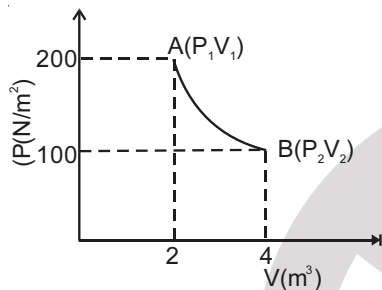
$$t_D = \sqrt{\frac{2\ell}{g(\sin\theta - \mu\cos\theta)}}$$

$$t_A = \frac{1}{2} t_D$$

$$\Rightarrow \mu = \frac{3}{5} \tan\theta = \frac{\sqrt{3}}{5}$$

6. One mole of an ideal gas at 27°C is taken from A to B as shown in the given PV indicator diagram. The work done by the system will be \_\_\_\_\_  $\times 10^{-1}$  J.

[Given :  $R = 8.3$  J / mole K,  $\ln 2 = 0.6931$ ] (Round off to the nearest integer)



**Answer (17258)**

**Sol.** Assuming process to be isothermal

$$W = nRT \ln \left( \frac{V_f}{V_i} \right)$$

$$= 8.3 \times 300 \times \ln (2)$$

$$= 1725.819 \text{ J}$$

$$= 17258 \times 10^{-1} \text{ J}$$

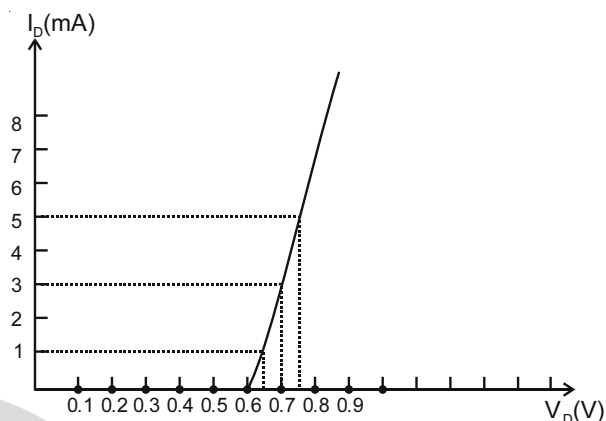
7. A series LCR circuit of  $R = 5 \Omega$ ,  $L = 20$  mH and  $C = 0.5 \mu\text{F}$  is connected across an AC supply of 250 V, having variable frequency. The power dissipated at resonance condition is \_\_\_\_\_  $\times 10^2$  W.

**Answer (125)**

**Sol.**  $P_r = \frac{V_{rms}^2}{R}$  as at resonance  $Z = R$

$$= \frac{250 \times 250}{50} = 125 \times 10^2 \text{ W}$$

8. For the forward biased diode characteristics shown in the figure, the dynamic resistance at  $I_D = 3$  mA will be \_\_\_\_\_  $\Omega$ .



**Answer (25)**

**Sol.**  $R_D = \frac{\Delta V}{\Delta I} = \frac{0.1}{4 \times 10^{-3}} = 25 \Omega$

9. A radioactive substance decays to  $\left(\frac{1}{16}\right)^{\text{th}}$  of its initial activity in 80 days. The half life of the radioactive substance expressed in days is \_\_\_\_\_.

**Answer (20)**

**Sol.**  $\frac{x}{16} = x \left(\frac{1}{2}\right)^n$

$$n = 4$$

$$\Rightarrow 4 \text{ half lives} = 80 \text{ days}$$

$$1 \text{ half life} = 20 \text{ days}$$

10. A body rotating with an angular speed of 600 rpm is uniformly accelerated to 1800 rpm in 10 sec. The number of rotations made in the process is \_\_\_\_\_.

**Answer (200)**

**Sol.**  $4\pi^2 \frac{(18^2 - 6^2)10^4}{60^2} = \frac{2 \times 1200}{10} \times \frac{\pi 2 \times \theta}{60}$

$$\theta = \frac{\pi \times 24 \times 12 \times 10^4}{120 \times 60} = \frac{\pi 24 \times 10^3}{60}$$

$$\theta = \frac{(12000)(2\pi)}{60}$$

$$= (200)2\pi$$

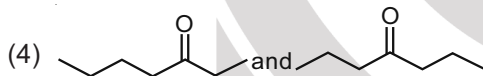
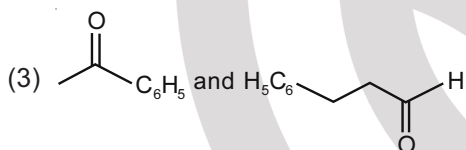
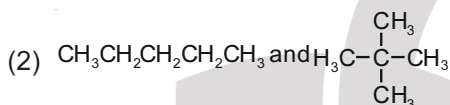
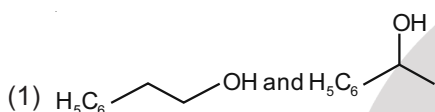
## PART-B : CHEMISTRY

### SECTION - I

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

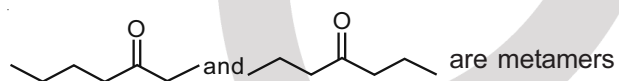
**Choose the correct answer :**

1. Which one of the following pairs of isomers is an example of metamerism?

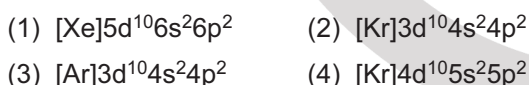


**Answer (4)**

**Sol.** Metamers have a common functional group and differ in the type of alkyl groups attached to the functional group



2. Outermost electronic configuration of a group 13 element, E, is  $4s^2, 4p^1$ . The electronic configuration of an element of p-block period-five placed diagonally to element, E is:



**Answer (4)**

**Sol.** The element E belongs to group-13 and period-4. The element belonging to period-5 and placed diagonally to E has the electronic configuration  $[\text{Kr}]4d^{10}5s^25p^2$

3. Which one of the following gases is reported to retard photosynthesis?
- (1)  $\text{NO}_2$                                       (2) CFCs  
 (3) CO    (4)  $\text{CO}_2$

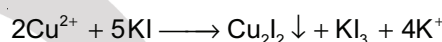
**Answer (1)**

**Sol.** The gas responsible to retard the rate of photosynthesis is  $\text{NO}_2$ . It is also known to damage plant leaves.

4.  $\text{Cu}^{2+}$  salt reacts with potassium iodide to give:
- (1)  $\text{Cu}_2\text{I}_2$                                       (2)  $\text{Cu}_2\text{I}_3$   
 (3)  $\text{Cu}(\text{I}_3)_2$                                   (4) CuI

**Answer (1) or (4)**

**Sol.**  $\text{Cu}^{2+}$  salt reacts with potassium iodide to form  $\text{Cu}_2\text{I}_2$  and  $\text{KI}_3$



$\text{Cu}_2\text{I}_2$  is sometimes also written as CuI.

5. Metallic sodium does not react normally with:
- (1) Ethyne  
 (2) Gaseous ammonia  
 (3) But-2-yne  
 (4) Tert-butyl alcohol

**Answer (3)**

**Sol.** Sodium metal is a very strong base. It can remove proton from all compounds having acidic hydrogen like water, alcohol, ammonia and terminal alkynes. But-2-yne does not have acidic hydrogen. Therefore, it will not react with sodium metal.

6. Which one of the following statements is not true about enzymes ?
- (1) The action of enzymes is temperature and pH specific  
 (2) Enzymes are non-specific for a reaction and substrate  
 (3) Enzymes work as catalysts by lowering the activation energy of a biochemical reaction  
 (4) Almost all enzymes are proteins

**Answer (2)**

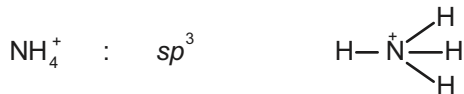
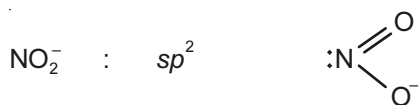
**Sol.** Enzymes are mostly proteins. They function as catalysts in biochemical reactions by lowering the energy of activation. They are highly specific w.r.t. temperature and pH in their action.

