Time : 3 hrs. M.M. : 300

# Answers & Solutions

*for*

# JEE (MAIN)-2021 (Online) Phase-3

## (Physics, Chemistry and Mathematics)

**IMPORTANT INSTRUCTIONS :**

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper A, B, C consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part has two sections.

(i) Section-I : This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

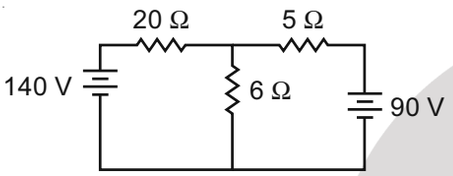
(ii) Section-II : This section contains 10 questions. In Section-II, attempt any **five questions out of 10.** There will be **no negative marking for Section-II**. The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and there is no negative marking for wrong answer.

# PART–A : PHYSICS

## SECTION - I

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer :**

1. 

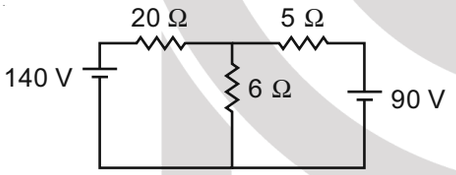

The value of current in the 6 Ω resistance is

(1) 8 A (2) 10 A

(3) 6 A (4) 4 A

**Answer (2)**

**Sol.** $\frac{E_1}{r_1}+\frac{E_2}{r_2}=\frac{E_{eq}}{r_{eq}}$




$\Rightarrow \frac{140}{20}+\frac{90}{5}=\frac{E_{eq}}{4}$

$\Rightarrow E_{eq} = 100\ V,\ r_{eq} = 4\ \Omega$

$\Rightarrow I=\frac{100}{6+4}=10\ A$

2. The entropy of any system is given by

$$S=\alpha^2\beta \ln\left[\frac{\mu kR}{J\beta^2}+3\right]$$

where $\alpha$ and $\beta$ are the constants, $\mu$, J, $k$ and R are no. of moles, mechanical equivalent of heat, Boltzmann constant and gas constant respectively.

[Take $S=\frac{dQ}{T}$]

Choose the **incorrect option** from the following

(1) S, $\beta$, $k$ and $\mu$R have the same dimensions

(2) $\alpha$ and $k$ have the same dimensions

(3) $\alpha$ and J have the same dimensions

(4) S and $\alpha$ have different dimensions

**Answer (2)**

**Sol.** $[\mu R]=\left[\frac{\text{Energy}}{\text{Temperature}}\right]=[k]=[S]$

$\Rightarrow [\beta]=[k]=[\mu R]=[S]=\frac{ML^2T^{-2}}{k}$

3. If $\vec{A}$ and $\vec{B}$ are two vectors satisfying the relation $\vec{A}\times\vec{B}=|\vec{A}\times\vec{B}|$. Then the value of $|\vec{A}-\vec{B}|$ will be

(1) $\sqrt{A^2+B^2}$

(2) $\sqrt{A^2+B^2-\sqrt{2}AB}$

(3) $\sqrt{A^2+B^2+2AB}$

(4) $\sqrt{A^2+B^2+\sqrt{2}AB}$

**Answer (2)**

**Sol.** $\vec{A}\cdot\vec{B}=|\vec{A}\times\vec{B}|$

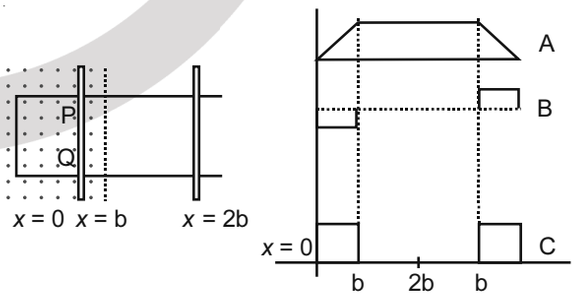
$\Rightarrow ABcos\theta = ABsin\theta$

$\Rightarrow cos\theta = sin\theta \quad \Rightarrow \theta = 45°$

$\therefore |\vec{A}-\vec{B}|=\sqrt{A^2+B^2-2ABcos\theta}$

$=\sqrt{A^2+B^2-\sqrt{2}AB}$

4. The arm PQ of a rectangular conductor is moving from $x$ = 0 to $x$ = 2b outwards and then inwards from $x$ = 2$b$ to $x$ = 0 as shown in the figure. A uniform magnetic field perpendicular to the plane is acting from $x$ = 0 to $x$ = b. Identify the graph showing the variation of different quantities with distance.




(1) A-Flux, B-EMF, C-Power dissipated

(2) A-Flux, B-Power dissipated, C-EMF

(3) A-Power dissipated, B-Flux, C-EMF

(4) A-EMF, B-Power dissipated, C-Flux

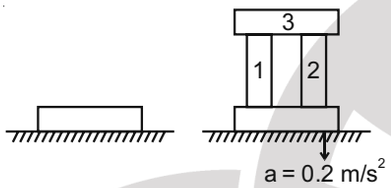
**Answer (1)**

**Sol.** Flux increases and then becomes constant and finally decreases.

Induced emf changes direction, power dissipation is identical.

5. A steel block of 10 kg rests on a horizontal floor as shown. When three iron cylinders are placed on it as shown, the block and cylinders go down with an acceleration 0.2 m/s$^2$. The normal reaction R′ by the floor if mass of the iron cylinders are equal and of 20 kg each, is _____ N.

[Take $g$ = 10 m/s$^2$ and $\mu_s$ = 0.2]

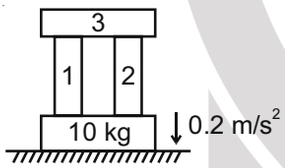



(1) 714 (2) 716

(3) 684 (4) 686

**Answer (4)**

**Sol.**




R′ = (10 + 3 × 20) × ($g$ – $a$)

= 70 × (9.8)

= 686 N

6. Consider a mixture of gas molecule of types A, B and C having masses $m_A < m_B < m_C$. The ratio of their root mean square speeds at normal temperature and pressure is :

(1) $\frac{1}{v_A} > \frac{1}{v_B} > \frac{1}{v_C}$ (2) $v_A = v_B \neq v_C$

(3) $v_A = v_B = v_C = 0$ (4) $\frac{1}{v_A} < \frac{1}{v_B} < \frac{1}{v_C}$

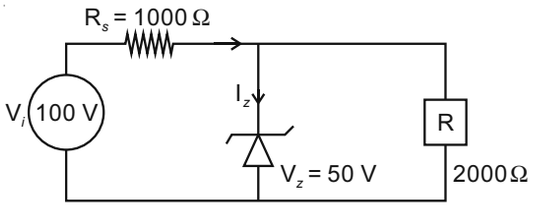
**Answer (4)**

**Sol.** $v_{rms} = \sqrt{\frac{3RT}{M}}$

$\Rightarrow \quad v \propto \frac{1}{\sqrt{m}} \Rightarrow \frac{1}{v} \propto \sqrt{m}$

$\therefore \quad \frac{1}{v_A} < \frac{1}{v_B} < \frac{1}{v_C}$

7. For the circuit shown below, calculate the value of $I_z$ :

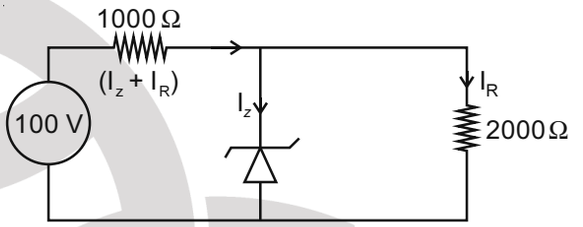



(1) 0.1 A (2) 25 mA

(3) 0.05 A (4) 0.15 A

**Answer (2)**

**Sol.**




$I_R = \frac{50}{2000} = 2.5 \times 10^{-2}$ A

$1000(I_R + I_z) = -50 + 100$

$\Rightarrow \quad (I_R + I_z) = 0.05$

$I_z = 0.025$ A

8. A butterfly is flying with a velocity $4\sqrt{2}$ m/s in North-East direction. Wind is slowly blowing at 1 m/s from North to South. The resultant displacement of the butterfly in 3 seconds is:

(1) $12\sqrt{2}$ m (2) 3 m

(3) 20 m (4) 15 m

**Answer (4)**

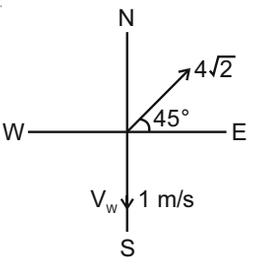
**Sol.** $V_x$ = 4 m/s

$V_y$ = 3 m/s

$x = 4 \times 3 = 12$ m

$y = 3 \times 3 = 9$ m

$D = \sqrt{x^2 + y^2} = 15$ m



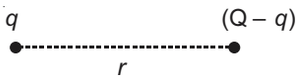

9. A certain charge Q is divided into two parts $q$ and (Q – $q$). How should the charges Q and $q$ be divided so that $q$ and (Q – $q$) placed at a certain distance apart experience maximum electrostatic repulsion?