7. The hybridisations of the atomic orbitals of nitrogen in  $\text{NO}_2^-$ ,  $\text{NO}_2^+$  and  $\text{NH}_4^+$  respectively are :
- (1)  $sp^3$ ,  $sp^2$  and  $sp$                       (2)  $sp$ ,  $sp^2$  and  $sp^3$   
 (3)  $sp^2$ ,  $sp$  and  $sp^3$                       (4)  $sp^3$ ,  $sp$  and  $sp^2$

**Answer (3)**



**Sol.** The type of hybridisation of atomic orbitals of nitrogen in the given species is

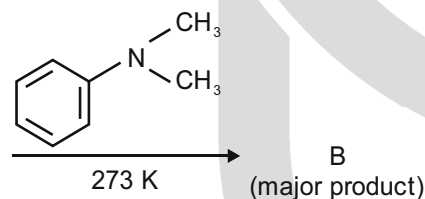
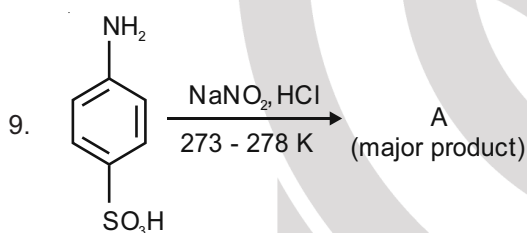


8. Bakelite is a cross-linked polymer of formaldehyde and :

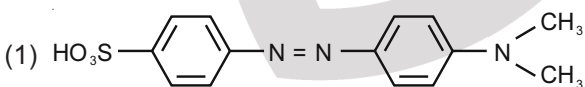
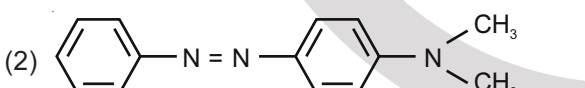
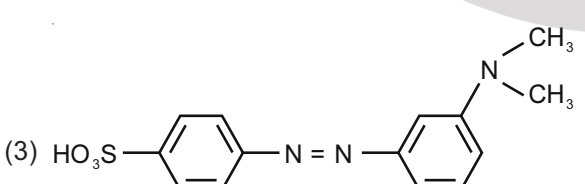
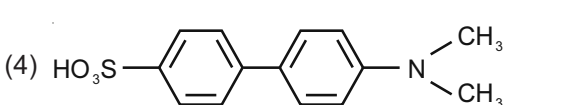
- (1) Buna-S (2) Novolac  
(3) Dacron (4) PHBV

**Answer (2)**

**Sol.** Novolac is a linear condensation polymer of phenol and formaldehyde. But, bakelite is a cross-linked polymer of formaldehyde and Novolac.

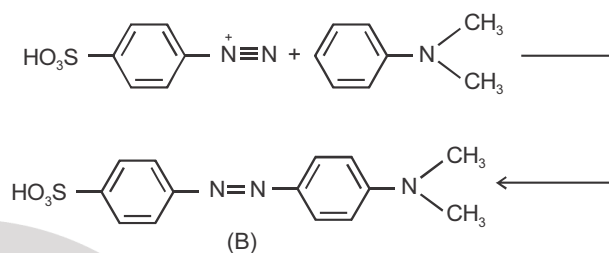
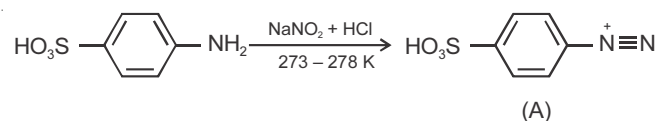


Consider the above reaction, compound B is :

- (1)   
 (2)   
 (3)   
 (4) 

**Answer (1)**

**Sol.**

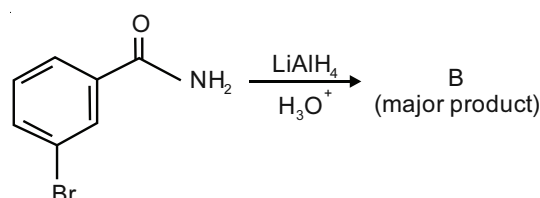
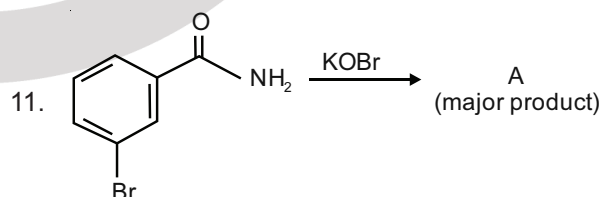
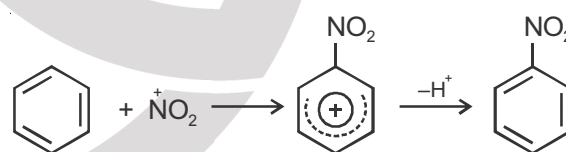
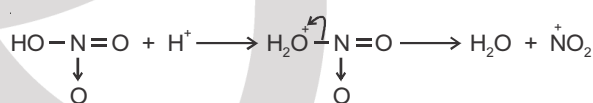
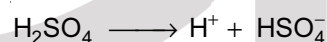


10. Benzene on nitration gives nitrobenzene in presence of  $\text{HNO}_3$  and  $\text{H}_2\text{SO}_4$  mixture, where :

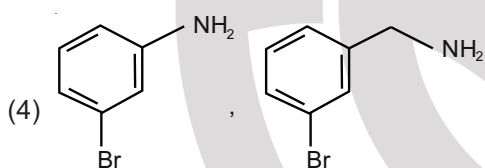
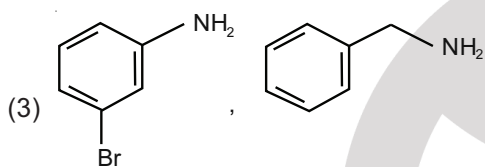
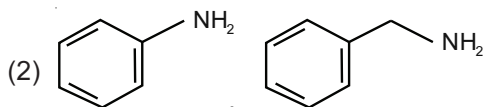
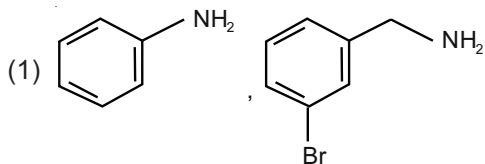
- (1)  $\text{HNO}_3$  acts as a base and  $\text{H}_2\text{SO}_4$  acts as an acid  
 (2) Both  $\text{H}_2\text{SO}_4$  and  $\text{HNO}_3$  act as acids  
 (3)  $\text{HNO}_3$  acts as an acid and  $\text{H}_2\text{SO}_4$  acts as a base  
 (4) Both  $\text{H}_2\text{SO}_4$  and  $\text{HNO}_3$  act as bases

**Answer (1)**

**Sol.** In the nitration of benzene using nitrating mixture,  $\text{HNO}_3$  acts as a base and  $\text{H}_2\text{SO}_4$  acts as an acid to generate  $\text{NO}_2^+$  ion.

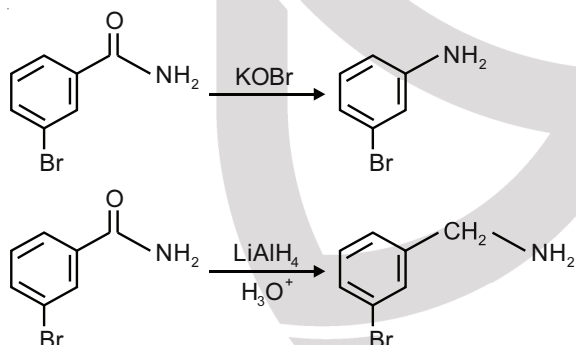


In the above reactions, product A and product B respectively are



**Answer (4)**

**Sol.**

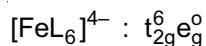


12. Spin only magnetic moment of an octahedral complex of  $\text{Fe}^{2+}$  in the presence of a strong field ligand in BM is

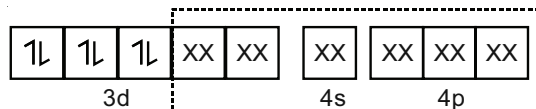
- (1) 3.46
- (2) 2.82
- (3) 0
- (4) 4.89

**Answer (3)**

**Sol.** The electronic configuration of  $\text{Fe}^{2+}$  is  $3d^64s^0$ . In presence of strong field ligand ( $L^{-1}$ ) resulting in the formation of inner orbital octahedral complex the electronic configuration of  $\text{Fe}^{2+}$  would be

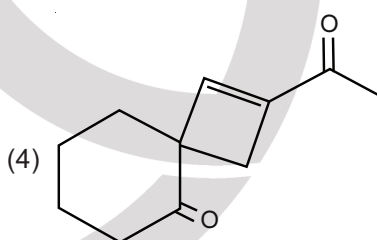
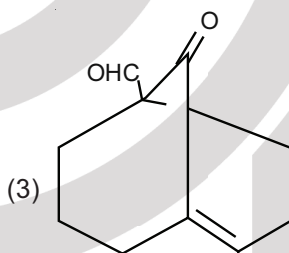
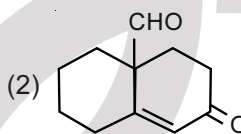
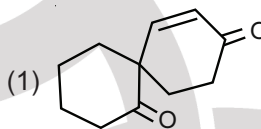
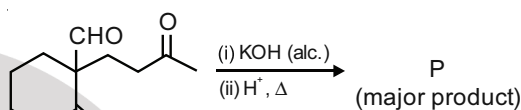


Magnetic moment,  $\mu = 0$



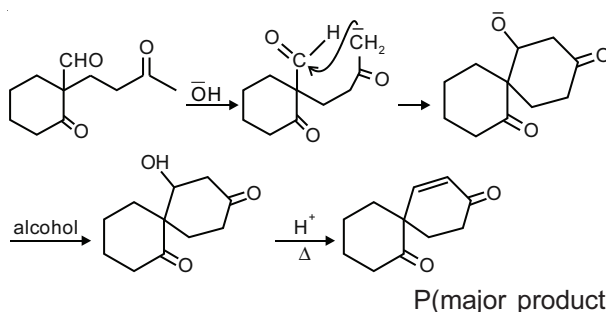
$d^2sp^3$  hybridisation

13. The major product (P) in the following reaction is



**Answer (1)**

**Sol.** This problem is based on intramolecular aldol condensation reaction.



14. Which one of the following species **doesn't** have a magnetic moment of 1.73 BM (spin only value)?

- (1)  $[\text{Cu}(\text{NH}_3)_4]\text{Cl}_2$       (2)  $\text{CuI}$   
 (3)  $\text{O}_2^-$                       (4)  $\text{O}_2^+$

**Answer (2)**

**Sol.**  $[\text{Cu}(\text{NH}_3)_4]\text{Cl}_2$  :  $\text{Cu}^{2+}$  : 1 unpaired electron;  $\mu = 1.73$  BM

$\text{CuI}$  :  $\text{Cu}^+$  : No unpaired electron ;  $\mu = 0$

$\text{O}_2^-$  : 1 unpaired electron;  $\mu = 1.73$  BM

$\text{O}_2^+$  : 1 unpaired electron;  $\mu = 1.73$  BM

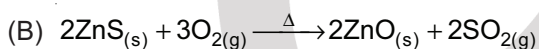
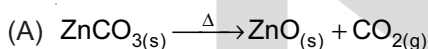
15. The single largest industrial application of dihydrogen is

- (1) In the synthesis of nitric acid  
 (2) Rocket fuel in space research  
 (3) In the synthesis of ammonia  
 (4) Manufacture of metal hydrides

**Answer (3)**

**Sol.** The single largest industrial application of dihydrogen is in the synthesis of ammonia which is mainly used in the manufacture of fertiliser.

16. Consider two chemical reactions (A) and (B) that take place during metallurgical process :



The correct option of names given to them respectively is

- (1) Both (A) and (B) are producing same product so both are calcination  
 (2) (A) is calcination and (B) is roasting  
 (3) Both (A) and (B) are producing same product so both are roasting  
 (4) (A) is roasting and (B) is calcination

**Answer (2)**

**Sol.** Heating of carbonate and hydroxide ore in absence or limited supply of air is calcination.

So, A is calcination

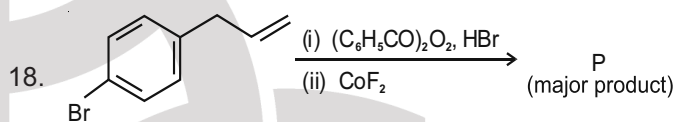
B is roasting

17. In Carius method, halogen containing organic compound is heated with fuming nitric acid in the presence of

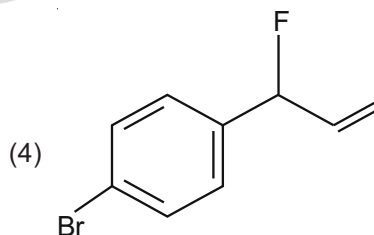
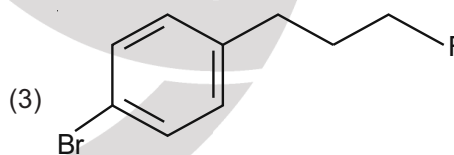
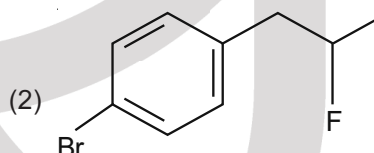
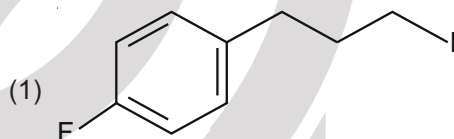
- (1)  $\text{HNO}_3$   
 (2)  $\text{CuSO}_4$   
 (3)  $\text{BaSO}_4$   
 (4)  $\text{AgNO}_3$

**Answer (4)**

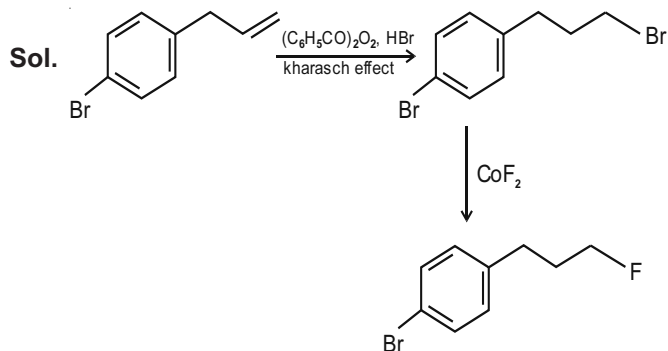
**Sol.** Halide ion reacts with silver nitrate to give silver halide precipitate.



Major product P of above reaction, is



**Answer (3)**



Select **correct** statement from the following

- (1) AgCl precipitates first because its  $K_{sp}$  is high.
- (2)  $Ag_2CrO_4$  precipitates first as its  $K_{sp}$  is low.
- (3)  $Ag_2CrO_4$  precipitates first because the amount of  $Ag^+$  needed is low.
- (4) AgCl will precipitate first as the amount of  $Ag^+$  needed to precipitate is low.

**Answer (4)**

**Sol.** Conc. of  $Cl^- = 0.1 \text{ M} = 10^{-1} \text{ M}$

Conc. of  $CrO_4^{2-} = 0.001 \text{ M} = 10^{-3} \text{ M}$

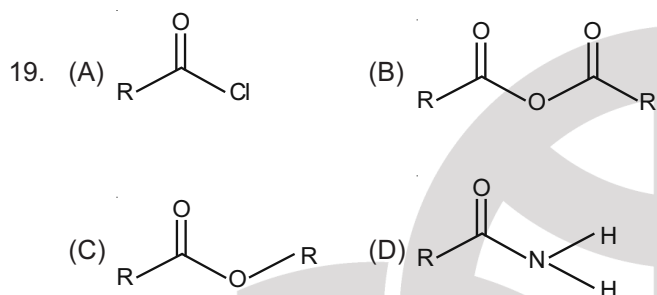
$$K_{sp}(AgCl) = [Ag^+][Cl^-]$$

$$[Ag^+]_{AgCl} = \frac{1.7 \times 10^{-10}}{10^{-1}} = 1.7 \times 10^{-9}$$

$$K_{sp}(Ag_2CrO_4) = [Ag^+]^2 [CrO_4^{2-}]$$

$$[Ag^+] = \sqrt{\frac{1.9 \times 10^{-12}}{10^{-3}}} = \sqrt{19} \times 10^{-4}$$

$\therefore$  AgCl will be precipitated first

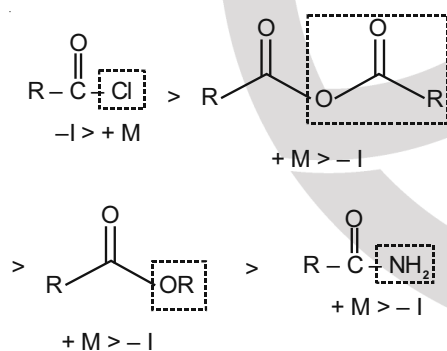


The **correct** order of their reactivity towards hydrolysis at room temperature is

- (1) (A) > (B) > (C) > (D)
- (2) (A) > (C) > (B) > (D)
- (3) (D) > (A) > (B) > (C)
- (4) (D) > (B) > (A) > (C)

**Answer (1)**

**Sol.** Order of hydrolysis



$-NH_2$  has greater donating power than  $-OR$  group making  $-C=O$  less electron deficient.

20. A solution is  $0.1 \text{ M}$  in  $Cl^-$  and  $0.001 \text{ M}$  in  $CrO_4^{2-}$ . Solid  $AgNO_3$  is gradually added to it. Assuming that the addition does not change in volume and  $K_{sp}(AgCl) = 1.7 \times 10^{-10} \text{ M}^2$  and  $K_{sp}(Ag_2CrO_4) = 1.9 \times 10^{-12} \text{ M}^3$ .