(1) Q = 4$q$ (2) $Q = \frac{q}{2}$

(3) Q = 3$q$ (4) Q = 2$q$

**Answer (4)**

**Sol.** $F = \frac{(k)(q)(Q-q)}{r^2}$




$f(q) = q(Q - q)$

$f'(q) = Q - 2q$

$f'(q) = 0 \Rightarrow \quad q = \frac{Q}{2}$

10. A nucleus of mass M emits γ-ray photon of frequency 'ν'. The loss of internal energy by the nucleus is:

[Take 'c' as the speed of electromagnetic wave]

(1) $h\nu\left[1-\frac{h\nu}{2Mc^2}\right]$ (2) $h\nu$

(3) $h\nu\left[1+\frac{h\nu}{2Mc^2}\right]$ (4) 0

**Answer (3)**

**Sol.** $\Delta E = h\nu + \frac{1}{2}MV^2$

$= h\nu + \frac{P^2}{2M}$

$= h\nu + \frac{(h\nu)^2}{2Mc^2}$

$= h\nu\left[1+\frac{h\nu}{2Mc^2}\right]$

11. A current of 5 A is passing through a non-linear magnesium wire of cross-section 0.04 $m^2$. At every point the direction of current density is at an angle of 60° with the unit vector of area of cross-section. The magnitude of electric field at every point of the conductor is:

(Resistivity of magnesium $\rho = 44 \times 10^{-8}\ \Omega m$)

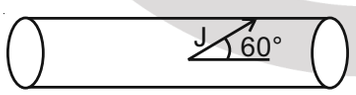
(1) $11 \times 10^{-5}$ V/m (2) $11 \times 10^{-2}$ V/m

(3) $11 \times 10^{-3}$ V/m (4) $11 \times 10^{-7}$ V/m

**Answer (1)**

**Sol.** I = 5A, A = 0.04 $m^2$

$\vec{J}\cdot\vec{A} = I$




$JA\cos60° = I$

$J = \frac{2I}{A} = \frac{2\times5}{4\times10^{-2}} = \frac{10^3}{4}\ A/m^2$

$J = \sigma E$

$J = \frac{E}{\rho} = \frac{10^3}{4}$

$E = 11 \times 10^{-5}$ V/m

12. A deuteron and an alpha particle having equal kinetic energy enter perpendicularly into a magnetic field. Let $r_d$ and $r_\alpha$ be their respective radii of circular path. The value of $\frac{r_d}{r_\alpha}$ is

(1) $\frac{1}{\sqrt{2}}$ (2) 1

(3) 2 (4) $\sqrt{2}$

**Answer (4)**

**Sol.** $r = \frac{mv}{qB}$

$r = \frac{\sqrt{2mk}}{qB}$

$\frac{r_d}{r_\alpha} = \frac{\sqrt{m_d}}{q_d} \times \frac{q_\alpha}{\sqrt{m_\alpha}} = \frac{2}{\sqrt{2}} = \sqrt{2}$

13. The normal reaction 'N' for a vehicle of 800 kg mass, negotiating a turn on a 30° banked road at maximum possible speed without skidding is_____× $10^3$ kg $m/s^2$.

[Given cos30° = 0.87, $\mu_s$ = 0.2]

(1) 6.96

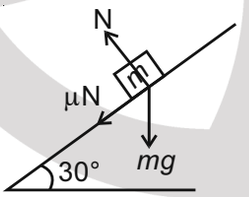
(2) 12.4

(3) 7.2

(4) 10.2

**Answer (4)**

**Sol.**




Along vertical

$N\cos30° = mg + \mu N\cos60°$

$N\frac{\sqrt{3}}{2} = 800\times9.8 + 0.2N\times\frac{1}{2}$

$\Rightarrow N\frac{\sqrt{3}}{2} - 0.1N = 800\times9.8$

$N = \frac{800\times9.8}{\frac{\sqrt{3}}{2} - 0.1}$

$\approx 10234$

$\approx 10.2 \times 10^3$ kg $m/s^2$

14. A radioactive material decays by simultaneous emissions of two particles with half lives of 1400 years and 700 years respectively. What will be the time after which one third of the material remains? (Take ln 3 = 1.1)

(1) 340 years (2) 1110 years

(3) 700 years (4) 740 years

**Answer (4)**

**Sol.** $t_{1/2} = \frac{\ln 2}{\lambda}$

$\lambda_{eq} = \lambda_1 + \lambda_2$

$\frac{1}{t_{eq}} = \frac{1}{t_1} + \frac{1}{t_2}$

$t_{eq} = \frac{700 \times 1400}{700 + 1400}$

$t_{eq} = \frac{1400}{3}$ years

$N = N_0 e^{-\lambda t}$

$\frac{N_0}{3} = N_0 e^{-\lambda t_0}$

$\ln 3 = \lambda t_0$

$t_0 = \frac{\ln 3}{\lambda}$

$= \frac{\ln 3}{\lambda_{eq}}$

$= \frac{1.1}{\frac{\ln 2}{t_{eq}}}$

$= \frac{1.1}{\ln 2} \times \frac{1400}{3}$

$= \frac{1.1}{0.3} \times \frac{1400}{3}$

= 740 years

15. AC voltage V(t) = 20 sinωt of frequency 50 Hz is applied to a parallel plate capacitor. The separation between the plates is 2 mm and the area is 1 $m^2$. The amplitude of the oscillating displacement current for the applied AC voltage is_____.

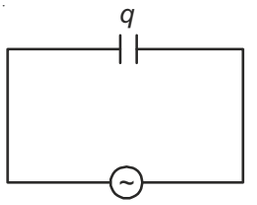
[Take $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m ]

(1) 21.14 μA (2) 83.37 μA

(3) 27.79 μA (4) 55.58 μA

**Answer (3)**

**Sol.**




$C = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 1}{2 \times 10^{-3}} = \frac{8.85 \times 10^{-9}}{2}$

$q = CV$

$i = C\frac{dv}{dt} = C20\omega \cos\omega t$

$i = 20\ \omega C \cos\omega t$

So, amplitude of displacement current

$= 20\omega C$

$= 20 \times 2\pi \times 50 \times \frac{\varepsilon_0 A}{d}$

$= 20 \times 2\pi \times 50 \times \frac{8.85 \times 10^{-9}}{2}$

$= \pi \times 8.85 \times 10^{-6}$

$= 27.79 \times 10^{-6}$ A = 27.79 μA

16. The radiation corresponding to 3 → 2 transition of a hydrogen atom falls on a gold surface to generate photoelectrons. These electrons are passed through a magnetic field of $5 \times 10^{-4}$ T. Assume that the radius of the largest circular path followed by these electrons is 7 mm, the work function of the metal is : (Mass of electron = $9.1 \times 10^{-31}$ kg)

(1) 1.36 eV (2) 1.88 eV

(3) 0.82 eV (4) 0.16 eV

**Answer (3)**

**Sol.** Energy of photon = $13.6\left[\frac{1}{4} - \frac{1}{9}\right]$

= 1.89 eV

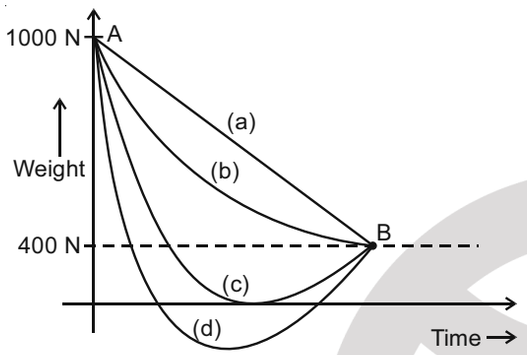
$r = \frac{mv}{qB}$

$K.E = \frac{r^2 q^2 B^2}{2m} = \frac{49 \times 10^{-6} \times 1.6 \times 10^{19} \times 25 \times 10^{-8}}{2 \times 9.1 \times 10^{-31}}$ eV

K.E = 1.08 eV

$\phi$ = 1.89 – 1.08 = 0.81 eV

17. A person whose mass is 100 kg travels from Earth to Mars in a spaceship. Neglect all other objects in sky and take acceleration due to gravity on the surface of the Earth and Mars as 10 m/s$^2$ and 4 m/s$^2$ respectively. Identify from the below figures, the curve that fits best for the weight of the passenger as a function of time.




(1) (a) (2) (c)

(3) (d) (4) (b)

**Answer (2)**

**Sol.** Weight of the person becomes zero only once.

Hence, option (2) is correct

18. The amount of heat needed to raise the temperature of 4 moles of a rigid diatomic gas from 0°C to 50°C when no work is done is _____. (R is the universal gas constant)

(1) 750 R (2) 500 R

(3) 250 R (4) 175 R

**Answer (2)**

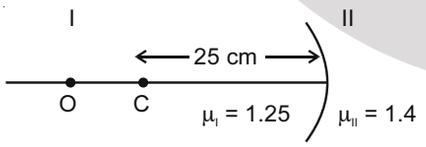
**Sol.** $\Delta Q = \Delta U + \Delta W$

$\Delta W = 0$

$$\Delta U = nR \times \frac{5}{2}\Delta T$$

$\Delta U = 500$ R

19. Region I and II are separated by a spherical surface of radius 25 cm. An object is kept in region I at a distance of 40 cm from the surface. The distance of the image from the surface is




(1) 18.23 cm (2) 9.52 cm

(3) 37.58 cm (4) 55.44 cm

**Answer (3)**

**Sol.** $$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1.4}{v} + \frac{1.25}{40} = \frac{1.4 - 1.25}{-25}$$

$$\Rightarrow \frac{1.4}{v} = \frac{0.15}{-25} - \frac{1.25}{40}$$

$v \simeq 37.33$ cm

20. The value of tension in a long thin metal wire has been changed from $T_1$ to $T_2$. The lengths of the metal wire at two different values of tension $T_1$ and $T_2$ are $l_1$ and $l_2$ respectively. The actual length of the metal wire is

(1) $\sqrt{T_1 T_2 l_1 l_2}$ (2) $\frac{T_1 l_1 - T_2 l_2}{T_1 - T_2}$

(3) $\frac{T_1 l_2 - T_2 l_1}{T_1 - T_2}$ (4) $\frac{l_1 + l_2}{2}$

**Answer (3)**

**Sol.** Let k be constant

$k(l_1 - l_0) = T_1$

$k(l_2 - l_0) = T_2$

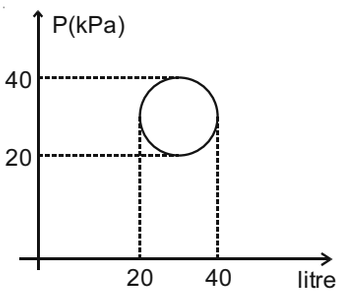
$$\frac{T_1}{(l_1 - l_0)} = \frac{T_2}{(l_2 - l_0)}$$

$$l_0 = \left[\frac{T_1 l_2 - T_2 l_1}{T_1 - T_2}\right]$$

## SECTION - II

**Numerical Value Type Questions:** This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. In the reported figure, heat energy absorbed by a system in going through a cyclic process is _____ $\pi$J.



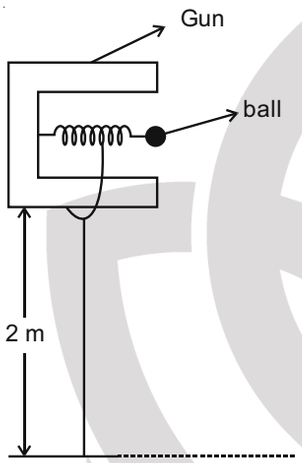

**Answer (100)**

**Sol.** W = Area

$$= \pi\left(\frac{40-20}{2}\right)\left(\frac{40-20}{2}\right)\text{J}$$

$$= 100\pi$$

2. In a spring gun having spring constant 100 N/m a small ball 'B' of mass 100 g is put in its barrel (as shown in figure) by compressing the spring through 0.05 m. There should be a box placed at a distance '*d*' on the ground so that the ball falls in it. If the ball leaves the gun horizontally at a height of 2 m above the ground. The value of *d* is _____ m. ($g$ = 10 m/s$^2$).