## SECTION - II

**Numerical Value Type Questions:** This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. 4 g equimolar mixture of NaOH and  $Na_2CO_3$  contains x g of NaOH and y g of  $Na_2CO_3$ . The value of x is \_\_\_\_\_ g.  
(Nearest integer)

**Answer (1)**

**Sol.** Mass of NaOH = x

$$\text{Moles of NaOH} = \frac{x}{40}$$

Mass of  $\text{Na}_2\text{CO}_3 = y$

Moles of  $\text{Na}_2\text{CO}_3 = \frac{y}{106}$

$$\frac{x}{40} = \frac{y}{106}$$

$$x + y = 4$$

$$x = 1.1, y = 2.9$$

$$x = 1.1 \approx 1 \text{ (nearest integer)}$$

2. The vapour pressures of A and B at  $25^\circ\text{C}$  are 90 mm Hg and 15 mm Hg respectively. If A and B are mixed such that the mole fraction of A in the mixture is 0.6, then the mole fraction of B in the vapour phase is  $x \times 10^{-1}$ . The value of x is \_\_\_\_\_. (Nearest integer)

**Answer (1)**

**Sol.**  $x_A = 0.6$

$$\begin{aligned} P_T &= x_A P_A^\circ + x_B P_B^\circ \\ &= 0.6 \times 90 + 0.4 \times 15 \\ &= 54 + 6 = 60 \end{aligned}$$

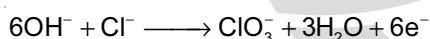
$$\begin{aligned} x_A P_A^\circ &= y_A P_T \\ 0.6 \times 90 &= y_A (60) \end{aligned}$$

$$\Rightarrow y_A = 0.9$$

$$y_B = 0.1 = 1 \times 10^{-1}$$

$$\therefore x = 1$$

3. Potassium chlorate is prepared by electrolysis of KCl in basic solution as shown by following equation.



A current of xA has to be passed for 10 h to produce 10.0 g of potassium chlorate. The value of x is \_\_\_\_\_. (Nearest integer)

(Molar mass of  $\text{KClO}_3 = 122.6 \text{ g mol}^{-1}$ ,  $F = 96500 \text{ C}$ )

**Answer (1)**

**Sol.** Mass of  $\text{KClO}_3 = 10 \text{ g}$

$$\Rightarrow \frac{10}{122.6} = \frac{x \times 10 \times 3600}{6 \times 96500}$$

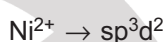
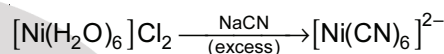
$$\Rightarrow x = 1.311 \approx 1 \text{ (nearest integer)}$$

4. An aqueous solution of  $\text{NiCl}_2$  was heated with excess sodium cyanide in presence of strong oxidizing agent to form  $[\text{Ni}(\text{CN})_6]^{2-}$ . The total change in number of unpaired electrons on metal centre is \_\_\_\_\_.

**Answer (2)**

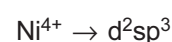
**Sol.**  $\text{NiCl}_2(\text{aq}) \rightarrow [\text{Ni}(\text{H}_2\text{O})_6]\text{Cl}_2$

$\text{H}_2\text{O}$  : weak field ligand



unpaired electrons

$$= 2$$



$\text{d}^6$

number of unpaired

electrons = 0

$\therefore$  Total change in number of unpaired electrons = 2

5. For a given chemical reaction  $\text{A} \rightarrow \text{B}$  at 300 K the free energy change is  $-49.4 \text{ kJ mol}^{-1}$  and the enthalpy of reaction is  $51.4 \text{ kJ mol}^{-1}$ . The entropy change of the reaction is \_\_\_\_\_  $\text{J K}^{-1} \text{ mol}^{-1}$ .

**Answer (336)**

**Sol.**  $\Delta G = -49.4 \text{ kJ/mol}$

$$\Delta H = 51.4 \text{ kJ/mol}$$

$$\Delta G = \Delta H - T\Delta S$$

$$-49400 = 51400 - 300\Delta S$$

$$\Delta S = \frac{+100800}{300} = 336 \text{ JK}^{-1} \text{ mol}^{-1}$$

6. When 0.15 g of an organic compound was analyzed using Carius method for estimation of bromine, 0.2397 g of  $\text{AgBr}$  was obtained. The percentage of bromine in the organic compound is \_\_\_\_\_. (Nearest integer)

Atomic mass :
Silver = 108
Bromine = 80

**Answer (68)**

**Sol.** % of Br =  $\frac{\text{Atomic mass of Br} \times m_1}{\text{molecular mass of AgBr} \times m} \times 100$

m = mass of organic compound taken

$m_1$  = mass of AgBr obtained.

$$\therefore \% \text{ of Br} = \frac{80 \times 0.2397}{188 \times 0.15} \times 100$$

$$= 68$$

7. The wavelength of electrons accelerated from rest through a potential difference of 40 kV is  $x \times 10^{-2}$  m. The value of x is \_\_\_\_\_. (Nearest integer)

Given: Mass of electron =  $9.1 \times 10^{-31}$  kg

Charge on an electron =  $1.6 \times 10^{-19}$  C

Planck's constant =  $6.63 \times 10^{-34}$  Js

**Answer (6)**

**Sol.** Wavelength of electron is given by

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

Here q = charge on electron, V = potential difference

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 40 \times 10^3}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{1164.8 \times 10^{-47}}} = 6.144 \times 10^{-12} \approx 6 \times 10^{-12}$$

x = 6

8. Diamond has a three dimensional structure of C atoms formed by covalent bonds. The structure of diamond has face centred cubic lattice where 50% of the tetrahedral voids are also occupied by carbon atoms. The number of carbon atoms present per unit cell of diamond is \_\_\_\_\_.

**Answer (8)**

**Sol.** Effective atoms of carbon from FCC lattice =

$$8 \times \frac{1}{8} + 6 \times \frac{1}{2} = 4$$

Number of atoms occupied in 1 unit cell (TV)

$$= 4 \times 1 = 4$$

$$\therefore \text{Total atoms} = 4 + 4 = 8$$

9.  $\text{PCl}_5(\text{g}) \rightarrow \text{PCl}_3(\text{g}) + \text{Cl}_2(\text{g})$

In the above first order reaction the concentration of  $\text{PCl}_5$  reduces from initial concentration  $50 \text{ mol L}^{-1}$  to  $10 \text{ mol L}^{-1}$  in 120 minutes at 300 K. The rate constant for the reaction at 300 K is  $x \times 10^{-2} \text{ min}^{-1}$ . The value of x is \_\_\_\_\_.

Given:  $\log 5 = 0.6989$

**Answer (1)**

**Sol.**  $a_0 = 50 \text{ mol L}^{-1}$

$a_t = 10 \text{ mol L}^{-1}$

$$K = \frac{1}{120} \times 2.303 \log \frac{50}{10}$$

$$= 0.01341$$

$$= 1.34 \times 10^{-2} \text{ min}^{-1}$$

$$x = 1.34 \approx 1 \text{ (nearest integer)}$$

10. 100 ml of 0.0018% (w/v) solution of  $\text{Cl}^-$  ion was the minimum concentration of  $\text{Cl}^-$  required to precipitate a negative sol in one h. The coagulating value of  $\text{Cl}^-$  ion is \_\_\_\_\_.

(Nearest integer)

**Answer (1) Bonus\***

Assuming coagulating of positive sol

**Sol.** 0.0018 g of  $\text{Cl}^-$  in 100 ml solution

$$\text{mmoles in 100 ml} = \frac{0.0018}{35.5} \times 1000$$

$$= 0.0507$$

$$\therefore \text{Coagulating value} = \frac{0.0507}{0.1}$$

$$= 0.507 = 0.51$$

$$\approx 1 \text{ (nearest integer)}$$

## PART-C : MATHEMATICS

### SECTION - I

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer :**

1. Let  $r_1$  and  $r_2$  be the radii of the largest and smallest circles, respectively, which pass through the point  $(-4, 1)$  and having their centres on the circumference of the circle  $x^2 + y^2 + 2x + 4y - 4 = 0$ . If

$$\frac{r_1}{r_2} = a + b\sqrt{2}, \text{ then } a + b \text{ is equal to}$$

- (1) 3    (2) 7  
(3) 11                                         (4) 5

**Answer (4)**

**Sol.**  $C \equiv (x + 1)^2 + (y + 2)^2 = 9$

Distance between  $(-1, -2)$  and  $(-4, 1)$

$$\sqrt{3^2 + 3^2} = \sqrt{18}$$

Maximum radius of required circle =  $\sqrt{18} + 3$

Minimum radius of required circle =  $\sqrt{18} - 3$

$$\frac{r_1}{r_2} = \frac{3\sqrt{2} + 3}{3\sqrt{2} - 1} = \frac{\sqrt{2} + 1}{\sqrt{2} - 1} = \frac{(\sqrt{2} + 1)^2}{1} = 3 + 2\sqrt{2}$$

2. If the mean and variance of six observations 7, 10, 11, 15, a, b are 10 and  $\frac{20}{3}$ , respectively, then the value of  $|a - b|$  is equal to

- (1) 7    (2) 1  
(3) 11                                         (4) 9

**Answer (2)**

**Sol.** Given  $\frac{7 + 10 + 11 + 15 + a + b}{6} = 10$

$$\Rightarrow a + b = 17 \quad \dots(i)$$

$$\& \frac{7^2 + 10^2 + 11^2 + 15^2 + a^2 + b^2}{6} - 10^2 = \frac{20}{3}$$

$$\frac{4095 + a^2 + b^2}{6} = \frac{320}{3}$$

$$\Rightarrow a^2 + b^2 = 145 \quad \dots(ii)$$

$$\therefore a^2 + b^2 + 2ab = 289$$

$$\Rightarrow 2ab = 144$$

$$(a - b)^2 = 145 - 144$$

$$\therefore (a - b) = 1$$

3. The value of  $\tan\left(2\tan^{-1}\left(\frac{3}{5}\right) + \sin^{-1}\left(\frac{5}{13}\right)\right)$  is equal to

- (1)  $\frac{220}{21}$                                       (2)  $\frac{151}{63}$   
(3)  $\frac{-181}{69}$                                         (4)  $\frac{-291}{76}$

**Answer (1)**

**Sol.**  $2\tan^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{\frac{6}{5}}{1 - \frac{9}{25}}\right) = \tan^{-1}\left(\frac{\frac{6}{5}}{\frac{16}{25}}\right) = \tan^{-1}\frac{15}{8}$

$$\therefore 2\tan^{-1}\left(\frac{3}{5}\right) + \sin^{-1}\left(\frac{5}{13}\right) = \tan^{-1}\left(\frac{15}{8}\right) + \tan^{-1}\left(\frac{5}{12}\right)$$

$$= \tan^{-1}\left(\frac{\frac{15}{8} + \frac{5}{12}}{1 - \frac{15}{8} \cdot \frac{5}{12}}\right)$$

$$= \tan^{-1}\left(\frac{180 + 40}{21}\right) = \tan^{-1}\left(\frac{220}{21}\right)$$

4. For the natural numbers m, n, if  $(1 - y)^m (1 + y)^n = 1 + a_1 y + a_2 y^2 + \dots + a_{m+n} y^{m+n}$  and  $a_1 = a_2 = 10$ , then the value of  $(m + n)$  is equal to

- (1) 64                                         (2) 80  
(3) 88                                         (4) 100

**Answer (2)**

**Sol.**  $(1 - y)^m (1 + y)^n = 1 + a_1 y + a_2 y^2 + \dots + a_{m+n} y^{m+n}$

Given  $(a_1 = a_2 = 10)$

$$(1 - my + {}^m C_2 y^2 + \dots)(1 + ny + {}^n C_2 y^2 + \dots) = 1 + a_1 y + a_2 y^2 + \dots$$

$$\Rightarrow n - m = 10 \quad \dots(i)$$

$$\Rightarrow {}^m C_2 + {}^n C_2 - mn = 10 \quad \dots(ii)$$

$$\frac{m(m-1)}{2} + \frac{n(n-1)}{2} - mn = 10$$

$$\Rightarrow \frac{m^2 - m}{2} + \frac{(10+m)(9+m)}{2} - m(10+m) = 10$$

$$\Rightarrow m^2 - m + m^2 + 19m + 90 - 2(m^2 + 10m) = 20$$

$$\Rightarrow 18m + 90 - 20m = 20$$

$$\Rightarrow 2m = 70$$

$$\Rightarrow m = 35 \& n = 45$$

$$m + n = 80$$

5. Let  $y = y(x)$  satisfies the equation  $\frac{dy}{dx} - |A| = 0$ , for

all  $x > 0$ , where  $A = \begin{bmatrix} y & \sin x & 1 \\ 0 & -1 & 1 \\ 2 & 0 & \frac{1}{x} \end{bmatrix}$ . If  $y(\pi) = \pi + 2$ ,

then the value of  $y\left(\frac{\pi}{2}\right)$  is

- (1)  $\frac{\pi}{2} - \frac{4}{\pi}$                       (2)  $\frac{\pi}{2} + \frac{4}{\pi}$   
 (3)  $\frac{3\pi}{2} - \frac{1}{\pi}$                       (4)  $\frac{\pi}{2} - \frac{1}{\pi}$

**Answer (2)**

**Sol.**  $|A| = \frac{-y}{x} - \sin x(-2) + 1(2)$

$$= 2 + 2\sin x - \frac{y}{x}$$

$$\frac{dy}{dx} + \frac{y}{x} = 2 + 2\sin x$$

I.f =  $e^{\int \frac{1}{x} dx} = x$

$$\int d(xy) = \int 2x(1 + \sin x) dx$$

$$\Rightarrow xy = x^2 - 2x \cos x + \int 2 \cos x dx$$

$$\Rightarrow xy = x^2 - 2x \cos x + 2 \sin x + c$$

$$\therefore y(\pi) = \pi + 2$$

$$\Rightarrow \pi(\pi + 2) = \pi^2 - 2\pi(-1) + 0 + c$$

$$\Rightarrow c = 0$$

For  $y\left(\frac{\pi}{2}\right)$

$$\frac{\pi}{2}y = \frac{\pi^2}{4} - \frac{2\pi}{2}(0) + 2$$

$$\Rightarrow y\left(\frac{\pi}{2}\right) = \frac{\pi}{2} + \frac{4}{\pi}$$

6. If sum of the first 21 terms of the series  $\log_{9^{1/2}} x + \log_{9^{1/3}} x + \log_{9^{1/4}} x + \dots$ , where  $x > 0$  is 504, then  $x$  is equal to

- (1) 81                                      (2) 243  
 (3) 9                                        (4) 7

**Answer (1)**

**Sol.**  $\log_{9^{1/2}} x + \log_{9^{1/3}} x + \log_{9^{1/4}} x + \dots$

$$\Rightarrow \log_9 x^2 + \log_9 x^3 + \log_9 x^4 + \dots$$

$$\Rightarrow \log_9 (x^{2+3+\dots+21\text{-terms}}) = 504$$

$$\Rightarrow 252 \log_9 x = 504$$

$$\Rightarrow x = 9^2 = 81$$

7. Let in a right angled triangle, the smallest angle be  $\theta$ . If a triangle formed by taking the reciprocal of its sides is also a right angled triangle, then  $\sin \theta$  is equal to :

- (1)  $\frac{\sqrt{5}+1}{4}$                                       (2)  $\frac{\sqrt{2}-1}{2}$   
 (3)  $\frac{\sqrt{5}-1}{2}$                                       (4)  $\frac{\sqrt{5}-1}{4}$

**Answer (3)**

**Sol.** Let a  $\Delta ABC$  having  $C = 90^\circ$  and  $A = \theta$

$$\frac{\sin \theta}{a} = \frac{\cos \theta}{b} = \frac{1}{c} \dots(i)$$

Also for triangle of reciprocals

$$\cos A = \frac{\left(\frac{1}{c}\right)^2 + \left(\frac{1}{b}\right)^2 - \left(\frac{1}{a}\right)^2}{2\left(\frac{1}{c}\right)\left(\frac{1}{b}\right)}$$

$$\frac{1}{c^2} + \frac{1}{(c \cos \theta)^2} = \frac{1}{(c \sin \theta)^2}$$

$$\Rightarrow 1 + \sec^2 \theta = \operatorname{cosec}^2 \theta$$

$$\Rightarrow \frac{1}{4} = \frac{\cos^2 \theta}{4 \sin^2 \theta \cos^2 \theta}$$

$$\Rightarrow \frac{1}{4} = \frac{\cos^2 \theta}{\sin^2 2\theta}$$

$$\Rightarrow 1 - \cos^2 2\theta = 4 \cos 2\theta$$

$$\cos^2 2\theta + 4 \cos 2\theta - 1 = 0$$

$$\cos 2\theta = \frac{-4 \pm \sqrt{16+4}}{2}$$

$$\cos 2\theta = -2 \pm \sqrt{5}$$

$$\cos 2\theta = \sqrt{5} - 2 = 1 - 2 \sin^2 \theta$$

$$\Rightarrow 2 \sin^2 \theta = 3 - \sqrt{5}$$

$$\Rightarrow \sin^2 \theta = \frac{3 - \sqrt{5}}{2}$$

$$\Rightarrow \sin \theta = \frac{\sqrt{5}-1}{2}$$

8. Let  $g(t) = \int_{-\pi/2}^{\pi/2} \cos\left(\frac{\pi}{4}t + f(x)\right) dx$ ,

where  $f(x) = \log_e\left(x + \sqrt{x^2 + 1}\right)$ ,  $x \in \mathbf{R}$ . Then which one of the following is correct?