**Answer (1)**

**Sol.** $\frac{1}{2}mv_0^2 = \frac{1}{2}kx_0^2$

$d = v_0 T$

$$= x_0\sqrt{\frac{k}{m}}\cdot\sqrt{\frac{2h}{g}}$$

$$= 0.05\sqrt{\frac{100}{0.1}}\sqrt{\frac{2\times 2}{10}}$$

= 1.0 m

3. A circular disc reaches from top to bottom of an inclined plane of length 'L'. When it slips down the plane, it takes time '$t_1$'. When it rolls down the plane, it takes time $t_2$. The value of $\frac{t_2}{t_1}$ is $\sqrt{\frac{3}{x}}$. The value of $x$ will be _____.

**Answer (2)**

**Sol.** Assuming frictionless

$$t_1 = \sqrt{\frac{2L}{g\sin\theta}}$$

When it rolls down

$$t_2 = \sqrt{\frac{\frac{2L}{g\sin\theta}}{\left(1+\frac{1}{2}\right)}} = \sqrt{\frac{3}{2}\cdot\frac{2L}{g\sin\theta}}$$

$$\frac{t_2}{t_1} = \sqrt{\frac{3}{2}}$$

4. An object viewed from a near point distance of 25 cm, using a microscopic lens with magnification '6', gives an unresolved image. A resolved image is observed at infinite distance with a total magnification double the earlier using an eyepiece along with the given lens and a tube of length 0.6 m, if the focal length of the eyepiece is equal to ____ cm.

**Answer (25)**

**Sol.** $m_1 = 1+\frac{D}{f} \Rightarrow f = 5$ cm

$$m_2 = \frac{LD}{f_o f_e} \Rightarrow f_e = \frac{LD}{f_o \times m_2} = 25 \text{ cm}$$

5. In an LCR series circuit, an inductor 30 mH and a resistor 1 Ω are connected to an AC source of angular frequency 300 rad/s. The value of capacitance for which, the current leads the voltage by 45° is $\frac{1}{x}\times 10^{-3}$ F. Then the value of $x$ is ____

**Answer (3)**

**Sol.** $\tan\phi = \frac{|X_L - X_C|}{R}$

$\Rightarrow X_C = X_L + R = 9+1 = 10\ \Omega$

$$C = \frac{1}{\omega X_C}$$

$$= \frac{1}{300\times 10} = \frac{1}{3}\times 10^{-3}\text{ F}$$

6. The frequency of a car horn encountered a change from 400 Hz to 500 Hz, when the car approaches a vertical wall. If the speed of sound is 330 m/s. Then the speed of car is ____ km/h.

**Answer (132)**

**Sol.** $f' = \frac{v + v_c}{v - v_c} f$

$\Rightarrow \frac{330 + v_c}{330 - v_c} \times 400 = 500$

$330 = 9v$

$\Rightarrow v = \frac{110}{3}$ m/s or 132 km/h

7. A rod of mass M and length L is lying on a horizontal frictionless surface. A particle of mass '*m*' travelling along the surface hits at one end of the rod with a velocity '*u*' in a direction perpendicular to the rod. The collision is completely elastic. After collision, particle comes to rest. The ratio of masses $\left(\frac{m}{M}\right)$ is $\frac{1}{x}$. The value of '*x*' will be ____.

**Answer (4)**

**Sol.** $mu = Mv$

$u = v + \frac{\omega L}{2}$

$0 = \frac{MvL}{2} - \frac{ML^2}{12}\omega$

$\Rightarrow \omega L = 6v$

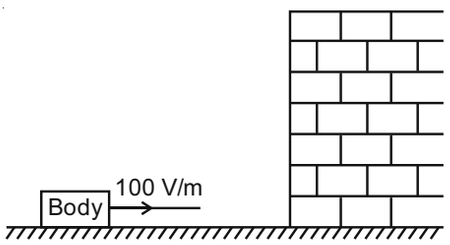
or $u = v + 3v$

$u = 4v$

$mu = Mv$

$\Rightarrow \frac{m}{M} = \frac{1}{4}$

8. A body having specific charge 8 μC/g is resting on a frictionless plane at a distance 10 cm from the wall (as shown in the figure). It starts moving towards the wall when a uniform electric field of 100 V/m is applied horizontally towards the wall. If the collision of the body with the wall is perfectly elastic, then the time period of the motion will be _____ s.




**Answer (1)**

**Sol.** $F = 100 \times 8m \times 10^{-3}$ N

$a = 800 \times 10^{-3}$ m/s$^2$

$d = 10 \times 10^{-2}$ m

$t = \sqrt{\frac{2 \times 10 \times 10^{-2}}{800 \times 10^{-3}}}\ s = \frac{1}{2}s$

Total time taken to come back = $2t$

= 1s

9. A carrier wave $V_C(t) = 160\sin(2\pi \times 10^6 t)$ volts is made to vary between $V_{max}$ = 200 V and $V_{min}$ = 120 V by a message signal $V_m(t) = A_m \sin(2\pi \times 10^3 t)$ volts. The peak voltage $A_m$ of the modulating signal is _____.

**Answer (40)**

**Sol.** $160 + A_m = 200$

$160 - A_m = 120$

$\Rightarrow A_m = 40$

10. The amplitude of wave disturbance propagating in the positive *x*-direction is given by $y = \frac{1}{(1+x)^2}$ at time $t = 0$ and $y = \frac{1}{1+(x-2)^2}$ at $t = 1$ s, where *x* and *y* are in metres. The shape of wave does not change during the propagation. The velocity of the wave will be _____ m/s.

**Answer (Bonus)**

**Sol. # Printing error in paper in this question.**

If $y = \frac{1}{1+x^2}$ at $t = 0$, instead of $\frac{1}{(1+x)^2}$

$y = \frac{1}{1+(x-2)^2}$ at $t = 1$

$\Rightarrow y = \frac{1}{1+(x-2t)^2}$ at any time $t$

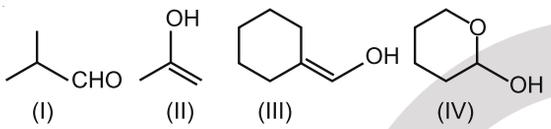
$v = 2$ m/s

# PART–B : CHEMISTRY

## SECTION - I

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer :**

1. 

(I) (II) (III) (IV)

Which among the above compound/s does/do not form Silver mirror when treated with Tollen's reagent?

(1) (I), (III) and (IV) only (2) (III) and (IV) only

(3) Only (IV) (4) Only (II)

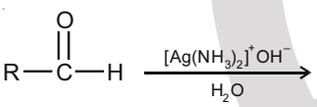
**Answer (4)**

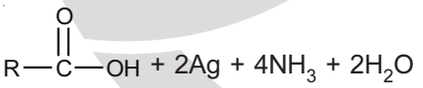
**Sol.** 

(II)

Aldehyde gives Silver mirror test with Tollen's reagent but not ketones.

(IV) gives test




2. The conditions given below are in the context of observing Tyndall effect in colloidal solutions:

(A) The diameter of the colloidal particles is comparable to the wavelength of light used.

(B) The diameter of the colloidal particles is much smaller than the wavelength of light used.

(C) The diameter of the colloidal particles is much larger than the wavelength of light used.

(D) The refractive indices of the dispersed phase and the dispersion medium are comparable.

(E) The dispersed phase has a very different refractive index from the dispersion medium.

Choose the most appropriate conditions from the options given below.

(1) (A) and (D) only

(2) (C) and (D) only

(3) (B) and (E) only

(4) (A) and (E) only

**Answer (4)**

**Sol.** The diameter of the dispersed particles is not much smaller than the wavelength of the light used; and

The refractive indices of the dispersed phase and the dispersion medium differ greatly in magnitude.

3. Orlon fibres are made up of :

(1) Cellulose

(2) Polyesters

(3) Polyamide

(4) Polyacrylonitrile

**Answer (4)**

**Sol.** Orlon fibres are made up of polyacrylonitrile.

4. In the given reaction 3-Bromo-2,2-dimethyl butane $\xrightarrow{C_2H_5OH}$ 'A' (Major Product)

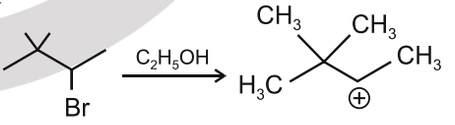
(1) 1-Ethoxy-3,3-dimethyl butane.

(2) 2-Hydroxy-3,3-dimethyl butane.

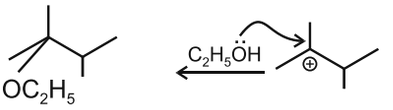
(3) 2-Ethoxy-2,3-dimethyl butane.

(4) 2-Ethoxy-3,3-dimethyl butane.

**Answer (3)**

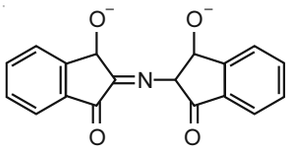
**Sol.** 

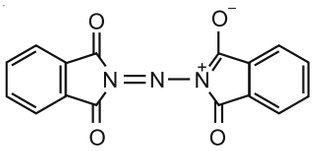

Rearrangement (methyl shift)

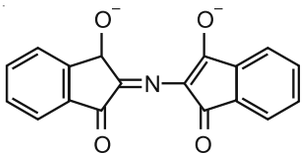


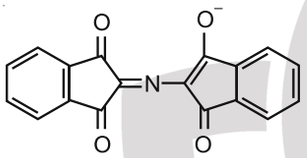

2-Ethoxy-2,3-dimethyl butane

5. The correct structure of Rhumann's Purple, the compound formed in the reaction of ninhydrin with proteins is:

(1) 

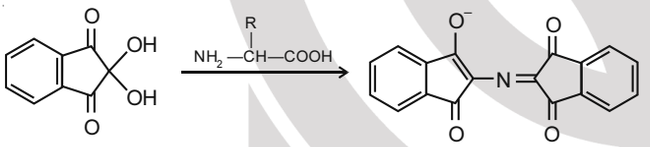
(2) 

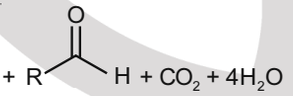
(3) 

(4) 

**Answer (4)**

**Sol.** Ninhydrin test





6. Identify the incorrect statement from the following

(1) Amylose is a branched chain polymer of glucose

(2) β-Glycosidic linkage makes cellulose polymer

(3) Glycogen is called as animal starch

(4) Starch is a polymer of α-D glucose

**Answer (1)**

**Sol.** Amylose is a unbranched chain with 200–1000 α-D-(+)-glucose units held together by C1-C4 glycosidic linkage.

7. Compound A is converted to B on reaction with $CHCl_3$ and KOH. The compound B is toxic and can be decomposed by C. A, B and C respectively are

(1) Secondary amine, isonitrile compound, conc. NaOH

(2) Secondary amine, nitrile compound, conc. NaOH

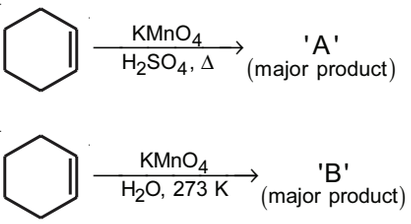
(3) Primary amine, isonitrile compound, conc. HCl

(4) Primary amine, nitrile compound, conc. HCl

**Answer (3)**

**Sol.** $\underset{\text{Primary amine}}{R-NH_2} \xrightarrow[KOH]{CHCl_3} \underset{\text{isonitrile}}{R-N\equiv C} + 3KCl + 3H_2O$

(Poisnous product)

8. 