- (1)  $g(1) + g(0) = 0$                       (2)  $g(1) = \sqrt{2}g(0)$   
 (3)  $\sqrt{2}g(1) = g(0)$                       (4)  $g(1) = g(0)$

**Answer (3)**



**Sol.**  $\therefore f(x) = \ln(x + \sqrt{x^2 + 1})$

$\therefore f(x) + f(-x) = \ln(\sqrt{x^2 + 1} + x) + \ln(\sqrt{x^2 + 1} - x)$

$\therefore f(x) + f(-x) = 0 \quad \dots(i)$

$\therefore g(t) = \int_{-\pi/2}^{\pi/2} \cos\left(\frac{\pi}{4}t + f(x)\right) dx.$

$= \int_0^{\pi/2} \left\{ \cos\left(\frac{\pi}{4}t + f(x)\right) + \cos\left(\frac{\pi}{4}t + f(-x)\right) \right\} dx.$

$= \int_0^{\pi/2} \left\{ \cos\left(\frac{\pi t}{4} + f(x)\right) + \cos\left(\frac{\pi t}{4} - f(x)\right) \right\} dx.$

$g(t) = 2 \int_0^{\pi/2} \cos \frac{\pi t}{4} \cdot \cos(f(x)) dx.$

$\therefore g(1) = \sqrt{2} \int_0^{\pi/2} \cos(f(x)) dx$

and  $g(0) = 2 \int_0^{\pi/2} \cos(f(x)) dx.$

$\therefore \sqrt{2} g(1) = g(0)$

9. In a triangle ABC, if  $|\overline{BC}| = 3$ ,  $|\overline{CA}| = 5$  and  $|\overline{BA}| = 7$ , then the projection of the vector  $\overline{BA}$  on  $\overline{BC}$  is equal to:

(1)  $\frac{13}{2}$

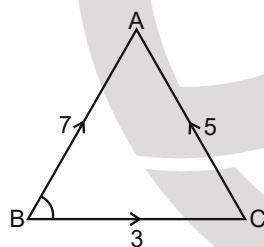
(2)  $\frac{19}{2}$

(3)  $\frac{15}{2}$

(4)  $\frac{11}{2}$

**Answer (4)**

**Sol.** Projection of  $\overline{BA}$  on  $\overline{BC}$



$= \frac{|\overline{BA} \cdot \overline{BC}|}{|\overline{BC}|}$

$= \frac{|\overline{BA}| \cdot |\overline{BC}| \cos B}{|\overline{BC}|}$

$= 7 \cdot \left( \frac{7^2 + 3^2 - 5^2}{2 \times 7 \times 3} \right)$

$= \frac{11}{2}$  units

10. If  $[x]$  denotes the greatest integer less than or equal to  $x$ , then the value of the integral

$\int_{-\pi/2}^{\pi/2} [[x] - \sin x] dx$  is equal to :

(1)  $-\pi$

(2) 0

(3)  $\pi$

(4) 1

**Answer (1)**

**Sol.**  $I = \int_{-\pi/2}^{\pi/2} [[x] - \sin x] dx$

$= \int_{-\pi/2}^{\pi/2} ([x] + [-\sin x]) dx$

$= \int_0^{\pi/2} ([x] + [-\sin x] + [-x] + [\sin x]) dx$

$= \int_0^{\pi/2} (-2) dx$

$= -\pi$

11. Let P be a variable point on the parabola  $y = 4x^2 + 1$ . Then, the locus of the mid-point of the point P and the foot of the perpendicular drawn from the point P to the line  $y = x$  is :

(1)  $(3x - y)^2 + 2(x - 3y) + 2 = 0$

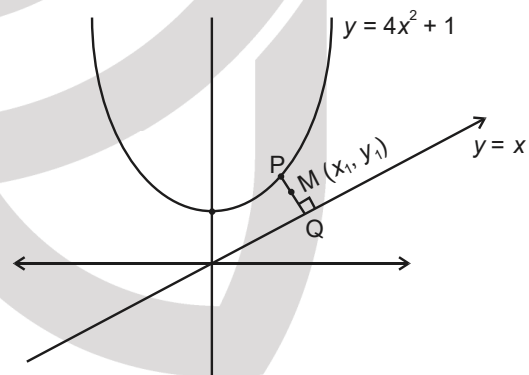
(2)  $2(3x - y)^2 + (x - 3y) + 2 = 0$

(3)  $2(x - 3y)^2 + (3x - y) + 2 = 0$

(4)  $(3x - y)^2 + (x - 3y) + 2 = 0$

**Answer (2)**

**Sol.**



Let coordinate of mid-point M of PQ be  $(x_1, y_1)$

Let coordinate of Q be  $(\alpha, \beta)$ .

$\therefore \frac{\alpha - x_1}{1} = \frac{\beta - y_1}{-1} = \frac{-(x_1 - y_1)}{2}$

$\therefore Q = (\alpha, \beta) = \left( \frac{x_1 + y_1}{2}, \frac{x_1 + y_1}{2} \right)$

and coordinate of P =  $\left( \frac{3x_1 - y_1}{2}, \frac{3y_1 - x_1}{2} \right)$

$\therefore$  P lies on parabola

$\therefore \frac{3y_1 - x_1}{2} = 4 \left( \frac{3x_1 - y_1}{2} \right)^2 + 1$

$\therefore$  Required locus is  $2(3x - y)^2 + (x - 3y) + 2 = 0$

12. If the real part of the complex number  $(1 - \cos\theta + 2i\sin\theta)^{-1}$  is  $\frac{1}{5}$  for  $\theta \in (0, \pi)$ , then the value of the integral  $\int_0^\theta \sin x \, dx$  is equal to :
- (1) 1 (2) 2  
(3) 0 (4) -1

**Answer (1)**

**Sol.**  $Z = \frac{1}{1 - \cos\theta + 2i\sin\theta} = \frac{(1 - \cos\theta) - 2i\sin\theta}{(1 - \cos\theta)^2 + 4\sin^2\theta}$

$\therefore \operatorname{Re}(Z) = \frac{1 - \cos\theta}{2 - 2\cos\theta + 3\sin^2\theta} = \frac{1}{5}$

$\therefore 5 - 5\cos\theta = 2 - 2\cos\theta + 3\sin^2\theta$   
 $3\cos\theta(1 - \cos\theta) = 0$

$\therefore \theta = \frac{\pi}{2}$ , when  $\theta \in (0, \pi)$

$\therefore \int_0^{\pi/2} \sin x \, dx = \int_0^{\pi/2} \sin x \, dx = 1$

13. The lines  $x = ay - 1 = z - 2$  and  $x = 3y - 2 = bz - 2$ , ( $ab \neq 0$ ) are coplanar, if
- (1)  $a = 2, b = 2$  (2)  $a = 2, b = 3$   
 (3)  $a = 1, b \in \mathbf{R} - \{0\}$  (4)  $b = 1, a \in \mathbf{R} - \{0\}$

**Answer (4)**

**Sol.** Lines are  $x = ay - 1 = z - 2$

$\therefore \frac{x}{1} = \frac{y - \frac{1}{a}}{\frac{1}{a}} = \frac{z - 2}{1} \dots (i)$

and  $x = 3y - 2 = bz - 2$

$\therefore \frac{x}{1} = \frac{y - \frac{2}{3}}{\frac{1}{3}} = \frac{z - \frac{2}{b}}{\frac{1}{b}} \dots (ii)$

$\therefore$  lines are co-planar

$\therefore \begin{vmatrix} 0 & -\frac{1}{a} + \frac{2}{3} & -2 + \frac{2}{b} \\ 1 & \frac{1}{a} & 1 \\ 1 & \frac{1}{3} & \frac{1}{b} \end{vmatrix} = 0$

$\therefore \begin{vmatrix} 0 & \frac{2}{3} - \frac{1}{a} & \frac{2}{b} - 2 \\ 0 & \frac{1}{a} - \frac{1}{3} & 1 - \frac{1}{b} \\ 1 & \frac{1}{3} & \frac{1}{b} \end{vmatrix} = 0$

$\therefore \frac{1}{a} - \frac{1}{ab} = 0$

$\Rightarrow b = 1$  and  $a \in \mathbf{R} - \{0\}$

14. Consider the following three statements

- (A) If  $3 + 3 = 7$  then  $4 + 3 = 8$ .  
 (B) If  $5 + 3 = 8$  then earth is flat.  
 (C) If both (A) and (B) are true then  $5 + 6 = 17$ .  
 Then, which of the following statements is correct?

- (1) (A) and (B) are false while (C) is true  
 (2) (A) is false, but (B) and (C) are true  
 (3) (A) and (C) are true while (B) is false  
 (4) (A) is true while (B) and (C) are false

**Answer (3)**

**Sol.**  $\therefore 3 + 3 = 7$  is false and  $4 + 3 = 8$  is false

then statement (A) is true

For (B)  $5 + 3 = 8$  is true and earth is flat is false.

Then statement (B) is false

For (C) if A and B are true then  $5 + 6 = 17$  is false, then (C) is true.

$\therefore$  (A) and (C) are true and (B) is false.

15. Let  $f: \mathbf{R} - \left\{\frac{\alpha}{6}\right\} \rightarrow \mathbf{R}$  be defined by  $f(x) = \frac{5x+3}{6x-\alpha}$ .