For above chemical reactions, identify the correct statement from the following.

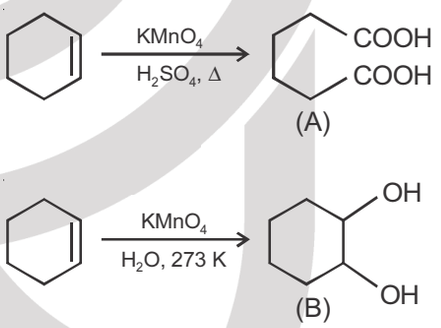
(1) Compound 'A' is dicarboxylic acid and compound 'B' is diol.

(2) Compound 'A' is diol and compound 'B' is dicarboxylic acid.

(3) Both compound 'A' and compound 'B' are dicarboxylic acids.

(4) Both compound 'A' and compound 'B' are diols.

**Answer (1)**

**Sol.** 

9. The correct order of intensity of colors of the compounds is

(1) $[Ni(CN)_4]^{2-} > [NiCl_4]^{2-} > [Ni(H_2O)_6]^{2+}$

(2) $[NiCl_4]^{2-} > [Ni(CN)_4]^{2-} > [Ni(H_2O)_6]^{2+}$

(3) $[Ni(H_2O)_6]^{2+} > [NiCl_4]^{2-} > [Ni(CN)_4]^{2-}$

(4) $[NiCl_4]^{2-} > [Ni(H_2O)_6]^{2+} > [Ni(CN)_4]^{2-}$

**Answer (4)**

**Sol.** Order of Intensity of colour is

| $[NiCl_4]^{2-}$ | > | $[Ni(H_2O)_6]^{2+}$ | > | $[Ni(CN)_4]^{2-}$ |
|---|---|---|---|---|
| → $d^8$ | | $d^8$ | | $d^8$ |
| → Tetrahedral | | Octahedral | | Diamagnetic |
| → High spin | | High spin | | |

10. An inorganic Compound 'X' on treatment with concentrated $H_2SO_4$ produces brown fumes and gives dark brown ring with $FeSO_4$ in presence of concentrated $H_2SO_4$. Also Compound 'X' gives precipitate 'Y', when its solution in dilute HCl is treated with $H_2S$ gas. The precipitate 'Y' on treatment with concentrated $HNO_3$ followed by excess of $NH_4OH$ further gives deep blue coloured solution, Compound 'X' is

(1) $Cu(NO_3)_2$ (2) $Pb(NO_2)_2$

(3) $Pb(NO_3)_2$ (4) $Co(NO_3)_2$

**Answer (1)**

**Sol.** $Cu(NO_3)_2 \xrightarrow{H_2SO_4} NO_2\uparrow$

(X)

$\xrightarrow[\text{conc. } H_2SO_4]{FeSO_4} [Fe(H_2O)_5NO]^{2+}$

$Cu(NO_3)_2 \xrightarrow[H_2S]{\text{dil. HCl}} CuS$

(X) (Y)

$CuS \xrightarrow{HNO_3} Cu(NO_3)_2 + S + NO\uparrow + H_2O$

(Y)

$Cu(NO_3)_2 \xrightarrow{NH_4OH} [Cu(NH_3)_4]^{2+}$

Deep blue coloured solution

11. Given below are two statements. One is labelled as **Assertion A** and the other is labelled as **Reason R.**

**Assertion A :** Sharp glass edge becomes smooth on heating it upto its melting point.

**Reason R :** The viscosity of glass decreases on melting.

Choose the most appropriate answer from the options given below.

(1) Both **A** and **R** are true and R is the correct explanation of A

(2) **A** is false but **R** is true.

(3) **A** is true but **R** is false.

(4) Both **A** and **R** are true but **R** is NOT the correct explanation of **A.**

**Answer (4)**

**Sol.** Sharp glass edges are heated for making them smooth. On heating, the glass melts and the surface of the liquid tends to take the rounded shape at the edges, which makes the edges smooth. This is called fire polishing of glass.

12. A s-block element (M) reacts with oxygen to form an oxide of the formula $MO_2$. The oxide is pale yellow in colour and paramagnetic. The element (M) is

(1) K (2) Na

(3) Mg (4) Ca

**Answer (1)**

**Sol.** $K + O_2 \rightarrow KO_2$ (Paramagnetic because of $O_2^-$)

13. The species given below that does NOT show disproportionation reaction is

(1) $BrO_3^-$ (2) $BrO^-$

(3) $BrO_2^-$ (4) $BrO_4^-$

**Answer (4)**

**Sol.** $BrO_4^-$ is in maximum oxidation state *i.e.*, + 7 so it can only reduce.

14. Given below are two statements : One is labelled as **Assertion A** and the other is labelled as **Reason R.**

**Assertion A :** The dihedral angles in $H_2O_2$ in gaseous phase is 90.2° and in solid phase is 111.5°.

**Reason R :** The change in dihedral angle in solid and gaseous phase is due to the difference in the intermolecular forces.

Choose the most appropriate answer from the options given below for **A** and **R**.

(1) **A** is not correct but **R** is correct.

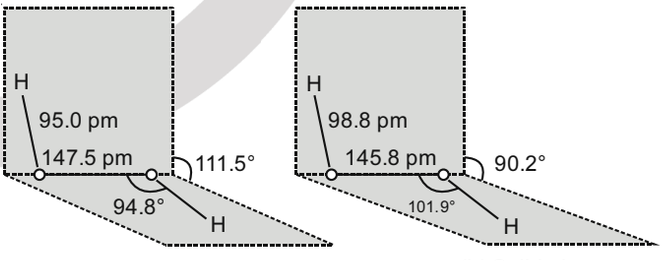
(2) **A** is correct but **R** is not correct.

(3) Both **A** and **R** are correct but **R** is not the correct explanation of **A**.

(4) Both **A** and **R** are correct and **R** is the correct explanation of **A**.

**Answer (1)**

**Sol.**




(a) Gas phase (b) Solid phase

(a) $H_2O_2$ structure in gas phase, dihedral angle is 111.5°.

(b) $H_2O_2$ structure in solid phase at 110 K, dihedral angle is 90.2°.

Assertion is wrong.

Reason is true.

15. According to the valence bond theory the hybridization of central metal atom is $dsp^2$ for which one of the following compounds?

(1) $NiCl_2.6H_2O$

(2) $[Ni(CO)_4]$

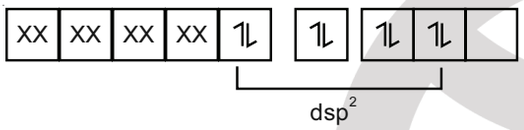
(3) $K_2[Ni(CN)_4]$

(4) $Na_2[NiCl_4]$

**Answer (3)**

**Sol.** $K_2[Ni(CN)_4] \rightarrow d^8$ + strong field ligand

$Ni^{2+}$




16. The set in which compounds have different nature is :

(1) $B(OH)_3$ and $H_3PO_3$

(2) $B(OH)_3$ and $Al(OH)_3$

(3) $Be(OH)_2$ and $Al(OH)_3$

(4) NaOH and $Ca(OH)_2$

**Answer (2)**

**Sol.** $B(OH)_3$ – Acidic

$H_3PO_3$ – Acidic

$Be(OH)_2$ – Amphoteric

$Al(OH)_3$ – Amphoteric

NaOH – Basic

$Ca(OH)_2$ – Basic

Option-2 contain acidic and amphoteric species

17. Chemical nature of the nitrogen oxide compound obtained from a reaction of concentrated nitric acid and $P_4O_{10}$(in 4 : 1 ratio) is :

(1) acidic

(2) basic

(3) neutral

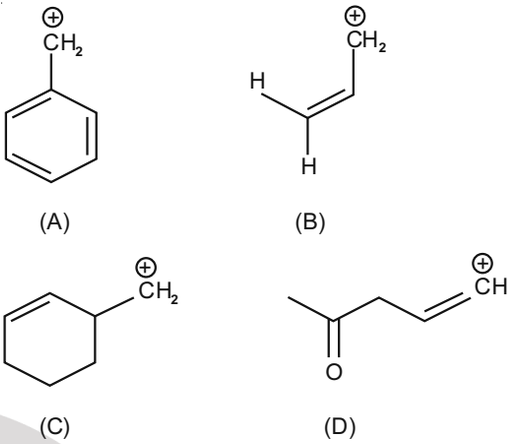
(4) amphoteric

**Answer (1)**

**Sol.** $4HNO_3 + P_4O_{10} \rightarrow 4HPO_3 + 2N_2O_5$

$N_2O_5$ is acidic in nature

18.




Among the given species the Resonance stabilised carbocations are :

(1) (C) and (D) only

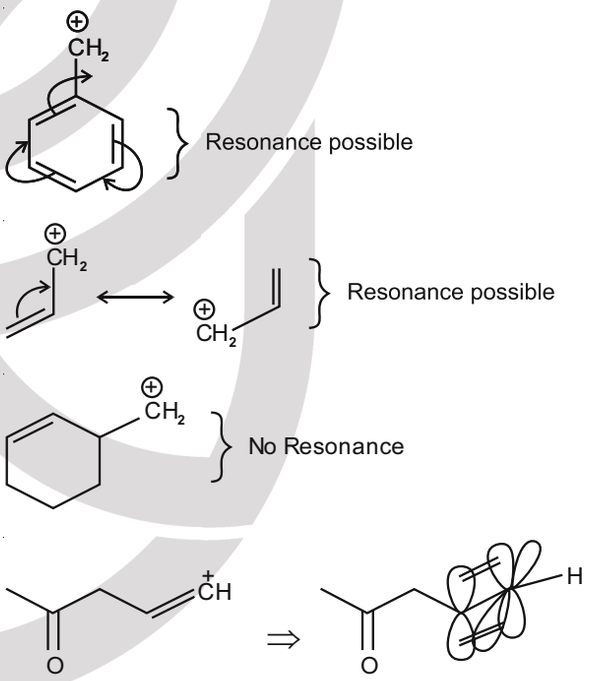
(2) (A), (B) and (C) only

(3) (A) and (B) only

(4) (A), (B) and (D) only

**Answer (3)**

**Sol.**




Empty P-orbital and π bond P-orbital are not in same phase to overlap. Hence no conjugation

19. Green Chemistry in day-to-day life is in the use of :

(1) Chlorine for bleaching of paper

(2) Liquified $CO_2$ for dry cleaning of clothes

(3) Large amount of water alone for washing clothes

(4) Tetrachloroethene for laundry

**Answer (2)**

**Sol.** Fact NCERT Pg no. 420 Part-2-XI

Liquified $CO_2$ is used as solvent for dry cleaning

20. The metal that can be purified economically by fractional distillation method is :

(1) Fe (2) Ni

(3) Cu (4) Zn

**Answer (4)**

**Sol.** Fact NCERT Pg no. 164 Part-1-XII

Zinc has low boiling point

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**SECTION - II**

**Numerical Value Type Questions:** This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

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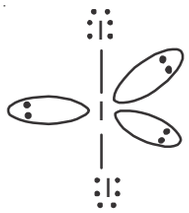
1. The number of lone pairs of electrons on the central I atom in $I_3^-$ is _____.

**Answer (3)**

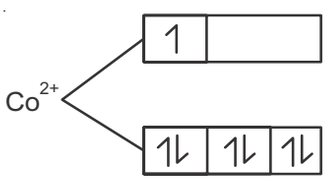
**Sol.**  central iodine atom has 3 lone pairs of electrons

2. The spin-only magnetic moment value for the complex $[Co(CN)_6]^{4-}$ is _____ BM.

[At. no. of Co = 27]

**Answer (2)**

**Sol.** $CN^-$ is strong field ligand and cause pairing of electrons.




Spin only magnetic moment $(\mu) = \sqrt{n(n+2)}$ BM

$\mu = \sqrt{3} = 1.732$ BM

3. An average person needs about 10000 kJ energy per day. The amount of glucose (molar mass = 180.0 g $mol^{-1}$) needed to meet this energy requirement is _____ g. (Nearest integer)

(Use: $\Delta_c H$(glucose) = –2700 kJ $mol^{-1}$)

**Answer (667)**

**Sol.** $C_6H_{12}O_6(s) + 6O_2(g) \longrightarrow 6CO_2(g) + 6H_2O(l)$ $\Delta_C H$ = –2700 kJ $mol^{-1}$

1 mole of $C_6H_{12}O_6$ gives 2700 kJ energy

$\frac{10000}{2700}$ mole of $C_6H_{12}O_6$ gives 10000 kJ of energy

Weight of $C_6H_{12}O_6$ needs to meet the energy

$= \frac{100}{27} \times 180$ = 666.67 g

$\approx$ 667 g

4. At 20°C, the vapour pressure of benzene is 70 torr and that of methyl benzene is 20 torr. The mole fraction of benzene in the vapor phase at 20°C above an equimolar mixture of benzene and methyl benzene is _____ × $10^{-2}$.

(Nearest integer)

**Answer (78)**

**Sol.** $P_{Total} = X_B P_B^\circ + X_{MB} P_{MB}^\circ$

$X_B$ – mole fraction of benzene in solution phase

$X_{MB}$ – mole fraction of methyl benzene in solution phase

$Y_B P_{Total} = X_B P_B^\circ$

$Y_B$ – mole fraction of benzene in vapor phase

$Y_B = \frac{0.5 \times 70}{0.5 \times 70 + 0.5 \times 20} = 0.7777$

= 77.77 × $10^{-2}$

$\approx 78 \times 10^{-2}$

5. $2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$

In an equilibrium mixture, the partial pressures are

$P_{SO_3} = 43\ kPa$; $P_{O_2} = 530\ Pa$ and $P_{SO_2} = 45\ kPa$.

The equilibrium constant $K_P$ = _____ × $10^{-2}$. (Nearest integer)

**Answer (172)**

**Sol.** $2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$

$$K_P = \frac{P_{SO_3}^2}{P_{SO_2}^2 P_{O_2}}$$

$$= \frac{(43)^2}{(0.53)(45)^2}$$

$$= 1.7228\ kPa^{-1}$$

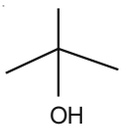
$$= 172.28 \times 10^{-2}\ kPa^{-1}$$

$$\approx 172 \times 10^{-2}\ kPa^{-1}$$

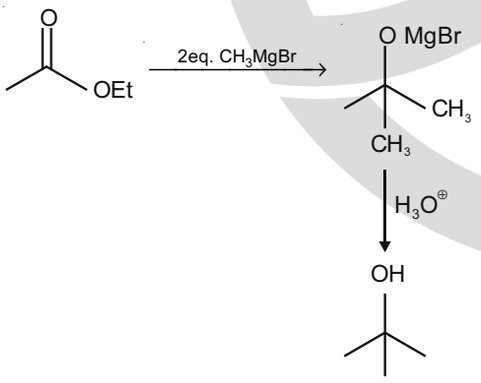
6. To synthesise 1.0 mole of 2-methylpropan-2-ol from Ethyle thanoate_____equivalents of $CH_3MgBr$ reagent will be required. (Integer value)

**Answer (2)**

**Sol.**




2-methylpropan-2-ol




7. 250 mL of 0.5 M NaOH was added to 500 mL of 1 M HCl. The number of unreacted HCl molecules in the solution after complete reaction is _____ ×$10^{21}$. (Nearest integer) ($N_A$ = 6.022 × $10^{23}$)

**Answer (226)**

**Sol.**

| NaOH | + HCl | → NaCl | + $H_2O$ |
|---|---|---|---|
| 125 m mol | 500 m mol | - | - |
| - | 375 m mol | 125 m mol | - |

Number of unreacted molecules of

HCl = 375 × $10^{-3}$ × 6.022 × $10^{23}$

= 2258.25 × $10^{20}$

= 225.825 × $10^{21}$

≈ 226 × $10^{21}$

8. The inactivation rate of a viral preparation is proportional to the amount of virus. In the first minute after preparation, 10% of the virus is inactivated. The rate constant for viral inactivation is _____ × $10^{-3}$ $min^{-1}$. (Nearest integer)

[Use : ln 10 = 2.303; $\log_{10}$ 3 = 0.477;

property of logarithm : log $x^y$ = y log x]

**Answer (106)**

**Sol.** Rate = k [Amount of virus]

$$k = \frac{1}{1\ min} \ln \frac{1}{0.9} \quad (\because \text{ 10\% of virus is inactivated})$$

= 2.303 (1– 0.954)

= 0.1059

= 105.9 × $10^{-3}$

≈ 106 × $10^{-3}$ $min^{-1}$ (nearest integer)

9. The Azimuthal quantum number for the valence electrons of $Ga^+$ ion is _____.

(Atomic number of Ga = 31)

**Answer (0)**

**Sol.** Ga – $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^2\ 3d^{10}\ 4p^1$

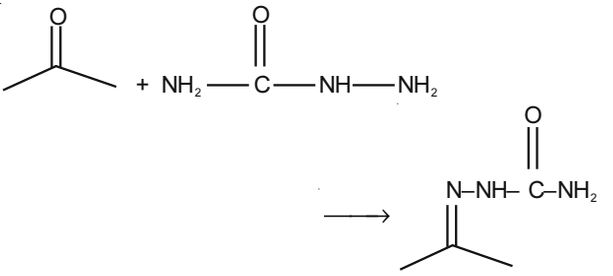
$Ga^+$ – $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^2\ 3d^{10}$

$4s^2$ are the valence electrons, so l = 0.

10. The number of nitrogen atoms in a semicarbazone molecule of acetone is _____.

**Answer (3)**

**Sol.**



# PART–C : MATHEMATICS

## SECTION - I

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer :**

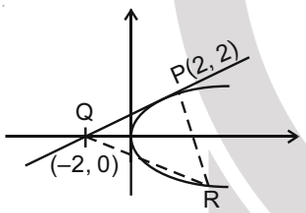
1. Let the tangent to the parabola S : $y^2 = 2x$ at the point P(2, 2) meet the $x$-axis at Q and normal at it meet the parabola S at the point R. Then the area (in sq. units) of the triangle PQR is equal to

(1) $\frac{25}{2}$ (2) $\frac{15}{2}$

(3) $\frac{35}{2}$ (4) 25

**Answer (1)**

**Sol.**




Tangent to $y^2 = 2x$ at P (2, 2)

$2y = x + 2$

$\therefore$ Q (–2, 0)

Normal at P (2, 2) meet at R $\left(\frac{9}{2}, -3\right)$

Area of $\Delta PQR = \left|\frac{1}{2}\begin{vmatrix} 2 & 2 & 1 \\ -2 & 0 & 1 \\ \frac{9}{2} & 3 & 1 \end{vmatrix}\right|$

$= \frac{25}{2}$ sq. units

2. If $\alpha$ and $\beta$ are the distinct roots of the equation $x^2 + (3)^{1/4}x + 3^{1/2} = 0$, then the value of $\alpha^{96}(\alpha^{12} - 1) + \beta^{96}(\beta^{12} - 1)$ is equal to

(1) $28 \times 3^{25}$ (2) $56 \times 3^{24}$

(3) $52 \times 3^{24}$ (4) $56 \times 3^{25}$

**Answer (3)**

**Sol.** $x = \frac{-3^{1/4} \pm \sqrt{3^{1/2} - 4 \cdot 3^{1/2}}}{2}$

$= \frac{3^{1/4}\left(-1 \pm \sqrt{3}i\right)}{2} = 3^{1/4}\omega$ or $3^{1/4}\omega^2$

$\alpha^{108} + \beta^{108} - (\alpha^{96} + \beta^{96})$

$= 3^{108/4}(\omega^{108} + \omega^{216}) - 3^{96/4}(\omega^{96} + \omega^{192})$