Then the value of  $\alpha$  for which  $(fof)(x) = x$ , for all

$x \in \mathbf{R} - \left\{\frac{\alpha}{6}\right\}$ , is

- (1) 5 (2) 8  
 (3) No such  $\alpha$  exists (4) 6

**Answer (1)**

**Sol.** For  $f(f(x)) = x$

$f(x) = f^{-1}(x)$

finding  $f^{-1}(x)$

$y = \frac{3x+3}{6x-\alpha}$

$\Rightarrow f^{-1}(x) = \frac{3+\alpha x}{6x-5}$

$\therefore f(x) = f^{-1}(x)$  gives

$\frac{3+\alpha x}{6x-5} = \frac{5x+3}{6x-\alpha}$

$\Rightarrow (30 - 6\alpha)x^2 + (\alpha^2 - 25)x + (3\alpha - 15) = 0$

$\therefore \alpha = 5$

16. If  $f: \mathbf{R} \rightarrow \mathbf{R}$  is given by  $f(x) = x + 1$ , then the value of  $\lim_{n \rightarrow \infty} \frac{1}{n} \left[ f(0) + f\left(\frac{5}{n}\right) + f\left(\frac{10}{n}\right) + \dots + f\left(\frac{5(n-1)}{n}\right) \right]$ , is

- (1)  $\frac{7}{2}$  (2)  $\frac{3}{2}$   
 (3)  $\frac{5}{2}$  (4)  $\frac{1}{2}$

**Answer (1)**

**Sol.**  $f(0) + f\left(\frac{5}{n}\right) + f\left(\frac{10}{n}\right) + \dots + f\left(\frac{5(n-1)}{n}\right)$

$$\Rightarrow 1 + 1 + \frac{5}{n} + 1 + \frac{10}{n} + \dots + 1 + \frac{5(n-1)}{n}$$

$$\Rightarrow n + \frac{5(n-1)n}{2} = \frac{2n + 5n - 5}{2} = \frac{7n - 5}{2}$$

$$\lim_{n \rightarrow \infty} \frac{1}{n} \left( \frac{7n - 5}{2} \right) = \frac{7}{2}$$

17. Consider the line L given by the equation  $\frac{x-3}{2} = \frac{y-1}{1} = \frac{z-2}{1}$ . Let Q be the mirror image of the point (2, 3, -1) with respect to L. Let a plane P be such that it passes through Q, and the line L is perpendicular to P. Then which of the following points is on the plane P?

- (1) (-1, 1, 2) (2) (1, 2, 2)  
 (3) (1, 1, 1) (4) (1, 1, 2)

**Answer (2)**

**Sol.** L is normal to P and plane P will pass through (2, 3, -1)

Equation of P is  $2x + y + z = \lambda = 6$  which is satisfied by (1, 2, 2)

18. The sum of all the local minimum values of the twice differentiable function  $f: \mathbf{R} \rightarrow \mathbf{R}$  defined by

$$f(x) = x^3 - 3x^2 - \frac{3f''(2)}{2}x + f''(1)$$

- (1) -22 (2) 5  
 (3) -27 (4) 0

**Answer (3)**

**Sol.**  $f'(x) = 6x - 6$

$$f'(2) = 6, f'(1) = 0$$

$$f(x) = x^3 - 3x^2 - 9x$$

$$f'(x) = 3x^2 - 6x - 9 = 3(x+1)(x-3)$$

Local min at  $x = 3$

$$\text{Local min value} = f(3) = -27$$

19. Let A, B and C be three events such that the probability that exactly one of A and B occurs is  $(1 - k)$ , the probability that exactly one of B and C occurs is  $(1 - 2k)$ , the probability that exactly one of C and A occurs is  $(1 - k)$  and the probability of all A, B and C occur simultaneously is  $k^2$ , where  $0 < k < 1$ . Then the probability that at least one of A, B and C occur is

- (1) Greater than  $\frac{1}{2}$   
 (2) Exactly equal to  $\frac{1}{2}$   
 (3) Greater than  $\frac{1}{8}$  but less than  $\frac{1}{4}$   
 (4) Greater than  $\frac{1}{4}$  but less than  $\frac{1}{2}$

**Answer (1)**

$$\text{Sol. } P(A) + P(B) - 2P(A \cap B) = 1 - k \quad \dots(i)$$

$$P(B) + P(C) - 2P(B \cap C) = 1 - 2k \quad \dots(ii)$$

$$P(C) + P(A) - 2P(C \cap A) = 1 - k \quad \dots(iii)$$

$$(i) + (ii) + (iii)$$

$$\Rightarrow \sum P(A) - \sum P(A \cap B) = \frac{3 - 4k}{2}$$

$$P(A \cup B \cup C) = \sum P(A) - \sum P(A \cap B) + P(A \cap B \cap C)$$

$$= \frac{3 - 4k}{2} + k^2$$

$$= (k - 1)^2 + \frac{1}{2} > \frac{1}{2}$$

20. The value of  $k \in \mathbf{R}$ , for which the following system of linear equations

$$3x - y + 4z = 3,$$

$$x + 2y - 3z = -2,$$

$$6x + 5y + kz = -3,$$

has infinitely many solutions, is

- (1) -3 (2) -5  
 (3) 5 (4) 3

**Answer (2)**

$$\text{Sol. } \Delta = \begin{vmatrix} 3 & -1 & 4 \\ 1 & 2 & -3 \\ 6 & 5 & k \end{vmatrix} = 0 \Rightarrow k = -5$$

$$\text{For } k = -5, \Delta_1 = \Delta_2 = \Delta_3 = 0$$

**SECTION - II**

**Numerical Value Type Questions:** This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. Let a curve  $y = y(x)$  be given by the solution of the differential equation

$$\cos\left(\frac{1}{2}\cos^{-1}(e^{-x})\right)dx = \sqrt{e^{2x}-1} dy$$

If it intersects  $y$ -axis at  $y = -1$ , and the intersection point of the curve with  $x$ -axis is  $(\alpha, 0)$ , then  $e^\alpha$  is equal to \_\_\_\_\_.

**Answer (2)**

**Sol.**  $\int dy = \int \frac{\cos\frac{1}{2}\cos^{-1}(e^{-x})}{\sqrt{e^{2x}-1}} dx$

Let  $\frac{1}{2}\cos^{-1}(e^{-x}) = \theta$

$e^{-x} = \cos 2\theta$

$x = \text{Insec} 2\theta$

$dx = 2 \tan 2\theta d\theta$

$y = \int 2 \cos \theta d\theta = 2 \sin \theta + C = \sqrt{2}\sqrt{1-\cos 2\theta} + C$

$= \sqrt{2}\sqrt{1-e^{-x}} + C$

$y(0) = -1 \Rightarrow C = -1$

$y = \sqrt{2(1-e^{-x})} - 1$

$y = 0 \Rightarrow e^\alpha = 2$

2. Let a function  $g : [0, 4] \rightarrow \mathbf{R}$  be defined as

$$g(x) = \begin{cases} \max \{t^3 - 6t^2 + 9t - 3\}, & 0 \leq x \leq 3 \\ 0 \leq t \leq x & , \text{ then} \\ 4 - x & , 3 < x \leq 4 \end{cases}$$

the number of points in the interval  $(0, 4)$  where  $g(x)$  is NOT differentiable, is \_\_\_\_\_.

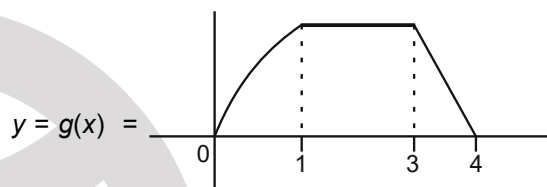
**Answer (1)**

**Sol.**  $f(t) = t^3 - 6t^2 + 9t - 3$

$f'(t) = 3(t-1)(t-3)$

Local max at  $x = 1, f(1) = 1$

$$g(x) = \begin{cases} f(x) = x^3 - 6x^2 + 9x - 3, & t \in [0, 1] \\ 1 & t \in (1, 3] \\ 4 - x & 3 < x \leq 4 \end{cases}$$



Not diff. at  $x = 3$

3. If the point on the curve  $y^2 = 6x$ , nearest to the point  $\left(3, \frac{3}{2}\right)$  is  $(\alpha, \beta)$ , then  $2(\alpha + \beta)$  is equal to \_\_\_\_\_.

**Answer (09.00)**

**Sol.** Let a point on  $y^2 = 6x$  is  $P\left(\frac{3}{2}t^2, 3t\right)$

The distance between P and  $\left(3, \frac{3}{2}\right)$  is D.

$\therefore D^2 = \left(\frac{3t^2}{2} - 3\right)^2 + \left(3t - \frac{3}{2}\right)^2$

$= 9\left\{\frac{t^4}{4} - t^2 + 1 + t^2 - t + \frac{1}{4}\right\}$

$= \frac{9}{4}(t^4 - 4t + 5)$

$\therefore 2D \cdot \frac{dD}{dt} = \frac{9}{4}(4t^3 - 4) = 9(t-1)(t^2 + t + 1)$

$\therefore$  For  $t = 1, D^2$  will be minimum.