$= 3^{27} \cdot 2 - 3^{24} \cdot 2$

$= 3^{24}(52) = 52 \times 3^{24}$

3. Let $y = y(x)$ be the solution of the differential equation

$$x\tan\left(\frac{y}{x}\right)dy = \left(y\tan\left(\frac{y}{x}\right) - x\right)dx,$$

$$-1 \le x \le 1, \; y\left(\frac{1}{2}\right) = \frac{\pi}{6}.$$

Then the area of the region bounded by the curves $x = 0$, $x = \frac{1}{\sqrt{2}}$ and $y = y(x)$ in the upper half plane is

(1) $\frac{1}{6}(\pi - 1)$ (2) $\frac{1}{12}(\pi - 3)$

(3) $\frac{1}{8}(\pi - 1)$ (4) $\frac{1}{4}(\pi - 2)$

**Answer (3)**

**Sol.** $\tan\left(\frac{y}{x}\right)\frac{dy}{dx} = \left(\frac{y}{x}\right)\tan\left(\frac{y}{x}\right) - 1$

Put $y = ux$

$\frac{dy}{dx} = u + x\frac{du}{dx}$

$\Rightarrow u + x\frac{du}{dx} = \frac{u\tan u}{\tan u} - \cot u$

$\Rightarrow x\frac{du}{dx} = -\cot u$

$\Rightarrow \frac{dx}{x} + \tan u \, du = 0$

$\Rightarrow \ln x + \ln\left|\sec\left|\frac{y}{x}\right|\right| = c$

Given $y\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$$\ln\left(\frac{1}{2}\right)+\ln\left|\sec\frac{\pi}{3}\right|=c$$

$\Rightarrow$ c = 0

$\therefore$ $y = x\cos^{-1}x$

$$\text{Area} = \int_0^{1/\sqrt{2}} x\cos^{-1}x\,dx$$

Put $x = \cos\theta$

$$\therefore \int_{\pi/2}^{\pi/4} \cos\theta\cdot\theta\cdot(-\sin\theta)\,d\theta = A$$

$$\therefore A = \int_{\pi/4}^{\pi/2}\left(\frac{\theta}{2}\right)\sin 2\theta\, d\theta$$

$$A = -\frac{\theta}{2}\frac{\cos 2\theta}{2}\Bigg|_{\pi/4}^{\pi/2}+\int_{\pi/4}^{\pi/2}\frac{1}{2}\frac{\cos 2\theta}{2}d\theta$$

$$= \left(\frac{-\pi}{4}\left(\frac{-1}{2}\right)\right)-\left(\frac{-\pi}{8}(0)\right)+\frac{1}{4}\frac{\sin 2\theta}{2}\Bigg|_{\pi/4}^{\pi/2}$$

$$= \frac{\pi}{8}-0-\frac{1}{8}=\frac{\pi-1}{8}$$

4. The coefficient of $x^{256}$ in the expansion of $(1-x)^{101}(x^2+x+1)^{100}$ is :

(1) $^{100}C_{15}$ (2) $^{100}C_{16}$

(3) $-^{100}C_{16}$ (4) $-^{100}C_{15}$

**Answer (1)**

**Sol.** $(1-x)((1-x)(1+x+x^2))^{100}$

$\Rightarrow$ $(1-x)(1-x^3)^{100}$

General term in $(1-x^3)^{100}$ is $^{100}C_r\left(-x^3\right)^r$

$\therefore$ $x^{256}$ occur if $3r = 256$ or $3r+1 = 256$

$r = \frac{256}{3}$ (not valid) $\quad r = \frac{255}{3} = 85$

$\therefore$ Coefficient of $x^{256}$ $\quad ^{100}C_{85} = ^{100}C_{15}$

5. If $z$ and $\omega$ are two complex numbers such that $|z\omega| = 1$ and $\arg(z) - \arg(\omega) = \frac{3\pi}{2}$, then $\arg\left(\frac{1-2\bar{z}\omega}{1+3\bar{z}\omega}\right)$ is

(Here arg($z$) denotes the principal argument of complex number $z$)

(1) $\frac{3\pi}{4}$ (2) $-\frac{3\pi}{4}$

(3) $\frac{\pi}{4}$ (4) $-\frac{\pi}{4}$

**Answer (2*)**

**Sol.** $z = re^{i\theta}$ $\quad\therefore\quad \omega = \frac{1}{r}e^{i(\theta-3\pi/2)}$

$$\frac{1-2\bar{z}\omega}{1+3\bar{z}\omega}=\frac{1-2e^{-i\theta}\cdot e^{i(-3\pi/2+\theta)}}{1+3e^{-i\theta}\cdot e^{i(-3\pi/2+\theta)}}$$

$$\therefore \text{Arg}\left(\frac{1-2i}{1+3i}\right)=-\frac{3\pi}{4}$$

6. Let $A=\begin{bmatrix}2 & 3\\ a & 0\end{bmatrix}$, $a\in R$ be written as P + Q where P is a symmetric matrix and Q is skew symmetric matrix. If det(Q) = 9, then the modulus of the sum of all possible values of determinant of P is equal to

(1) 24

(2) 18

(3) 45

(4) 36

**Answer (4)**

**Sol.** $Q=\frac{A-A^T}{2}=\frac{\begin{bmatrix}2 & 3\\ a & 0\end{bmatrix}-\begin{bmatrix}2 & a\\ 3 & 0\end{bmatrix}}{2}$

$$=\frac{1}{2}\begin{bmatrix}0 & 3-a\\ a-3 & 0\end{bmatrix}$$

$\det(Q) = \frac{1}{4}(a-3)^2 = 9 \Rightarrow a-3 = \pm 6$

a = 9, -3

$$P=\frac{A+A^T}{2}=\frac{1}{2}\begin{bmatrix}4 & 3+a\\ a+3 & 0\end{bmatrix}$$

$$\det(P)=\frac{1}{4}-(a+3)^2$$

$\therefore$ det(P) = 0 or $\frac{-144}{4}=36$

$\therefore$ Required sum 36

7. Let $y = y(x)$ be the solution of the differential equation $e^x\sqrt{1-y^2}\,dx+\left(\frac{y}{x}\right)dy=0$, $y(1)=-1$.

Then the value of $(y(3))^2$ is equal to

(1) $1+4e^6$ (2) $1-4e^6$

(3) $1-4e^3$ (4) $1+4e^3$

**Answer (2)**

**Sol.** $e^x\sqrt{1-y^2}\,dx=-\frac{y}{x}dy$

$$\Rightarrow \int xe^x dx=\int\frac{-y}{\sqrt{1-y^2}}dy$$

$\Rightarrow \quad xe^x - e^x = \sqrt{1-y^2} + c$

$y(1) = -1$

$\Rightarrow \quad 0 = 0 + c \quad \Rightarrow \quad c = 0$

$\therefore \quad xe^x - e^x = \sqrt{1-y^2}$

for $y(3)$ put $x = 3$

$3e^3 - e^3 = \sqrt{1-y^2}$

$4e^6 = 1 - y^2$

$\Rightarrow \quad (y(3))^2 = 1 - 4e^6$

8. The probability of selecting integers $a \in [-5, 30]$ such that $x^2 + 2(a + 4)x - 5a + 64 > 0$, for all $x \in \mathbf{R}$ is

(1) $\frac{1}{4}$ (2) $\frac{1}{6}$

(3) $\frac{7}{36}$ (4) $\frac{2}{9}$

**Answer (4)**

**Sol.** For $x^2 + 2(a + 4)x - 5a + 64 > 0$

$\therefore \quad D < 0$

$4(a + 4)^2 - 4(64 - 5a) < 0$

$\Rightarrow \quad a^2 + 8a + 16 - 64 + 5a < 0$

$\Rightarrow \quad a^2 + 13a - 48 < 0$

$\Rightarrow \quad a^2 + 16a - 3a - 48 < 0$

$\Rightarrow \quad (a + 16)(a - 3) < 0$

$a \in (-16, 3)$

in set $[-5, 30]$ total integers 36

favourable integers 8

$\Pr = \frac{8}{36} = \frac{2}{9}$

9. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. If $\vec{c}$ is a vector such that $\vec{a} \cdot \vec{c} = |\vec{c}|$, $|\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between $(\vec{a} \times \vec{b})$ and $\vec{c}$ is $\frac{\pi}{6}$, then the value of $\left|(\vec{a} \times \vec{b}) \times \vec{c}\right|$ is

(1) 4 (2) $\frac{2}{3}$

(3) $\frac{3}{2}$ (4) 3

**Answer (3)**

**Sol.** $\because \quad \vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$

$\Rightarrow \quad |\vec{a}| = \sqrt{4+1+4} = 3$

Now, $|\vec{c} - \vec{a}| = 2\sqrt{2}$

$\Rightarrow \quad (\vec{c} - \vec{a})^2 = 8$

$|\vec{c}|^2 + |\vec{a}|^2 - 2\vec{c} \cdot \vec{a} = 8$

$|\vec{c}|^2 + 9 - 2|\vec{c}| = 8 \qquad (\because \ \vec{a} \cdot \vec{c} = |\vec{c}|)$

$|\vec{c}|^2 - 2|\vec{c}| + 1 = 0$

$\therefore \quad |\vec{c}| = 1$

and angle between $\vec{a} \times \vec{b}$ and $\vec{c}$ is $\theta$ then

$\left|(\vec{a} \times \vec{b}) \times \vec{c}\right| = |\vec{a} \times \vec{b}| \cdot |\vec{c}| \cdot \sin\theta$

$\left|2\hat{i} - 2\hat{j} + \hat{k}\right| \cdot 1 \cdot \sin\frac{\pi}{6}$

$= \frac{3}{2}$

10. Let a function $f : \mathbf{R} \to \mathbf{R}$ be defined as

$$f(x) = \begin{cases} \sin x - e^x & \text{if } x \le 0 \\ a + [-x] & \text{if } 0 < x < 1 \\ 2x - b & \text{if } x \ge 1 \end{cases}$$

where $[x]$ is the greatest integer less than or equal to $x$. If f is continuous on $\mathbf{R}$, then $(a + b)$ is equal to :

(1) 5 (2) 3

(3) 4 (4) 2

**Answer (2)**

**Sol.** $\because \ f(x) = \begin{cases} \sin x - e^x, & x \le 0 \\ a + [-x], & 0 < x < 1 \\ 2x - b, & x \ge 1 \end{cases}$

$\because$ f(x) is continuous everywhere

$\therefore$ f(x) is continuous at x = 0

$\therefore \quad f(0^-) = f(0) = f(0^+)$

$\therefore \quad -1 = -1 = a - 1 \Rightarrow a = 0$

f(x) is continuous at x = 1

$\therefore \quad f(1^-) = f(1) = f(1^+)$

$a - 1 = 2 - b = 2 - b$

$\therefore \quad b = 3$

$\therefore \quad a + b = 3$

11. The mean of 6 distinct observations is 6.5 and their variance is 10.25. If 4 out of 6 observations are 2, 4, 5 and 7, then the remaining two observations are:

(1) 1, 20 (2) 10, 11

(3) 3, 18 (4) 8, 13

**Answer (2)**

**Sol.** Let the observations be 2, 4, 5, 7, x and y

$$\bar{x} = \frac{18 + x + y}{6} = 6.5 \Rightarrow x + y = 21 \quad ...(i)$$

and $\sigma^2 = \frac{2^2 + 4^2 + 5^2 + 7^2 + x^2 + y^2}{6} - (6.5)^2 = 10.25$

$\Rightarrow$ $x^2 + y^2 = 221$ ...(ii)

From (i) and (ii), we get

(x, y) = (10, 11) or (11, 10)

12. The Boolean expression $(p \wedge \sim q) \Rightarrow (q \vee \sim p)$ is equivalent to

(1) $q \Rightarrow p$ (2) $p \Rightarrow q$

(3) $p \Rightarrow \sim q$ (4) $\sim q \Rightarrow p$

**Answer (2)**

**Sol.** $\because$ $p \Rightarrow q$ is $\sim p \vee q$

$\therefore$ $(p \wedge \sim q) \Rightarrow (q \vee \sim p)$

$= \sim (p \wedge \sim q) \vee (q \vee \sim p)$

$= (\sim p \vee q) \vee (\sim p \vee q)$

$= \sim p \vee q$

$= p \Rightarrow q$

13. Words with or without meaning are to be formed using all the letters of the word EXAMINATION. The probability that the letter M appears at the fourth position in any such word is

(1) $\frac{1}{66}$ (2) $\frac{1}{11}$

(3) $\frac{1}{9}$ (4) $\frac{2}{11}$

**Answer (2)**

**Sol.** Total number of words formed by letters of word "EXAMINATION" is

$$n(S) = \frac{11!}{2! \cdot 2! \cdot 2!}$$

When 'M' at fourth position then number of words formed = $n(E) = \frac{10!}{2! \cdot 2! \cdot 2!}$

$\therefore$ Required probability = $\frac{n(E)}{n(S)} = \frac{1}{11}$

14. Let *a* be a positive real number such that

$$\int_0^a e^{x-[x]}dx = 10e - 9$$

where [*x*] is the greatest integer less than or equal to *x*. Then *a* is equal to :

(1) $10 + \log_e 3$ (2) $10 - \log_e (1 + e)$

(3) $10 + \log_e (1 + e)$ (4) $10 + \log_e 2$

**Answer (4)**

**Sol.** $\because$ $\int_0^a e^{x-[x]}dx = 10e - 9$

Here $e^{x-[x]}$ is periodic function of period 1

$\therefore$ $\int_0^{[a]+\{a\}} e^{\{x\}}dx = 10e - 9$

$\Rightarrow$ $[a]\int_0^1 e^x dx + \int_0^{\{a\}} e^x dx = 10e - 9$

$\Rightarrow$ $[a](e - 1) + (e^{\{a\}} - 1) = 10e - 9$

$\Rightarrow$ $[a]e - e^{\{a\}} - [a] - 1 = 10e - 9$

$\therefore$ Possible value of $a = 10 + \log_e 2$

15. Let 'a' be a real number such that the function $f(x) = ax^2 + 6x - 15$, $x \in \mathbb{R}$ is increasing in $\left(-\infty, \frac{3}{4}\right)$ and decreasing in $\left(\frac{3}{4}, \infty\right)$. Then the function $g(x) = ax^2 - 6x + 15$, $x \in \mathbb{R}$ has a :

(1) Local maximum at $x = \frac{3}{4}$

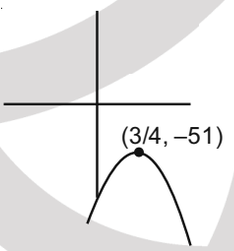
(2) Local maximum at $x = -\frac{3}{4}$

(3) Local minimum at $x = -\frac{3}{4}$

(4) Local minimum at $x = \frac{3}{4}$

**Answer (2)**

**Sol.**




$\because$ $f(x) = ax^2 + 6x - 15$

$\therefore$ $D = 36 + 60a$

vertex = $\left(-\frac{3}{a}, -\frac{36 + 60a}{a}\right) = \left(-\frac{3}{a}, -\frac{36}{a} - 60\right)$

Here $-\frac{3}{a} = \frac{3}{4}$ $\Rightarrow$ $a = -4$

$\therefore$ f(x) is increasing in $\left(-\infty, \frac{3}{4}\right)$

and decreasing in $\left(\frac{3}{4}, \infty\right)$

Now $g(x) = -4x^2 - 6x + 15$

$\therefore$ g(x) has local maxima

at $x = -\frac{3}{4}$

16. If in a triangle ABC, AB = 5 units, $\angle B = \cos^{-1}\left(\frac{3}{5}\right)$ and radius of circumcircle of $\Delta ABC$ is 5 units, then the area (in sq. units) of $\Delta ABC$ is :

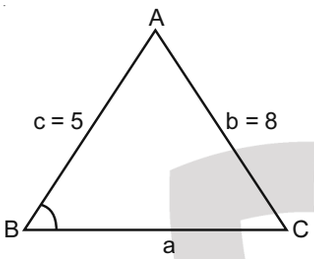
(1) $10 + 6\sqrt{2}$ (2) $6 + 8\sqrt{3}$

(3) $8 + 2\sqrt{2}$ (4) $4 + 2\sqrt{3}$

**Answer (2)**

**Sol.** In $\Delta ABC$, $\cos B = \frac{3}{5}$

$\therefore \quad \sin B = \frac{4}{5}$




$\because \quad \frac{5}{\sin C} = \frac{a}{\sin A} = \frac{b}{\frac{4}{5}} = 10$ [Sine rule]

$\Rightarrow$ C = 30° and b = 8

$\because \quad \sin A = \sin(B + C) = \frac{4}{5}\cdot\frac{\sqrt{3}}{2} + \frac{3}{5}\cdot\frac{1}{2} = \frac{3+4\sqrt{3}}{10}$

$\because \quad \Delta = \frac{1}{2}bc\sin A = \frac{1}{2}\cdot 8\cdot 5\cdot\left(\frac{3+4\sqrt{3}}{10}\right) = 6 + 8\sqrt{3}$

17. The number of real roots of the equation $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{4}$ is :

(1) 2

(2) 1

(3) 4

(4) 0

**Answer (4)**

**Sol.** $x(x + 1) \geq 0$ ...(i)

$x^2 + x + 1 \leq 1$

$\Rightarrow \quad x^2 + x \leq 0$ ...(ii)

$\Rightarrow \quad x^2 + x = 0 \Rightarrow x = 0, -1$

When x = 0 or –1

$\text{LHS} = \frac{\pi}{2} \Rightarrow$ No solution

18. Let $A = [a_{ij}]$ be a 3 × 3 matrix, where

$$a_{ij} = \begin{cases} 1 & , \quad \text{if } i = j \\ -x & , \quad \text{if } |i - j| = 1 \\ 2x+1 & , \quad \text{otherwise.} \end{cases}$$

Let a function f : R → R be defined as f(x) = det(A). Then the sum of maximum and minimum values of f on R is equal to :

(1) $-\frac{20}{27}$ (2) $\frac{88}{27}$

(3) $\frac{20}{27}$ (4) $-\frac{88}{27}$

**Answer (4)**

**Sol.** $f(x) = \begin{vmatrix} 1 & -x & 2x+1 \\ -x & 1 & -x \\ 2x+1 & -x & 1 \end{vmatrix} = 4x^3 - 4x^2 - 4x$

$f'(x) = 12x^2 - 8x - 4 = 0 \Rightarrow x = \frac{-1}{3}, 1$

$f\left(\frac{-1}{3}\right) + f(1) = \frac{20}{7} - \frac{108}{7} = \frac{-88}{7}$

19. The value of the integral $\int_{-1}^{1} \log_e\left(\sqrt{1-x} + \sqrt{1+x}\right)dx$ is equal to

(1) $2\log_e 2 + \frac{\pi}{4} - 1$ (2) $\log_e 2 + \frac{\pi}{2} - 1$

(3) $\frac{1}{2}\log_e 2 + \frac{\pi}{4} - \frac{3}{2}$ (4) $2\log_e 2 + \frac{\pi}{2} - \frac{1}{2}$

**Answer (2*)**

**Sol.** Let $x = \cos 2\theta$

$dx = -2\sin 2\theta$

$I = \int_0^{\frac{\pi}{2}} 2\sin 2\theta \ln\left(\sqrt{2}(\sin\theta + \cos\theta)\right)$

$= 2\left[\frac{-\cos 2\theta}{2} \times \ln\left(\sqrt{2}(\sin\theta + \cos\theta)\right)\Big|_0^{\pi/2} - \int_0^{\frac{\pi}{2}}\left(\frac{-\cos 2\theta}{2}\right)\times\left(\frac{\cos\theta - \sin\theta}{\sin\theta + \cos\theta}\right)d\theta\right]$

$= 2\left[\ln\sqrt{2} + \frac{1}{2}\int_0^{\frac{\pi}{2}}(\cos\theta - \sin\theta)^2 d\theta\right]$

$= \ln 2 + \int_0^{\frac{\pi}{2}}(1 - \sin 2\theta)d\theta = \ln 2 + \frac{\pi}{2} - 1$

20. Let [x] denote the greatest integer $\le x$, where $x \in R$. If the domain of the real valued function $f(x) = \sqrt{\frac{|[x]|-2}{|[x]|-3}}$ is $(-\infty, a), \cup [b, c) \cup (4, \infty)$, $a < b < c$, then the value of $a + b + c$ is

(1) –2 (2) 1

(3) 8 (4) –3

**Answer (1)**

**Sol.** $\frac{|[x]|-2}{|[x]|-3} \ge 0 \Rightarrow |[x]| \le 2$ Or $|[x]| > 3$

$\Rightarrow -2 \le [x] \le 2$ Or $[x] < -3$ Or $[x] > 3$

$\Rightarrow -2 \le x < 3$ Or $x < -3$ or $x \ge 4$

$\Rightarrow x \in (-\infty, -3) \cup [-2, 3) \cup [4, \infty)$

$a = -3, b = -2, c = 3$

## SECTION - II

**Numerical Value Type Questions:** This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. Let a, b, c, d be in arithmetic progression with common difference $\lambda$. If $\begin{vmatrix} x+a-c & x+b & x+a \\ x-1 & x+c & x+b \\ x-b+d & x+d & x+c \end{vmatrix} = 2$, then value of $\lambda^2$ is equal to _____.