$\therefore P = \left(\frac{3}{2}, 3\right) = (\alpha, \beta)$

$\therefore 2(\alpha + \beta) = 9$

4. If  $\lim_{x \rightarrow 0} \frac{\alpha x e^x - \beta \log_e(1+x) + \gamma x^2 e^{-x}}{x \sin^2 x} = 10$ ,  $\alpha, \beta, \gamma \in \mathbf{R}$ , then the value of  $\alpha + \beta + \gamma$  is \_\_\_\_\_.

**Answer (3)**

$$\alpha x \left( 1 + \frac{x}{1} + \frac{x^2}{2} + \dots \right) - \beta \left( x - \frac{x^2}{2} + \frac{x^3}{3} + \dots \right)$$

$$\text{Sol. } \lim_{x \rightarrow 0} \frac{\alpha x \left( 1 + \frac{x}{1} + \frac{x^2}{2} + \dots \right) - \beta \left( x - \frac{x^2}{2} + \frac{x^3}{3} + \dots \right) + \gamma x^2 \left( 1 - \frac{x}{1} + \frac{x^2}{2} + \dots \right)}{x^3 \left( \frac{\sin x}{x} \right)^2} = 10$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{x(\alpha - \beta) + x^2 \left( \alpha + \frac{\beta}{2} + \gamma \right) + x^3 \left( \frac{\alpha}{2} - \frac{\beta}{3} - \gamma \right) + \dots}{x^3} = 10$$

$$\Rightarrow \alpha - \beta = 0, \alpha + \frac{\beta}{2} + \gamma = 0, \frac{\alpha}{2} - \frac{\beta}{3} - \gamma = 10$$

$$\Rightarrow \alpha = 6, \beta = 6, \gamma = -9$$

5. For  $k \in \mathbf{N}$ , let  $\frac{1}{\alpha(\alpha+1)(\alpha+2)\dots(\alpha+20)} = \sum_{k=0}^{20} \frac{A_k}{\alpha+k}$ ,

where  $\alpha > 0$ . Then the value of  $100 \left( \frac{A_{14} + A_{15}}{A_{13}} \right)^2$  is equal to \_\_\_\_\_.

**Answer (9)**

$$\text{Sol. } \alpha(\alpha+1)(\alpha+2)\dots(\alpha+20) \sum_{k=0}^{20} \frac{A_k}{\alpha+k} = 1$$

$$\text{Put } \alpha = -13, -A_{13} \cdot \underline{13} \underline{7} = 1 \Rightarrow A_{13} = \frac{1}{\underline{7} \underline{13}}$$

$$\text{Put } \alpha = -14, -A_{14} \cdot \underline{14} \underline{6} = 1 \Rightarrow A_{14} = \frac{-1}{\underline{14} \underline{6}}$$

$$\text{Put } \alpha = 15, -A_{15} \cdot \underline{15} \underline{5} = 1 \Rightarrow A_{15} = \frac{-1}{\underline{15} \underline{5}}$$

$$100 \left( \frac{A_{14} + A_{15}}{A_{13}} \right)^2 = \frac{100 \left( \frac{1}{\underline{14} \underline{6}} - \frac{1}{\underline{15} \underline{5}} \right)^2}{\left( \frac{1}{\underline{7} \underline{13}} \right)^2}$$

$$= 100 \left( \frac{\frac{9}{\underline{15} \underline{6}}}{\frac{1}{\underline{7} \underline{13}}} \right)^2 = 9$$

6. For  $p > 0$ , a vector  $\vec{v}_2 = 2\hat{i} + (p+1)\hat{j}$  is obtained by rotating the vector  $\vec{v}_1 = \sqrt{3}p\hat{i} + \hat{j}$  by an angle  $\theta$  about origin in counter clockwise direction. If

$$\tan \theta = \frac{(\alpha\sqrt{3} - 2)}{(4\sqrt{3} + 3)}, \text{ then the value of } \alpha \text{ is equal to}$$

**Answer (6)**

$$\text{Sol. } \therefore \cos \theta = \frac{\vec{v}_1 \cdot \vec{v}_2}{|\vec{v}_1| \cdot |\vec{v}_2|} \text{ and } |\vec{v}_1| = |\vec{v}_2|$$

$$\Rightarrow \cos \theta = \frac{2\sqrt{3}p + p + 1}{|\vec{v}_1|^2} \text{ and } 4 + (p+1)^2 = 3p^2 + 1$$

$$\Rightarrow p = 2$$

$$\Rightarrow \cos \theta = \frac{4\sqrt{3} + 3}{13} \Rightarrow \tan \theta = \frac{6\sqrt{3} - 2}{4\sqrt{3} + 3}$$

7. The number of solutions of the equation  $\log_{(x+1)}(2x^2 + 7x + 5) + \log_{(2x+5)}(x+1)^2 - 4 = 0$ ,  $x > 0$ , is \_\_\_\_\_.

**Answer (1)**

$$\text{Sol. } \log_{(x+1)}(x+1)(2x+5) + \log_{(2x+5)}(x+1)^2 = 4$$

$$\Rightarrow 1 + \log_{(x+1)}(2x+5) + 2 \log_{(2x+5)}(x+1) = 4$$

$$\text{Let } \log_{(x+1)}(2x+5) = t$$

$$\text{then } t + \frac{2}{t} = 3 \Rightarrow t = 1, 2$$

$$\Rightarrow 2x+5 = x+1 \text{ or } 2x+5 = (x+1)^2$$

$$\Rightarrow x = -4, +2, -2 \text{ out of which only } x = 2 \text{ is acceptable.}$$

8. Consider a triangle having vertices  $A(-2, 3)$ ,  $B(1, 9)$  and  $C(3, 8)$ . If a line  $L$  passing through the circum-center of triangle  $ABC$ , bisects line  $BC$ , and

intersects  $y$ -axis at point  $\left( 0, \frac{\alpha}{2} \right)$ , then the value of real number  $\alpha$  is \_\_\_\_\_.

**Answer (9)**

**Sol.** Line  $L$  is perpendicular bisector of  $BC$ , which is

$$L : 4x - 2y + 9 = 0$$

$$L \text{ cuts the } y\text{-axis at } \left( 0, \frac{9}{2} \right)$$

$$\text{Clearly } \alpha = 9$$

9. Let  $A = \{a_{ij}\}$  be a  $3 \times 3$  matrix, where

$$a_{ij} = \begin{cases} (-1)^{j-i} & \text{if } i < j, \\ 2 & \text{if } i = j, \\ (-1)^{i+j} & \text{if } i > j, \end{cases}$$

then  $\det(3 \operatorname{Adj}(2A^{-1}))$  is equal to \_\_\_\_\_.

**Answer (108)**

**Sol.**  $\operatorname{adj}(2A^{-1}) = |2A^{-1}|(2A^{-1})^{-1} = \frac{8}{|A|} \cdot \frac{1}{2}A = \frac{4A}{|A|}$

So,  $|3 \operatorname{adj}(2A^{-1})| = \left| 12 \frac{A}{|A|} \right| = \left( \frac{12}{|A|} \right)^3 \cdot |A| = \frac{12^3}{|A|^2}$

$\therefore A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \Rightarrow |A| = 4$

Hence,  $|3 \operatorname{adj}(2A^{-1})| = \frac{12^3}{4^2} = 108$

10. Let  $\{a_n\}_{n=1}^{\infty}$  be a sequence such that  $a_1 = 1$ ,  $a_2 = 1$  and  $a_{n+2} = 2a_{n+1} + a_n$  for all  $n \geq 1$ . Then the value

of  $47 \sum_{n=1}^{\infty} \frac{a_n}{2^{3n}}$  is equal to \_\_\_\_\_.

**Answer (7)**

**Sol.**  $a_{n+2} = 2a_{n+1} + a_n$  has its characteristic equation as

$$x^2 = 2x + 1 \Rightarrow x = 1 \pm \sqrt{2}$$

So  $a_n = a(1 + \sqrt{2})^{n-1} + b(1 - \sqrt{2})^{n-1}$

$\therefore a_1 = 1 \Rightarrow a + b = 1$

and  $a_2 = 1 \Rightarrow (a + b) + \sqrt{2}(a - b) = 1$

$\Rightarrow a = \frac{1}{2}$  and  $b = \frac{1}{2}$

So,  $a_n = \frac{(1 + \sqrt{2})^{n-1} + (1 - \sqrt{2})^{n-1}}{2}$

$$\sum_{n=1}^{\infty} \frac{a_n}{2^{3n}} = \frac{1}{16} \left[ \sum_{n=1}^{\infty} \left( \frac{1 + \sqrt{2}}{8} \right)^{n-1} + \sum_{n=1}^{\infty} \left( \frac{1 - \sqrt{2}}{8} \right)^{n-1} \right]$$

$$= \frac{1}{16} \left[ \frac{8}{7 - \sqrt{2}} + \frac{8}{7 + \sqrt{2}} \right]$$

$$= \frac{7}{47}$$