**Answer (1)**

**Sol.** $R_1 \to R_1 + R_3 - 2R_2$

$$D = \begin{vmatrix} 2 & 0 & 0 \\ x-1 & x+c & x+b \\ x-b+d & x+d & x+c \end{vmatrix}$$

$= 2((x^2 + 2cx + c^2) - (x^2 + (b + d)x + bd))$

$= 2(c^2 - bd) = 2(c^2 - (c - \lambda)(c + \lambda))$

$= 2\lambda^2$

$D = 2 \Rightarrow \lambda^2 = 1$

2. If the value of $\lim\limits_{x\to 0}\left(2 - \cos x\sqrt{\cos 2x}\right)^{\left(\frac{x+2}{x^2}\right)}$ is equal to $e^a$, then a is equal to _____.

**Answer (3)**

**Sol.** $L = e^{\lim\limits_{x\to 0}(1-\cos x\sqrt{\cos 2x})\times\frac{x+2}{x^2}}$

$= e^{\lim\limits_{x\to 0}\left(\frac{1-\cos^2 x(\cos 2x)}{1+\cos x\sqrt{\cos 2x}}\right)\times\frac{x+2}{x^2}}$

$= e^{\lim\limits_{x\to 0}\frac{1-(1-\sin^2 x)(1-2\sin^2 x)}{x^2}\times\frac{x+2}{1+\cos x\sqrt{\cos 2x}}}$

$= e^{\lim\limits_{x\to 0}\frac{1-(1-3\sin^2 x+2\sin^4 x)}{x^2}\times\frac{2}{1+1}}$

$= e^3$

$\Rightarrow a = 3$

3. Let $A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$ and $B = 7A^{20} - 20A^7 + 2I$, where I is an identity matrix of order 3 × 3. If $B = [b_{ij}]$, then $b_{13}$ is equal to _____.

**Answer (910)**

**Sol.** Let $A = I + C$ where $C = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}$

$C^2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ and $C^n = 0$ for $n \ge 3$

So $A^n = (I + C)^n = I + nC + {}^nC_2.C^2$.

So $(b_{13})_n = 0 + 0 + {}^nC_2.1 = {}^nC_2$

Now $b_{13}$ element of $7.A^{20} - 20.A^7 + 2I$

$= 7({}^{20}C_2) - 20({}^7C_2) + 0$

$= 7 \times 190 - 20 \times 21$

$= 70[19 - 6] = 910$

4. Let $y = mx + c$, $m > 0$ be the focal chord of $y^2 = -64x$, which is tangent to $(x + 10)^2 + y^2 = 4$. Then, the value of $4\sqrt{2}(m + c)$ is equal to _____.

**Answer (34)**

**Sol.** $\because$ $y = mx + c$ passes through (–16, 0)

then $c = 16\,m$ ...(1)

also $y = mx + c$ touches the given circle

So, $\left|\frac{-10\,m + c}{\sqrt{1+m^2}}\right| = 2$ ...(2)

$\Rightarrow |3\,m| = \sqrt{1+m^2}$

$\Rightarrow m = \frac{1}{2\sqrt{2}}$ and $c = 4\sqrt{2}$

then $4\sqrt{2}(m + c) = 2 + 32 = 34$

5. The number of rational terms in the binomial expansion of $\left(4^{\frac{1}{4}}+5^{\frac{1}{6}}\right)^{120}$ is _____.

**Answer (21)**

**Sol.** $T_{r+1}={}^{120}C_r.\left(4^{\frac{1}{4}}\right)^{120-r}.\left(5^{\frac{1}{6}}\right)^{r}$

$={}^{120}C_r.2^{60-\frac{r}{2}}.5^{\frac{r}{6}}$

For $T_{r+1}$ to be rational, r must be divisible by 6.

r = 0, 6, 12, …, 120

Number of rational terms = 21

6. If the shortest distance between the lines $\vec{r_1}=\alpha\hat{i}+2\hat{j}+2\hat{k}+\lambda\left(\hat{i}-2\hat{j}+2\hat{k}\right)$, $\lambda\in\mathbf{R}$, $\alpha>0$ and $\vec{r_2}=-4\hat{i}-\hat{k}+\mu\left(3\hat{i}-2\hat{j}-2\hat{k}\right)$, $\mu\in\mathbf{R}$ is 9, then $\alpha$ is equal to _____.

**Answer (6)**

**Sol.** $\bar{a}_1-\bar{a}_2=(\alpha+4)\hat{i}+2\hat{j}+3\hat{k}$

$$\bar{b}_1\times\bar{b}_2=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&-2&2\\3&-2&-2\end{vmatrix}=8\hat{i}+8\hat{j}+4\hat{k}=4(2\hat{i}+2\hat{j}+\hat{k})$$

Shortest distance

$$=\left|\frac{(\bar{a}_1-\bar{a}_2).(\bar{b}_1\times\bar{b}_2)}{|\bar{b}_1\times\bar{b}_2|}\right|=\left|\frac{2(\alpha+4)+4+3}{3}\right|=9$$

$\Rightarrow$ $(2\alpha + 15) = 27$

$\Rightarrow$ $\alpha = 6$

7. Let P be a plane passing through the points (1, 0, 1), (1, –2, 1) and (0, 1, –2). Let a vector $\vec{a}=\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$ be such that $\vec{a}$ is parallel to the plane P, perpendicular to $\left(\hat{i}+2\hat{j}+3\hat{k}\right)$ and $\vec{a}\cdot\left(\hat{i}+\hat{j}+2\hat{k}\right)=2$, then $(\alpha-\beta+\gamma)^2$ equals _____.

**Answer (81)**

**Sol.** Let $\bar{n}$ be the normal vector of the given plane.

$$\bar{n}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\0&2&0\\1&-1&3\end{vmatrix}=6\hat{i}-2\hat{k}$$

$\because$ $\bar{a}$ is perpendicular to $\bar{n}$ and $\hat{i}+2\hat{j}+3\hat{k}$

So, $\bar{a}=\lambda\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\3&0&-1\\1&2&3\end{vmatrix}=\lambda(2\hat{i}-10\hat{j}+6\hat{k})$

$\because$ $\bar{a}.(i+j+2k)=2\Rightarrow\lambda(2-10+12)=2\Rightarrow\lambda=\frac{1}{2}$

hence $\bar{a}=\hat{i}-5\hat{j}+3\hat{k}$

So $(\alpha - \beta + \gamma)^2 = (1 + 5 + 3)^2 = 81$

8. There are 15 players in a cricket team, out of which 6 are bowlers, 7 are batsmen and 2 are wicketkeepers. The number of ways, a team of 11 players be selected from them so as to include at least 4 bowlers, 5 batsmen and 1 wicketkeeper, is _____.

**Answer (777)**

**Sol.** There will be total three cases.

(i) 4 Bowlers + 5 Batsmen + 2 WK

No. of ways = ${}^6C_4.{}^7C_5.{}^2C_2$ = 315

(ii) 4 Bowlers + 6 Batsmen + 1 WK

No. of ways = ${}^6C_4.{}^7C_6.{}^2C_1$ = 210

(iii) 5 Bowlers + 5 Batsmen + 1WK

No. of ways = ${}^6C_5.{}^7C_5.{}^2C_1$ = 252

Total number of ways = 777

9. Let $\vec{a},\vec{b},\vec{c}$ be three mutually perpendicular vectors of the same magnitude and equally inclined at an angle $\theta$, with the vector $\vec{a}+\vec{b}+\vec{c}$. Then $36\cos^2 2\theta$ is equal to _____.

**Answer (4)**

**Sol.** $|\bar{a}+\bar{b}+\bar{c}|^2=|\bar{a}|^2+|\bar{b}|^2+|\bar{c}|^2+2(\bar{a}.\bar{b}+\bar{b}.\bar{c}+\bar{c}.\bar{a})$

$\Rightarrow$ $|\bar{a}+\bar{b}+\bar{c}|=\sqrt{3}|\bar{a}|$

Now $\cos\theta=\frac{\bar{a}.(\bar{a}+\bar{b}+\bar{c})}{|\bar{a}||\bar{a}+\bar{b}+\bar{c}|}=\frac{|\bar{a}|^2}{|\bar{a}|(\sqrt{3}|\bar{a}|)}=\frac{1}{\sqrt{3}}$

So, $36\cos^2 2\theta = 36(2\cos^2\theta - 1)^2$

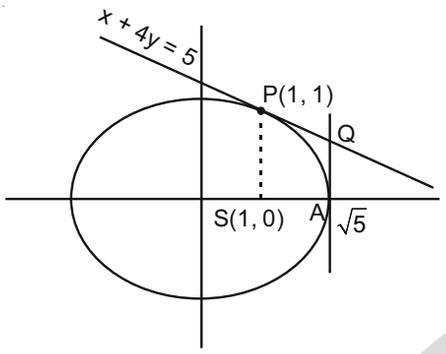
$$=36\left(\frac{2}{3}-1\right)^2=4$$

10. Let T be the tangent to the ellipse E : $x^2 + 4y^2 = 5$ at the point P(1, 1). If the area of the region bounded by the tangent T, ellipse E, lines $x = 1$ and $x=\sqrt{5}$ is $\alpha\sqrt{5}+\beta+\gamma\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$, then $|\alpha+\beta+\gamma|$ is equal to _____.

**Answer (1.25*)**

**Sol.** E : $x^2 + 4y^2 = 5$

Required area = Area of trapezium SPQA – Area under the segment of ellipse




$$= \frac{1}{2}\left(1+\frac{5-\sqrt{5}}{4}\right)\left(\sqrt{5}-1\right) - \int_1^{\sqrt{5}} \frac{1}{2}\sqrt{5-x^2}\,dx$$

$$= \frac{5\sqrt{5}-7}{4} - \frac{1}{4}\left[x\sqrt{5-x^2} + 5\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)\right]_1^{\sqrt{5}}$$

$$\frac{5\sqrt{5}-7}{4} - \frac{5\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)-2}{4} =$$

$$\frac{5\sqrt{5}}{4} - \frac{5}{4} - \frac{5}{4}\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$$

Clearly, $\alpha = \frac{5}{4}, \beta = -\frac{5}{4}$ and $\gamma = -\frac{5}{4}$

$\Rightarrow \quad |\alpha+\beta+\gamma| = \frac{5}{4}$

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